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Rethinking phase height model of fringe projection profilometry: A geometry transformation viewpoint

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ABSTRACT

The phase height model is developed in the phase measuring profilometry, while the geometry transformation model is proposed in the structured light method and the active stereovision. However the inner connection or coherence of them has not been clearly discussed. In this paper, the gap between them is bridged. The phase height model is reasoned from the geometry transformation model, which can be denoted as an explicit function of the geometry parameters. The reasoned phase height model is consistent with two traditional phase height models, whether the system is coplanar or non-coplanar optical axes. On the other hand, this generic phase height model explains the connection of two traditional phase height models. The simulation and experiments affirm that the reasoned model is valid and the proposed calibration method is feasible.

1. Introduction

Phase measuring profilometry (PMP), a well-known optical metrology based on active stereovision, has been developed to meet the demands of various applications [1,2]. It adopts a time-varying phase shift to generate a group of sinusoidal waveforms. Then, these phase-shifted intensity patterns are projected onto the scene, reflected off the target object, and viewed by a camera from a shifted position over time. Finally, the underlying phase distribution of the acquired fringe images is calculated through fringe analysis techniques [3–6] and phase unwrapping methods [7–10]. In order to perform accurate 3D measurement, calibration of the relationship between the phase distribution and the depth map is the pre-requisite. This mapping relationship is a nonlinear function, whose parameters are dependent on the positions and orientations of the camera, the projector and the reference plane. Until now, two phase height models have been proposed. One is the relative phase-height mapping(RPHM) [11–16], and the other is the absolute phase height model (APHM) [17–21].

RPHM describes the mapping between the phase difference and the out-off-plane height for each camera pixel. It chooses a reference plane as the X-Y coordinate plane. The phase distribution for the reference plane is set the origin, and the phase difference is computed for any target object through subtracting the origin phase. The out-off-plane height is the distance between the target with the reference plane. It is equivalent to the Z coordinate. Zhou and Su [11] first indicate the linear relationship for RPHM, if the height and phase difference is represented with their reciprocals. An explicit equation is present to describe the linear parameters, a and b, when the optical axes of the projector and the camera are coplanar. They also predicted that the RPHM is the same in the two non-coplanar optical axes system. However, how the system parameters influence a and b is unavailable. With regard to coplanar axes system, Xiao et al. [12] reasoned a similar equation with another geometry model to describe a and b. For non-coplanar optical axes system, Guo et al. [13] and Wen et al. [14] proceeded geometric analysis from two different view points, and both gave explicit equations a and b. However, above mentioned methods did not adopt the geometry parameters to calibrate the phase height model and directly estimated a and b for each pixel.

APHM denotes the relationship between the absolute phase and the out-off-plane height. It adopts polynomial division instead of the linear function, and the polynomials for numerator and denominator both contain six parameters. APHM also sets the world coordinate system on a reference plane, but the absolute phase is the measured phase, which equals the original projected phase. Du and Wang [17] first presented the APHM for an arbitrarily arranged fringe projection profilometry system. Cui and Zhu [18] adopted matrices instead of the geometry method to

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reason the phase height model. Further, the lens distortion is considered in the works [19,20].

On the other hand, PMP is believed as a branch of the structured light method. The light transmission and the image formation are denoted with the projective geometry. Some works [22–25] take the projector as the inverse of the camera, and calibrate the PMP using the geometry transformation model. This geometry transformation model includes all system parameters. Besides it is much more convenient to understand the light projection and the image formation. Instead of phase height model, the 3D shape is computed with the triangulation of the camera pixel and the projected phase.

Actually, the geometry transformation model and the phase height model, using different theories and ways, both successfully describe the same PMP and achieve the 3D metrology. However the inner connection or coherence of them has not been clearly discussed. Similarly, two phase height model should be transformed to each other, however, no work is present, to the best of our knowledge. They are two main contributions of this paper. First, the phase height model is reasoned from the geometry transformation model. The proposed phase height model is an explicit function of the geometry parameters of the reference plane, the camera, and the projector. If the reference plane is changed, the new model is easily computed without another calibration. This generic model can describe the coplanar and non-coplanar optical axes system. Second, the proposed phase height model can be transformed to the RPHM and the APHM. The parameters in the RPHM and the APHM are also explicit determined by the geometry parameters.

2. The analysis of the phase height model

2.1. Geometry transformation from 3D point to images

In this paper, we also see the PMP as one of active stereovision. The projector is modeled as same as the camera, since the projector can be conceptually regarded as a camera acting in reverse. A 3D point $\mathbf{p_w} = [X_\mathbf{w}, Y_\mathbf{w}, Z_\mathbf{w}]^\mathbf{T}$ is mapped to the 2D pixel $\mathbf{m}^* = [u^*, v^*]^\mathbf{T}$ on the camera or projector image through the projective transformation. The sign $*$ denotes the specific device, such as $\mathbf{m^c}$ for the camera and $\mathbf{m^p}$ for the projector. The geometric transformations are as follows.

