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Polarization singularities of random partially coherent electromagnetic beams in atmospheric turbulence

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ABSTRACT

A transverse-longitudinal cross-spectral density matrix of partially coherent electromagnetic beams was proposed, and polarization singularities of random partially coherent electromagnetic beams can be applied to study with this cross-spectral density matrix. Take random partially coherent electromagnetic beams in atmospheric turbulence as an example, polarization singularities are studied in this paper. The numerical calculation show that the atmospheric turbulence can speed up the evolution process of polarization singularities of partially coherent electromagnetic beams.

1. Introduction

There has been substantial interest in polarization singularities which as typical singularities in vector wave fields have been extensively studied both analytically and experimentally since Nye's work [1–10]. The polarization singularities contain singularities including *C*-points and *L*-lines [1] and can be analyzed by using Stokes parameters [5,8,9]. Theoretical results of polarization singularities can find promising applications in singular Stokes-polarimetry [4]. However, these studies were restricted in the paraxial field. The purpose of the present paper is to study polarization singularities of random partially coherent electromagnetic beams in the general situation. A transverse-longitudinal cross-spectral density matrix of partially coherent electromagnetic beams was proposed, and polarization singularities of random partially coherent electromagnetic beams can be applied to study with this cross-spectral density matrix. In Section 2 the Stokes parameters of polarization singularities of partially coherent electromagnetic beams in atmospheric turbulent are derived. Section 3 takes a typical example of polarization singularities of partially coherent vortex electromagnetic beams propagating in atmospheric turbulence to illustrated polarization singularities. Finally, Section 4 contains a summary of the main results obtained in this paper.

2. A transverse-longitudinal cross-spectral density matrix and polarization singularities of random partially coherent electromagnetic beams in atmospheric turbulence

Consider two electric fields $\mathbf{E}(\boldsymbol{\rho_v}, z)$ ($v = 1, 2$) at the plane z in propagation zone of the incident electric field $\mathbf{E}_s(\boldsymbol{\rho}_{0v}, 0) = E_x(\boldsymbol{\rho}_{0v}, 0)\mathbf{i} + E_y(\boldsymbol{\rho}_{0v}, 0)\mathbf{j}$ (where $\mathbf{i}$ and $\mathbf{j}$ respectively denote the unit vector along the *x*- and *y*- axis) propagating in atmospheric turbulence can be expressed as

$$E(\rho_v, z) = E_s(\rho_v, z)\mathbf{s} + E_z(\rho_v, z)\mathbf{k} \tag{1a}$$

where $E_s(\boldsymbol{\rho_v}, z)$ and $E_z(\boldsymbol{\rho_v}, z)$ respectively denote amplitudes of the transverse and longitudinal components of the electromagnetic field, $\mathbf{k}$, $\mathbf{s}$ denote the unit vector along the *z*-axis and the synthesis direction of $E_x(\boldsymbol{\rho_v}, z)$ and $E_y(\boldsymbol{\rho_v}, z)$, and

$$E_s(\rho_v, z) = \left[E_x^2(\rho_v, z) + E_y^2(\rho_v, z)\right]^{1/2} \tag{1b}$$

denotes the synthesis amplitude of $E_x(\boldsymbol{\rho_v}, z)$ and $E_y(\boldsymbol{\rho_v}, z)$, and $\boldsymbol{\rho}_v(= x_v\mathbf{i} + y_v\mathbf{j})$ denotes a position vector in the plane z. This method can be considered that the two components (***x*** and ***y***) in the cross section of the field are synthesized as a component **s**. Thus, this expression of the field decreases from the three-dimensional (***xyz***) to the two-one (**sk**). It can be decreased the difficult to study the field problem in certain extent. Certainly, traditional theory of the two-dimensional field can be applied to this two-one. Thus, the stochastic electromagnetic beam whose fluctuations are statistically stationary, the second-order correlation properties of the electromagnetic beam in the space-frequency domain are

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characterized by a cross-spectral density matrix of the form can be expressed as

$$W^{(t)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,z)=\left[\left\langle E_\alpha{}^*(\boldsymbol{\rho}_1,z)E_\beta(\boldsymbol{\rho}_2,z)\right\rangle\right]\quad(\alpha,\beta=s,z), \tag{2}$$

where $\langle\,\rangle$ specifies the ensemble average, * is the complex conjugate, and Eq. (2) can be rewritten as

$$W^{(t)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,z)=\begin{bmatrix}W_{ss}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,z) & W_{sz}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,z)\\ W_{zs}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,z) & W_{zz}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,z)\end{bmatrix} \tag{3}$$

