



Full length article

Shaped beam scattering by an object with a uniaxial anisotropic inclusion

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ABSTRACT

The differential scattering characteristics of an object having a uniaxial anisotropic inclusion, for incidence of an arbitrarily shaped beam, are theoretically investigated. The scattering problem is solved in a spherical basis by using the boundary conditions, method of moments technique and Schelkunoff's equivalence theorem, which leads to a system of linear equations for the expansion coefficients of the scattered and internal fields when the description of the incident shaped beam is known. The theoretical formulation is reviewed, and the numerical problems are discussed. As examples, for a Gaussian beam, zero-order Bessel beam and Hertzian electric dipole radiation striking a spheroid with a uniaxial anisotropic spheroid inclusion and a circular cylinder with a uniaxial anisotropic circular cylinder inclusion, the normalized differential scattering cross sections are calculated, and the scattering properties are analyzed concisely.

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1. Introduction

A number of natural and artificial materials are optically anisotropic, and the investigation of their scattering characteristics is an interesting and important problem, which has found a wide variety of applications in such areas as chemistry, biology, optics, and microwave technology, etc. For some fundamental canonical problems, exact analytical or semi-analytical solutions have been obtained by using the expansion representations of the electromagnetic (EM) fields within an anisotropic medium in terms of the vector wave functions (VWFs). The scattering problem has been researched for an incident plane wave on an anisotropic circular cylinder [1], uniaxial or gyrotropic anisotropic sphere [2–4], and aggregate of uniaxial anisotropic spheres [5]. Schmidt et al. applied the extended boundary condition method (EBCM) to the analysis of the EM scattering by a biaxial anisotropic non-axisymmetric particle [6]. For the case of an incident Gaussian beam, the scattered and internal fields of a uniaxial anisotropic sphere or circular cylinder are calculated within the generalized Lorenz-Mie theory (GLMT) framework [7–9]. Recently, the EBCM has been implemented to compute the EM scattering from an optically anisotropic particle illuminated by a shaped beam [10–12]. In these studies, it is necessary to expand the incident EM fields in

terms of the VWFs series. In this paper, following the method of moments (MoM) procedure and Schelkunoff's equivalence theorem, we present an exact semi-analytical solution to the shaped beam scattering by an object with a uniaxial anisotropic inclusion, in which what we need is to know the explicit expressions of the incident EM fields.

The body of this paper is organized as follows. Section 2 provides the theoretical procedure for determining the scattered fields of an incident EM beam by an object with a uniaxial anisotropic inclusion. In Section 3, numerical results of the scattering properties are presented for a Gaussian beam, zero-order Bessel beam (ZOBB) and Hertzian electric dipole (HED) radiation. Section 4 is the conclusion.

2. Formulation

2.1. Expansions of the scattered and internal fields in terms of the spherical VWFs

The scattering problem is schematically illustrated in Fig. 1 for a two-layered theoretical model. An arbitrarily shaped EM beam propagates in free space and from the negative z' to the positive z' axis in the Cartesian coordinate system $O'x'y'z'$ (EM beam coordinate system). To define the location and orientation of a two-layered scatterer with respect to the incident EM beam, an accessory system $Ox''y''z''$ (parallel to $O'x'y'z'$) is introduced, with

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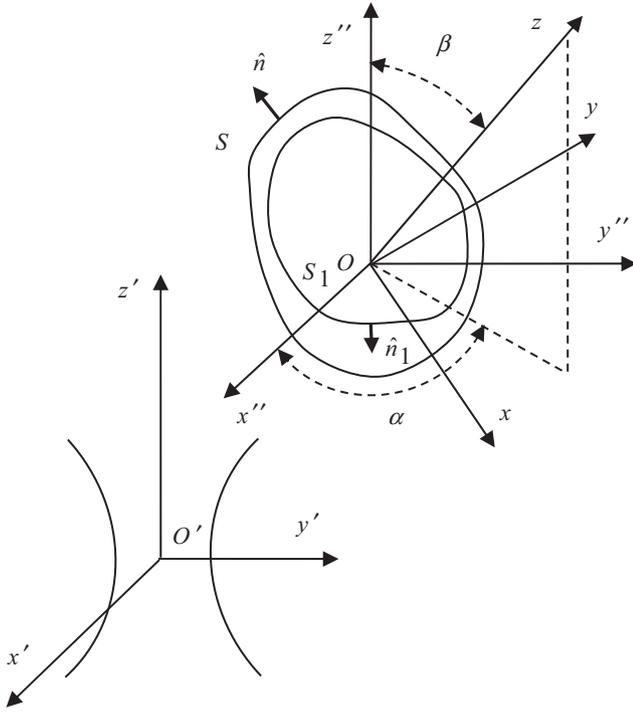


Fig. 1. A dielectric object with a uniaxial anisotropic inclusion illuminated by an EM beam.

its origin O having the Cartesian coordinates (x_0, y_0, z_0) in $O'x'y'z'$. The two-layered scatterer, characterized by a dielectric coating (outer layer) and a uniaxial anisotropic inclusion (inner object), is natural to the system $Oxyz$ which is obtained by rotating $Ox'y'z'$ through Euler angles α and β [13]. In this paper, a time dependence of the form $\exp(-i\omega t)$ is assumed for the EM fields.