Step 1: Transformation from the world coordinate system to the device coordinate system

$$\tilde{\mathbf{p}}_* = \begin{bmatrix} \mathbf{R}_* & \mathbf{t}_* \\ 0 & 1 \end{bmatrix} \tilde{\mathbf{p}}_\mathbf{w} \tag{1}$$

where $\tilde{\mathbf{p}}_\mathbf{w} = [X_\mathbf{w}, Y_\mathbf{w}, Z_\mathbf{w}, 1]^\mathbf{T}$ and $\tilde{\mathbf{p}}_* = [X_*, Y_*, Z_*, 1]^\mathbf{T}$ denote the homogeneous coordinates of the 3D point in world coordinate system and the device coordinate system, respectively. $\mathbf{R}_*$ is the rotation matrix and $\mathbf{t}_*$ is the translation vector. $[\mathbf{R}_*, \mathbf{t}_*]$ are called the extrinsic parameters.

Step 2: Normalized (pinhole) projection

$$\begin{bmatrix} u_n^* \\ v_n^* \end{bmatrix} = \begin{bmatrix} X_*/Z_* \\ Y_*/Z_* \end{bmatrix} \tag{2}$$

$\mathbf{m}_n^* = [u_n^*, v_n^*]^\mathbf{T}$ is the normalized image point.

Step 3: Conversion into the pixel coordinate

$$\tilde{\mathbf{m}}^* = \mathbf{K}_* \tilde{\mathbf{m}}_n^*, \quad \text{where} \quad \mathbf{K}_* = \begin{bmatrix} \alpha_* & \gamma_* \cdot \beta_* & u_0^* \\ 0 & \beta_* & v_0^* \\ 0 & 0 & 1 \end{bmatrix} \tag{3}$$

$\tilde{\mathbf{m}}^*$ and $\tilde{\mathbf{m}}_n^*$ are the homogeneous coordinates. $\mathbf{K}_*$ is called the camera or projector intrinsic matrix. $[u_0^*, v_0^*]$ are the coordinates of the principal point, α_* and β_* are the scale factors in image u and v axes, and γ_* describes the skewness of the two image axes.

2.2. Lateral coordinate model

PMP captures the 3D shape, only if the phase-height model and the lateral coordinate model are available and calibrated. The lateral coordinate model describes the mapping from the camera pixel coordinate to the lateral (X-Y) coordinate in the world coordinate system, while the phase-height model presents how to compute the out-off-plane height (Z coordinate) from the phase at one camera pixel.

Assuming the camera is calibrated, the lateral coordinate $[X_\mathbf{w}, Y_\mathbf{w}]^T$ is dependent on the camera pixel coordinate $\tilde{\mathbf{m}}^c$ and the out-off-height. As shown in Fig. 1, the $\tilde{\mathbf{m}}^c$ is corresponding to P_o on the reference plane when the out-off-height is zero, and it is mapped to P_W for Z_w. In this paper, the lateral coordinate model is reasoned from the geometry transformation.

Step 1: Extracting the lateral coordinate for any 3D point.

Without loss of generality, the 3D point can be represented as:

$$\begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix} = \begin{bmatrix} \mathbf{I} & \mathbf{t}_h \\ 0 & 1 \end{bmatrix} \begin{bmatrix} X_w \\ Y_w \\ 0 \\ 1 \end{bmatrix}, \quad \text{where} \quad \mathbf{t}_h = \begin{bmatrix} 0 \\ 0 \\ Z_w \end{bmatrix} \tag{4}$$

If the 3D point, e.g P_o is on the reference plane, t_h equals zero. Or not, e.g P_w, it is equivalent to a in-plane point after translation with t_h. This in-plane point contains the same lateral coordinate.

Step 2: Mapping between the lateral coordinate with the projection point.

Denote the i^{th} column of the rotation matrix $\mathbf{R}$ by $\mathbf{r}_i$. From Eqs. (1), (2), and (4) we have

$$\begin{bmatrix} u_n^c \\ v_n^c \\ 1 \end{bmatrix} = \frac{1}{Z_c} [\mathbf{r}_1^c \quad \mathbf{r}_2^c \quad Z_w \mathbf{r}_3^c + \mathbf{t}_c] \begin{bmatrix} X_w \\ Y_w \\ 1 \end{bmatrix} \tag{5}$$

Inverting Eq. (5), we can get

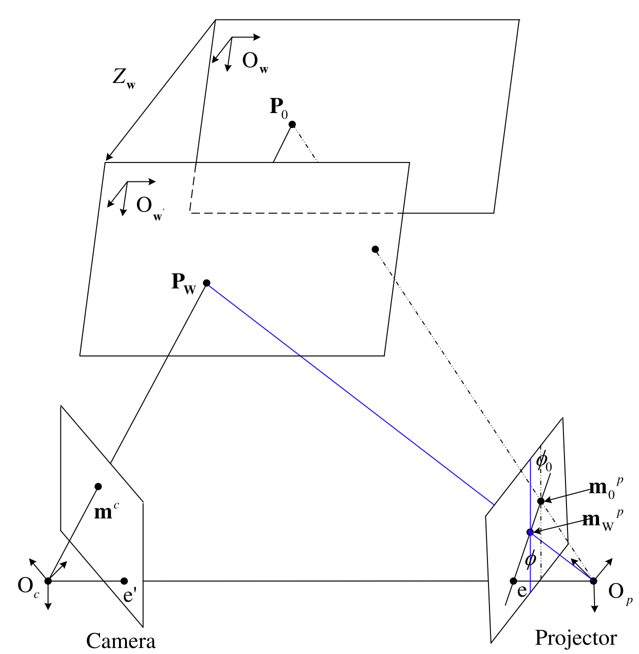

Fig. 1. Triangle theory.