Eqs. (2) and (3) denote the coherence between the quantity $E_s^*(\boldsymbol{\rho}_1,0)$ and $E_z(\boldsymbol{\rho}_2, 0)$. The matrix $\mathbf{W}^{(t)}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, z)$ is called as the transverse-longitudinal cross-spectral density matrix of random partially coherent electromagnetic beams. The matrix expressed by Eq. (3) represents the correlation properties between the transverse and the longitudinal fields of random partially coherent beams in free space when the random part $\Psi_a(\boldsymbol{\rho}_v, \boldsymbol{\rho}_{0v}, z)$ of the complex phase is zero, and we know that the column of the cross-spectral density matrix decreases from 3 × 3 to 2 × 2.

Based on the vector Rayleigh—Sommerfeld diffraction integrals [11] and the Eq. (1), after a simplify mathematical arrangement, $\mathbf{E}_s(\boldsymbol{\rho}_v, z)$ and $\mathbf{E}_z(\boldsymbol{\rho}_v, z)$ can be written as

$$E_s(\boldsymbol{\rho}_v,z)\frac{1}{2\pi}\left[\left[\iint_{z=0}E_x(\boldsymbol{\rho}_{0v},0)\exp\left[\psi_x(\boldsymbol{\rho}_v,\boldsymbol{\rho}_{0v},z)\right]\times\frac{\partial}{\partial z}\left(\frac{\exp(ikR_v)}{R_v}\right)d^2\boldsymbol{\rho}_{0v}\right]^2+\left[\iint_{z=0}E_y(\boldsymbol{\rho}_{0v},0)\exp\left[\psi_y(\boldsymbol{\rho}_v,\boldsymbol{\rho}_{0v},z)\right]\frac{\partial}{\partial z}\left(\frac{\exp(ikR_v)}{R_v}\right)d^2\boldsymbol{\rho}_{0v}\right]^2\right]^{12} \tag{4a}$$

$$E_z(\boldsymbol{\rho}_v,z)=\frac{1}{2\pi}\iint_{(z=0)}\left[E_x(\boldsymbol{\rho}_{0v},z)\frac{\partial}{\partial x_v}\left(\frac{\exp(ikR_v)}{R_v}\right)\exp\left[\psi_x(\boldsymbol{\rho}_v,\boldsymbol{\rho}_{0v},z)\right]+E_y(\boldsymbol{\rho}_{0v},0)\frac{\partial}{\partial y_v}\left(\frac{\exp(ikR_v)}{R_v}\right)\exp\left[\psi_y(\boldsymbol{\rho}_v,\boldsymbol{\rho}_{0v},z)\right]\right]d^2\boldsymbol{\rho}_{0v} \tag{4b}$$

where

$$\boldsymbol{\rho}_{0v}=x_{0v}\mathbf{i}+y_{0v}\mathbf{j},R_v{}^2=(x_v-x_{0v})^2+(y_v-y_{0v})^2+z_2,k=2\pi/\lambda \tag{5}$$

and $\Psi_a(\boldsymbol{\rho}_v, \boldsymbol{\rho}_{0v}, z)$ $(a = x, y)$ stands for the random part of the complex phase of a spherical wave due to the turbulence along the α-axis. and in which $\boldsymbol{W}_{ss}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2,z)$, $\boldsymbol{W}_{sz}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2,z)$ and $\boldsymbol{W}_{zs}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2,z)$ can be expressed as following

$$W_{zz}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,z)=\frac{z}{4\pi^2}\iint\iint_{(z=0)}\begin{bmatrix}W_{xx}(\boldsymbol{\rho}_{01},\boldsymbol{\rho}_{02},0)(x_1-x_{01})(x_2-x_{02})\Phi_{xx}\\+W_{xy}(\boldsymbol{\rho}_{01},\boldsymbol{\rho}_{02},0)(y_1-y_{01})(y_2-y_{02})\Phi_{xy}\\+W_{yx}(\boldsymbol{\rho}_{01},\boldsymbol{\rho}_{02},0)(x_1-x_{01})(x_2-x_{02})\Phi_{yx}\\+W_{yy}(\boldsymbol{\rho}_{01},\boldsymbol{\rho}_{02},0)(y_1-y_{01})(y_2-y_{02})\Phi_{yy}\end{bmatrix}P_1P_2d^2\boldsymbol{\rho}_{01}d^2\boldsymbol{\rho}_{02} \tag{6d}$$

