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State machine based nonlinear hysteresis model

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ABSTRACT

A wide range of systems are characterized by nonlinear hysteresis effects. This especially appears in moving or deforming mechanical systems. In an effort to model these effects, a novel approach is presented in this work. The proposed model is based on a state machine imposing very few restrictions on the modeled system and therefore allowing a wide range of applications. The state machine represents the memory of the system. The size of the hysteresis loop and the local dependence is described by envelop functions while the shape of the loop is affected by the transition function. In this work we use polynomial functions as the envelop function and a hyperbolic tangent function to model the transition. The separation of these two parts and hence the free choice of functions allows a very good and easy fit to real system behavior. The identification of two exemplary mechanical systems with hysteresis is described and the results are presented.

1. Introduction

Hysteresis systems are characterized by a memory effect, so their behavior does not only depend on the input, but also on the previous states of the system. A simple two-dimensional characteristic function yields a non-injective function, where a certain input vector does not yield a unique output. This nonlinear effect appears in several kinds of systems, such as mechanical deformation, friction, electronic relay circuits, smart actuators or magnetism. Precise identification and modeling is necessary for simulation, control design and prototyping.

Several models have been developed to describe this effect. In 1935, Preisach developed a hysteresis model based on the mechanism of magnetics [1–3]. The Preisach operator is a simple switching operator with only two states and two switching thresholds. If the input signal fluctuates between these limits, the previous output value will be held. In 1971, Krasnoselskii and Pokrovskii developed the hysteron outlining the mathematical principle of the phenomenological Preisach model and extending it to make it applicable not only to magnetic but to any physical hysteresis [4].

Another popular hysteresis model is the Prandtl–Ishlinskii operator [5,6]. Its basic principle is a play operator as described in [7]. This model applies a superposition of play or stop operator and is parameterized by one threshold variable. In contrast to the previously mentioned models, this approach allows an analytic design of a compensator in control design tasks [8,9]. The models have been validated with several ferromagnetic materials and smart actuators [10–12].

The well-known Bouc–Wen model was first proposed in 1967 [13] and generalized in 1976 [14]. It is characterized by the position of an oscillator and a hysteretic restoring force that acts on the oscillator. The model is computationally simple as it uses only one auxiliary differential equation. Further advantages of the model are its outstanding versatility and mathematical tractability. However, the model parameters are comparably hard to identify. The model is used to reflect nonlinear random vibration analysis [15], degrading of stiffness, strength, and pinching effects [16], biaxial hysteresis [17], and asymmetry of the peak restoring force [18].

The above mentioned models describe rate-independent hysteresis assuming a static memory effect. Variations in the periodic input–output map for different input frequencies cannot be modeled. To account for rate-dependent hysteresis effects adapted Preisach models have been presented [19–21]. A modified Prandtl–Ishlinskii formalism is introduced in [22] in order to solve this problem. In [23] an extension of the Bouc–Wen model with a Hammerstein method using a static nonlinear block followed by a linear dynamic block is proposed.

The outlined basic Preisach, Prandtl–Ishlinskii and Bouc–Wen models also fail to describe asymmetric hysteresis as they appear in hysteresis loops of structural elements and output saturation effects. In [8] the authors developed a dead zone operator in order to solve this issue. Another approach proposed by [7,9] features nonlinear play operators to allow the use of asymmetric and

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saturated hysteresis loops. This is also done for a rate-dependent model in [24]. [25] present a generalized Bouc–Wen-model.

All mentioned exemplary hysteresis models consist of a characteristic function and one or more auxiliary hysteresis parameters. Since these variables offers only a low degree of flexibility the transition behavior can hardly be influenced. Furthermore, it is complicated to change the direction in which the hysteresis loop is passed through. This prohibits the use of the same hysteresis model for direct and inverse tasks. For an empirical design of a hysteresis model the above formalisms are not appropriate.

Therefore a novel modeling scheme is introduced in this work. In the proposed model a state machine controls the current state of the hysteresis and thus it decides which envelop and transition function is used for calculation of the output value. The benefit of this strategy is that the rising, falling and transition behavior can be identified and calculated separately. This allows the combination of two arbitrary envelop functions (increasing and decreasing) and a nearly unrestricted transition function which describes the behavior of the hysteresis during a change of the input direction. The separation of the functions ensures a simple and reliable determination of the model parameters. After identification all parameters are static and only few variables have to be memorized for the model execution. Furthermore, the output can be calculated directly without integration or differentiation and this results directly in a low computational effort.