Since the scattered EM fields have to be compliant with the radiation condition of an outgoing wave, their expansion representations, for the assumed time dependence, can be obtained in terms of the spherical VWFs of the third kind with respect to the system $Oxyz$, as follows:

$$\mathbf{E}^s = E_0 \sum_{m=-\infty}^{\infty} \sum_{n=|m|}^{\infty} [\alpha_{mn} \mathbf{M}_{mn}^{(3)}(k_0 \mathbf{r}) + \beta_{mn} \mathbf{N}_{mn}^{(3)}(k_0 \mathbf{r})] \quad (1)$$

$$\mathbf{H}^s = -iE_0 \frac{1}{\eta_0} \sum_{m=-\infty}^{\infty} \sum_{n=|m|}^{\infty} [\alpha_{mn} \mathbf{N}_{mn}^{(3)}(k_0 \mathbf{r}) + \beta_{mn} \mathbf{M}_{mn}^{(3)}(k_0 \mathbf{r})] \quad (2)$$

where $k_0 = \omega \sqrt{\mu_0 \epsilon_0}$ and $\eta_0 = \sqrt{\mu_0 / \epsilon_0}$ are respectively the free space wave number and wave impedance, and α_{mn}, β_{mn} are the unknown expansion coefficients to be determined.

In this paper, we follow the descriptions of the spherical VWFs by Stratton in [14].

The EM fields within the outer dielectric layer can be expanded as

$$\mathbf{E}^w = E_0 \sum_{m=-\infty}^{\infty} \sum_{n=|m|}^{\infty} [c_{mn} \mathbf{M}_{mn}^{(1)}(k \mathbf{r}) + c'_{mn} \mathbf{M}_{mn}^{(3)}(k \mathbf{r}) + d_{mn} \mathbf{N}_{mn}^{(1)}(k \mathbf{r}) + d'_{mn} \mathbf{N}_{mn}^{(3)}(k \mathbf{r})] \quad (3)$$

$$\mathbf{H}^w = -i \frac{1}{\eta} E_0 \sum_{m=-\infty}^{\infty} \sum_{n=|m|}^{\infty} [c_{mn} \mathbf{N}_{mn}^{(1)}(k \mathbf{r}) + c'_{mn} \mathbf{N}_{mn}^{(3)}(k \mathbf{r}) + d_{mn} \mathbf{M}_{mn}^{(1)}(k \mathbf{r}) + d'_{mn} \mathbf{M}_{mn}^{(3)}(k \mathbf{r})] \quad (4)$$

where $\eta = \eta_0 / \tilde{n}$, $k = k_0 \tilde{n}$, and \tilde{n} is the refractive index of the material in the dielectric layer relative to that of free space.

The constitutive relations of the medium of a uniaxial anisotropic inclusion in Fig. 1 are described by a permittivity tensor $\bar{\epsilon} = (\hat{x}\hat{x} + \hat{y}\hat{y})\epsilon_t + \hat{z}\hat{z}\epsilon_z$ in the system $Oxyz$ and a scalar permeability μ_0 (free space permeability). As discussed in [10], the basis VWFs can be used to expand the EM fields within the uniaxial anisotropic inclusion, in the following form

$$\mathbf{E}^{w(1)} = E_0 \sum_{m=-\infty}^{\infty} \sum_{n=|m|}^{\infty} [F_{mn1} \mathbf{X}_{mn1}^e(k_1 \mathbf{r}) + F_{mn2} \mathbf{X}_{mn2}^e(k_2 \mathbf{r})] \quad (5)$$

$$\mathbf{H}^{w(1)} = -iE_0 \frac{1}{\eta_0} \sum_{m=-\infty}^{\infty} \sum_{n=|m|}^{\infty} [F_{mn1} \mathbf{X}_{mn1}^h(k_1 \mathbf{r}) + F_{mn2} \mathbf{X}_{mn2}^h(k_2 \mathbf{r})] \quad (6)$$

where the basis VWFs $\mathbf{X}_{mn}^{e,h}(k, \mathbf{r})$ ($t = 1, 2$) defined by the spherical VWFs expansions are provided in [10].