where

$$\Phi_{ab}=<\exp[\psi_a{}^*(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,z)+\psi_b(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,z)]>\quad(a,b=x,y) \tag{7a}$$

$$P_1=(-1-ikR_1)\exp(-ikR_1)/R_1{}^3,\quad P_2=(ikR_2-1)\exp(ikR_2)/R_2{}^3 \tag{7b}$$

and $\mathbf{W}_{ab}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, 0)$ denotes cross-spectral density matrix elements of the incident partially coherent electromagnetic beam at the plane z = 0.

From Eqs. (6b) and (6c), we obtain

$$W_{sz}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,z)=[W_{zs}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,z)]^*\textit{and}\ \ W_{ss}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,z)=W_{xx}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,z)+W_{yy}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,z) \tag{8}$$

We define the Stokes parameters for the electric field in atmospheric turbulence as in Ref. [13], that is

$$s_0(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,z)=W_{ss}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,z)+W_{zz}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,z) \tag{9a}$$

$$s_1(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,z)=W_{ss}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,z)-W_{zz}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,z) \tag{9b}$$

$$s_2(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,z)=W_{sz}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,z)-W_{zs}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,z) \tag{9c}$$

$$s_3(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,z)=i[W_{zs}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,z)-W_{sz}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,z)] \tag{9d}$$

where *i* denote the image unit. Just like the $\mathbf{W}^{(t)}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, z)$, the Stokes parameters contain the information about both the polarization and coherence properties of the stochastic electromagnetic beams. The generalized Stokes parameters of random electromagnetic beams expressed by Eq. (9) can not only be applied in paraxial field, but also the general vector field. The Stokes parameters at any point $\boldsymbol{\rho}$ in the plane *z* are obtained from Eq. (9) by letting $\boldsymbol{\rho}_1 = \boldsymbol{\rho}_2 = \boldsymbol{\rho}$, and the Stokes parameters in the plane *z* are rewritten as

$$SP_1=[W_{ss}(\boldsymbol{\rho},\boldsymbol{\rho},z)-W_{zz}(\boldsymbol{\rho},\boldsymbol{\rho},z)]/[W_{ss}(\boldsymbol{\rho},\boldsymbol{\rho},z)+W_{zz}(\boldsymbol{\rho},\boldsymbol{\rho},z)] \tag{10a}$$

$$W_{ss}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,z)=\frac{z^2}{4\pi^2}\left[\begin{matrix}\left[\iint_{z=0}W_{xx}(\boldsymbol{\rho}_{01},\boldsymbol{\rho}_{02},0)\Phi_{xx}P_1P_2d^2\boldsymbol{\rho}_{01}d^2\boldsymbol{\rho}_{02}\right]^2+\left[\iint_{z=0}W_{xy}(\boldsymbol{\rho}_{01},\boldsymbol{\rho}_{02},0)\Phi_{xy}P_1P_2d^2\boldsymbol{\rho}_{01}d^2\boldsymbol{\rho}_{02}\right]^2\\+\left[\iint_{z=0}W_{yx}(\boldsymbol{\rho}_{01},\boldsymbol{\rho}_{02},0)\Phi_{yx}P_1P_2d^2\boldsymbol{\rho}_{01}d^2\boldsymbol{\rho}_{02}\right]^2+\left[\iint_{z=0}W_{yy}(\boldsymbol{\rho}_{01},\boldsymbol{\rho}_{02},0)\Phi_{yy}P_1P_2d^2\boldsymbol{\rho}_{01}d^2\boldsymbol{\rho}_{02}\right]^2\end{matrix}\right]^{1/2} \tag{6a}$$