The first advantage of the presented approach is the direct and clear reference between model parameters and experimental observation. The second advantage is the possibility to (re-)design each model part in terms of structure and complexity separately. This enables the possibility to balance between model accuracy, identification complexity and computational cost. This modular framework highly simplifies the design and realization of identification experiments and grants easy and flexible handling.

The contribution is structured as follows: In Section 2 the model's structure, implementation, and limitation is explained in detail. Section 3 presents two experimental set-ups, which are used to show the applicability of the presented model. Before drawing conclusions, experimental results in Section 4 demonstrate the effectiveness of the proposed approach.

2. Hysteresis model

Hysteresis effects are characterized by an output which does not only depend on the current but also on past input values. This behavior is expressed in different trajectories for rising and falling input (see Fig. 1). To model these effects the characteristic curves for increasing and decreasing input and the transition between them need to be developed. In addition, one has to define which of these curves is used for actual output.

In the developed hysteresis model this is realized by a state machine. The easy separation of the increasing and the decreasing curve allows more flexibility in the choice of modeling functions used to approximate the measured curves. In the following section, the state machine, the envelop functions and the transition function are described.

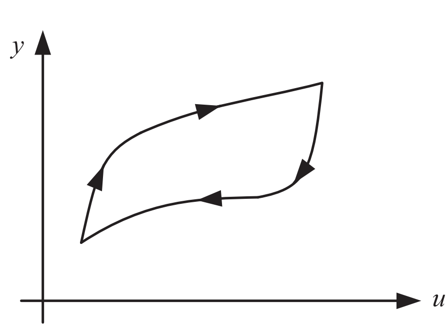

Fig. 1. Hysteresis.

2.1. State machine

The task of the state machine is to determine the output function of the hysteresis model. In the play operator or the Prandtl–Ishlinskii model this decision is made depending on the model output. In this work, the model input is used to make that decision. This independency from the output of the model enables the use of a wider range of increasing and decreasing functions. Hence, the distance between these functions can highly vary and the functions can also cross each other. The transition from increasing to decreasing function or vise versa is made, when the deviation of the input $\dot{u}$ changes its sign. In case of a time-discrete description the former input u_{k-1} is compared to the actual input u_k. If the current input is higher than the previous one, the first output function f_1 will generate the output y. If the current input is lower than the previous one, the second output function f_2 is used. If the input u remains constant, the output is held. In summary this yields:

$$y_k = \begin{cases} f_1, & u_k > u_{k-1} \\ y_{k-1}, & u_k = u_{k-1} \\ f_2, & u_k < u_{k-1} \end{cases} \tag{1}$$

This can be realized by using the state machine shown in Fig. 2. A further benefit of such a state machine is the free choice of direction, in which the hysteresis is passed through. The direction can simply be changed by reversing the relation signs. Hence this model can easily be used for direct or inverted description of the system behavior.

2.2. Envelop functions

The envelop functions consist of the increasing function f_i and the decreasing function f_d limiting the output of the hysteresis to its maximum and minimum level (see gray area in Fig. 3). The space between the envelop functions is defined by the transition function.

Both functions characterize the major hysteresis loop behavior and the size of hysteresis gap is defined by the distance between both functions. The hysteresis gap can be asymmetric and may vary substantially over the input range. The state machine's selection of the path is based on the model input allowing to use a huge range of functions as envelop function.

$$\begin{aligned} &f_i \in C^0(\mathbb{R} \to \mathbb{R}); \\ &f_d \in C^0(\mathbb{R} \to \mathbb{R}); \end{aligned} \tag{2}$$

Even contact or intersection of the increasing and decreasing function is possible. With this, a wide range of nonlinear effects as for example saturation can be modeled. In this work polynomial functions are used as follows:

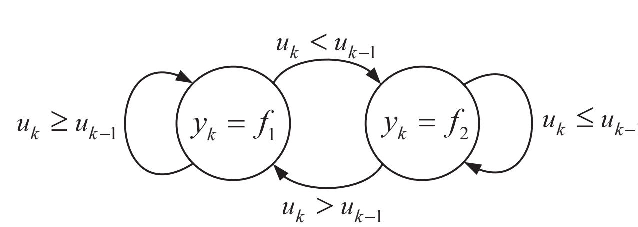

Fig. 2. State machine.