Mechatronics 31 (2015) 264–275

Contents lists available at ScienceDirect

Mechatronics

journal homepage: www.elsevier.com/locate/mechatronics

Technical note

Rotation rate estimation in parametrically excited micro gyroscopes

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ARTICLE INFO

Article history:
Received 20 July 2013
Accepted 23 July 2015
Available online 1 December 2015

Keywords:
Micro gyroscope
Parametric excitation
Extended Kalman filter
Unscented Kalman filter

ABSTRACT

This paper reports the estimation of angular velocity in micro gyroscopes with parametric excitation. The identification procedure is done into two consecutive steps: In the first step, the physical parameters of the gyroscope (stiffness, damping, and actuator parameters) are estimated via continuous time Extended and Unscented Kalman filter. In the second step, a separate Kalman filter is dedicated to estimate the time varying rotation rate, using the output of the first step. Using numerical simulations, it is found that by introducing an artificial noise in the observer equations and tuning its variance, arbitrary temporal variation of angular velocity can be well tracked by the proposed observer.

1. Introduction

Micro gyroscopes, aimed at measuring angular position or velocity, have revolutionized the industry of inertial sensors by offering several advantages over conventional gyroscopes, such as small size and weight, IC integration capability, and low cost [1–4]. Their use, however, is still restricted due to their low sensitivity and dependence on ambient conditions in comparison to fiber optic gyroscopes. Micro gyroscopes still need improvement in terms of sensitivity and robustness against volatile operating conditions.

The operation principle of a generic micro gyroscope is based on inducement of Coriolis effect. First, a two-degree-of-freedom (DOF for short) micro-scale mass is actuated in one direction called the drive axis (denoted by x). When the frame of the sensor, attached to the rotating body, is undergone an external angular velocity normal to its plane of vibration, the induced Coriolis force excites the mass along its second degree of freedom, called the sense mode (denoted by y). This induced vibration is measured to estimate the angular velocity. Any technique that contributes to higher amplitudes in the sense mode would decrease the minimum detectable angular rate, translating into higher mechanical sensitivity of the sensor.

Viewing the drive and the sense axes as two second order lightly damped filters, it can be argued that the induced sense amplitude is maximized if the resonant frequencies of both axes are exactly matched. Due to high quality factor and narrow bandwidth, however, even small split between the natural frequencies degrades the sense amplitude. Theoretically, symmetrical designs can nullify the frequency mismatching. In practice, mismatching is always present as a result of micromachining tolerances and variable operating conditions, temperature in particular. To overcome this problem, the sensor should operate in either non-resonant mode, which deteriorates the sensitivity, or in closed-loop, which requires additional elements for implementation. Some structurally robust designs such as multi-DOF micro gyroscopes [2] or parametric excitation [5,6] have also been suggested to enhance the sensitivity at no additional cost.

As the name implies, in harmonic excitation the exciting force is a harmonic signal. In parametric excitation, on the other hand, this force is a bivariate function of time and displacement. This property results in a nonlinear Mathieu equation in the drive axis. In this configuration, the linear and nonlinear stiffness are periodically modulated in time. The periodic modulation of stiffness increases both the drive amplitude and the bandwidth of this mode, allowing for unavoidable mismatching to be partially tolerated, even in open loop operation.

The concept of parametric excitation in micro gyroscopes was first introduced by Oropeza-Ramos et al. [6]. In this experimental work, inspired by the previous works of Turner in the context of parametric excitation [7–9], non-interdigitated comb fingers were used in the drive axis to generate parametric amplification. The sensor was fabricated and calibrated using rate table setup. Stability analysis and parametric design of parametrically excited gyroscopes were further investigated by Pakniyat et al. [10,11]. Some other works have been reported on parametric amplification and

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http://dx.doi.org/10.1016/j.mechatronics.2015.07.012

damping in either the drive or sense axis [12,13]. To the knowledge of the authors, no article has reported so far on the estimation of angular velocity in micro gyroscopes with parametric excitation. Online estimation of angular velocity from the governing dynamics has only been discussed in harmonic gyroscopes [13,14]. Also, only few papers have considered the general case of time varying angular velocity [12,13]. In this paper, we design Extended Kalman–Bucy filter (EKBF) and Unscented Kalman–Bucy filter (UKBF) for estimating time varying angular velocity. However, since the physical parameters of the sensor are uncertain due to fabrication imperfections, they are also estimated prior to estimation of the angular velocity.

In summary, the main contributions of this paper are as follows:

- Development of a comprehensive model of micro gyroscope with parametric excitation.
- Using continuous time Extended and Unscented Kalman filter to estimate the time varying angular velocity in presence of noise and parameter uncertainty.

The rest of this paper is organized as follows: In Section 2, a complete dynamic model of parametrically excited micro gyroscope is developed. In Section 3 the problem is formally stated. Section 4 explores the identification algorithms used in the estimation process. In Section 5, numerical simulations are presented to illustrate the effectiveness of the parameter estimators. Section 6 presents the online estimator for angular velocity estimation. Section 7 concludes the paper along with future directions and open questions.

2. Dynamic modeling

The mechanical structure of the micro gyroscope considered in this paper consists of a two DOF mass suspended above a silicon substrate by means of symmetric micro beams. The symmetrical structure of the device is similar to the design in [15] (see Fig. 1). The main advantage of this structure is its mode decoupling property as well as providing nearly identical natural frequencies in both axes.