$$W_{sz}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,z)=\frac{z}{4\pi^2}\left[\begin{matrix}\left[\iint_{z=0}W_{xx}(\boldsymbol{\rho}_{01},\boldsymbol{\rho}_{02},0)(x_2-x_{02})\Phi_{xx}P_1P_2d^2\boldsymbol{\rho}_{01}d^2\boldsymbol{\rho}_{02}\right]^2+\left[\iint_{z=0}W_{xy}(\boldsymbol{\rho}_{01},\boldsymbol{\rho}_{02},0)(y_2-y_{02})\Phi_{xy}P_1P_2d^2\boldsymbol{\rho}_{01}d^2\boldsymbol{\rho}_{02}\right]^2\\+\left[\iint_{z=0}W_{yx}(\boldsymbol{\rho}_{01},\boldsymbol{\rho}_{02},0)(x_2-x_{02})\Phi_{yx}P_1P_2d^2\boldsymbol{\rho}_{01}d^2\boldsymbol{\rho}_{02}\right]^2+\left[\iint_{z=0}W_{yy}(\boldsymbol{\rho}_{01},\boldsymbol{\rho}_{02},0)(y_2-y_{02})\Phi_{yy}P_1P_2d^2\boldsymbol{\rho}_{01}d^2\boldsymbol{\rho}_{02}\right]^2\\+2\iint_{z=0}W_{yx}(\boldsymbol{\rho}_{01},\boldsymbol{\rho}_{02},0)W_{yy}(\boldsymbol{\rho}_{01},\boldsymbol{\rho}_{02},0)(x_2-x_{02})(y_2-y_{02})\Phi_{yy}\Phi_{yx}P_1P_2P_1P_2d^2\boldsymbol{\rho}_{01}d^2\boldsymbol{\rho}_{02}\\+2\iint_{z=0}W_{xy}(\boldsymbol{\rho}_{01},\boldsymbol{\rho}_{02},0)W_{xx}(\boldsymbol{\rho}_{01},\boldsymbol{\rho}_{02},0)(x_2-x_{02})(y_2-y_{02})\Phi_{xx}\Phi_{xy}P_1P_2P_1P_2d^2\boldsymbol{\rho}_{01}d^2\boldsymbol{\rho}_{02}\end{matrix}\right]^{1/2} \tag{6b}$$

$$W_{zs}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,z)=\frac{z}{4\pi^2}\left[\begin{matrix}\left[\iint_{z=0}W_{xx}(\boldsymbol{\rho}_{01},\boldsymbol{\rho}_{02},0)(x_1-x_{01})\Phi_{xx}P_1P_2d^2\boldsymbol{\rho}_{01}d^2\boldsymbol{\rho}_{02}\right]^2+\left[\iint_{z=0}W_{xy}(\boldsymbol{\rho}_{01},\boldsymbol{\rho}_{02},0)(x_1-x_{01})\Phi_{xy}P_1P_2d^2\boldsymbol{\rho}_{01}d^2\boldsymbol{\rho}_{02}\right]^2\\+\left[\iint_{z=0}W_{yy}(\boldsymbol{\rho}_{01},\boldsymbol{\rho}_{02},0)(y_1-y_{01})\Phi_{yy}P_1P_2d^2\boldsymbol{\rho}_{01}d^2\boldsymbol{\rho}_{02}\right]^2+\left[\iint_{z=0}W_{yx}(\boldsymbol{\rho}_{01},\boldsymbol{\rho}_{02},0)(y_1-y_{01})\Phi_{yx}P_1P_2d^2\boldsymbol{\rho}_{01}d^2\boldsymbol{\rho}_{02}\right]^2\\+2\iint_{z=0}W_{yx}(\boldsymbol{\rho}_{01},\boldsymbol{\rho}_{02},0)W_{xx}(\boldsymbol{\rho}_{01},\boldsymbol{\rho}_{02},0)(y_1-y_{01})(x_1-x_{01})\Phi_{xx}\Phi_{yx}P_1P_2P_1P_2d^2\boldsymbol{\rho}_{01}d^2\boldsymbol{\rho}_{02}\\+2\iint_{z=0}W_{xy}(\boldsymbol{\rho}_{01},\boldsymbol{\rho}_{02},0)W_{yy}(\boldsymbol{\rho}_{01},\boldsymbol{\rho}_{02},0)(x_1-x_{01})(y_1-y_{01})\Phi_{yy}\Phi_{xy}P_1P_2P_1P_2d^2\boldsymbol{\rho}_{01}d^2\boldsymbol{\rho}_{02}\end{matrix}\right]^{1/2} \tag{6c}$$