2.2. Determination of the scattered fields by the MOM scheme and Schelkunoff's equivalence theorem

The boundary conditions over the interface between the outer dielectric layer and free space, i.e., S require that the tangential components of the EM fields be continuous

$$\hat{n} \times (\mathbf{E}^s + \mathbf{E}^{inc}) = \hat{n} \times \mathbf{E}^w \quad (7)$$

$$\hat{n} \times (\mathbf{H}^s + \mathbf{H}^{inc}) = \hat{n} \times \mathbf{H}^w \quad (8)$$

where \mathbf{E}^{inc} and \mathbf{H}^{inc} represent the electric and magnetic fields of the incident EM beam, and \hat{n} denotes the outward normal to the surfaces S .

Substituting Eqs. (1)–(4) into Eqs. (7) and (8), we can obtain

$$\begin{aligned} \hat{n} \times E_0 \sum_{m=-\infty}^{\infty} \sum_{n=|m|}^{\infty} [\alpha_{mn} \mathbf{M}_{mn}^{(3)}(k_0 \mathbf{r}) + \beta_{mn} \mathbf{N}_{mn}^{(3)}(k_0 \mathbf{r})] + \hat{n} \times \mathbf{E}^{inc} \\ = \hat{n} \times E_0 \sum_{m=-\infty}^{\infty} \sum_{n=|m|}^{\infty} [c_{mn} \mathbf{M}_{mn}^{(1)}(k \mathbf{r}) + c'_{mn} \mathbf{M}_{mn}^{(3)}(k \mathbf{r}) + d_{mn} \mathbf{N}_{mn}^{(1)}(k \mathbf{r}) \\ + d'_{mn} \mathbf{N}_{mn}^{(3)}(k \mathbf{r})] \end{aligned} \quad (9)$$

$$\begin{aligned} \hat{n} \times E_0 \sum_{m=-\infty}^{\infty} \sum_{n=|m|}^{\infty} [\alpha_{mn} \mathbf{N}_{mn}^{(3)}(k_0 \mathbf{r}) + \beta_{mn} \mathbf{M}_{mn}^{(3)}(k_0 \mathbf{r})] + \hat{n} \times i\eta_0 \mathbf{H}^{inc} \\ = \hat{n} \times E_0 \frac{\eta_0}{\eta} \sum_{m=-\infty}^{\infty} \sum_{n=|m|}^{\infty} [c_{mn} \mathbf{N}_{mn}^{(1)}(k \mathbf{r}) + c'_{mn} \mathbf{N}_{mn}^{(3)}(k \mathbf{r}) + d_{mn} \mathbf{M}_{mn}^{(1)}(k \mathbf{r}) \\ + d'_{mn} \mathbf{M}_{mn}^{(3)}(k \mathbf{r})] \end{aligned} \quad (10)$$

In the MoM scheme, it can be interpreted that Eqs. (9) and (10) are obtained by choosing appropriate spherical VWFs as basis functions to expand the scattered and internal fields.

Multiplication of Eqs. (9) and (10) with $\mathbf{M}_{(-m')n'}^{(1)}(k_0 \mathbf{r})$, $\mathbf{N}_{(-m')n'}^{(1)}(k_0 \mathbf{r})$ respectively (dot product) and then integration over the surface S yield

$$\begin{aligned} - \oint_S \mathbf{M}_{(-m')n'}^{(1)}(k_0 \mathbf{r}) \times \mathbf{E}^{inc} \cdot \hat{n} dS = E_0 \sum_{m=-\infty}^{\infty} \sum_{n=|m|}^{\infty} [U_{m'n'mn} \alpha_{mn} + V_{m'n'mn} \beta_{mn}] \\ - E_0 \sum_{m=-\infty}^{\infty} \sum_{n=|m|}^{\infty} [U_{m'n'mn}^{(1)} c_{mn} + U_{m'n'mn}^{(3)} c'_{mn} + V_{m'n'mn}^{(1)} d_{mn} + V_{m'n'mn}^{(3)} d'_{mn}] \end{aligned} \quad (11)$$

$$\begin{aligned} - \oint_S \mathbf{N}_{(-m')n'}^{(1)}(k_0 \mathbf{r}) \times \mathbf{E}^{inc} \cdot \hat{n} dS = E_0 \sum_{m=-\infty}^{\infty} \sum_{n=|m|}^{\infty} [K_{m'n'mn} \alpha_{mn} + L_{m'n'mn} \beta_{mn}] \\ - E_0 \sum_{m=-\infty}^{\infty} \sum_{n=|m|}^{\infty} [K_{m'n'mn}^{(1)} c_{mn} + K_{m'n'mn}^{(3)} c'_{mn} + L_{m'n'mn}^{(1)} d_{mn} + L_{m'n'mn}^{(3)} d'_{mn}] \end{aligned} \quad (12)$$

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