Assuming an external angular rate applied perpendicular to the plane of vibration, one can write the dynamic equations as follows [11] (see Fig. 2):

$$m\ddot{x} = m\Omega^2 x + m\dot{\Omega}y + 2m\Omega\dot{y} + f_x^s + f_x^d + f_x^a + \delta f_x \tag{1}$$
$$m\ddot{y} = m\Omega^2 y - m\dot{\Omega}x - 2m\Omega\dot{x} + f_y^s + f_y^d + f_y^a + \delta f_y$$

In (1), x and y are the positions of the proof mass in the rotating frame, f^s, f^d, f^a and δf accounts for spring recovery force, damping force, actuation force and external disturbances, respectively. Terms containing angular velocity Ω are induced Coriolis forces, and terms containing $\dot{\Omega}$ are the contributions of the angular acceleration. δf_x and δf_y represent mechanical–thermal noise which is modeled as white noise with normal distribution [16]. Spring force–displacement relation is modeled as a third order odd polynomial.[1] Although the suspension system is theoretically decoupled, fabrication tolerances cause cross coupling stiffness between the two axes which is referred to as quadrature error. Hence, f^s and f^d can be expressed as

$$f_x^s = -(k_{x1}x + k_{x3}x^3 + k_{yx}y) \tag{2}$$
$$f_y^s = -(k_{y1}y + k_{y3}y^3 + k_{xy}x)$$
$$f_x^d = -c\dot{x}$$
$$f_y^d = -c\dot{y}$$

where k_{q1} and $k_{q3}(q = x, y)$ are the linear and cubic stiffness, respectively, and k_{xy} and k_{yx} are cross coupling stiffnesses which are assumed to be symmetric, i.e. $k_{xy} = k_{yx}$. In non-vacuum conditions, the overall damping is dominated by viscous damping of the surrounding air, which is identical in both directions [17].

The parametric excitation force in the drive axis is generated via non-interdigitated comb fingers (see Fig. 3). In this type of actuator, the non-overlapping comb fingers are in stable equilibrium in aligned position. As the fingers are perturbed from the aligned position, the fringing field between the moving comb fingers provokes a recovery electrostatic force. This force can be mathematically expressed as $f_x^a(t) = -(r_1x(t) + r_3x(t)^3)V(t)^2$ where r_1 and r_3 are geometric dependent constants of the actuators, and $V(t)$ is the potential difference across the actuator terminals.

In order to isolate the parametric resonance from harmonic resonance, the actuating voltage is chosen as $V(t) = V_A\sqrt{1 + \cos 2\omega_A t}$ [6]. Therefore, the actuating forces can be expressed as

$$f_x^a = -V_A^2(r_1x + r_3x^3)(1 + \cos 2\omega_A t) \tag{3}$$
$$f_y^a = 0$$

substitution of (2) and (3) back in (1), and nondimensionalization with scaling frequency (ω_0) and length (L_0) results in the following dynamics

$$\begin{aligned} &x'' + \mu x' + (\omega_x^2 - \gamma^2 + 2\beta_{x1}(1 + \cos 2\omega\tau))x \\ &\quad + (\alpha_x + 2\beta_{x3}(1 + \cos 2\omega\tau))x^3 + \left(\omega_{xy}^2 - \gamma'\right)y - 2\gamma y' \\ &\quad = \delta\tilde{f}_x \end{aligned} \tag{4}$$

$$y'' + \mu y' + \left(\omega_y^2 - \gamma^2\right)y + \alpha_y y^3 + \left(\omega_{xy}^2 + \gamma'\right)x + 2\gamma x' = \delta\tilde{f}_y \tag{5}$$

Nondimensional parameters of (4) and (5) are listed in Table 1. The governing equation of the drive axis is in the form of nonlinear Mathieu equation and the governing equation of the sense mode has the structure of Duffing equation. In the rest of the paper, for simplicity in notation, nondimensional time and derivative with respect to it will be denoted by "t" and "dot" accent, respectively.

3. Problem statement

In practice, physical parameters of the gyroscope, i.e. damping, stiffness and parametric excitation coefficients are either unknown or dependent on ambient conditions. Even FEM solutions cannot give exact values of the parameters due to fabrication imperfections. Hence, for accurate estimation of angular velocity from the dynamical equations, these parameters need to be estimated.

According to (4) and (5), there are totally ten parameters: ω_x, ω_y, ω_{xy}, μ, α_x, α_y, β_{x1}, β_{x3}, γ, $\dot{\gamma}$. For this purpose, we divide the identification process into two separate steps:

- In the first step, we assume that the sensor is not rotating $(\gamma = \dot{\gamma} = 0)$. The eight remaining parameters are estimated in the turn-on time of the sensor, i.e. from the initial application of power until the sensor produces a specified useful output [18].
- In the second step, we assume that the exact values of the physical parameters of the gyroscope are available and the external angular velocity and acceleration are the only unknown quantities to be estimated.

In each step, we augment the position and velocity of the proof mass with the unknown parameters and then we will use a

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[1] Higher order nonlinearities are nulled by using folded micro beams instead of fixed–fixed ones to achieve almost linear behavior. The cubic nonlinearity, however, has significant effect on the dynamic behavior and should be considered.