Optics and Laser Technology xxx (2018) xxx–xxx

Contents lists available at ScienceDirect

Optics and Laser Technology

journal homepage: www.elsevier.com/locate/optlastec

Full length article

# The effect of geometric parameters of a single-gap SRR metamaterial on its electromagnetic properties as a unit cell of interior invisibility cloak in the microwave regime

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ARTICLE INFO

*Article history:*
Received 29 March 2018
Received in revised form 10 June 2018
Accepted 12 July 2018
Available online xxxx

*Keywords:*
Split ring resonator
Effective permeability
Metamaterial
Invisibility cloak
Plasmonics
Transformation optics

ABSTRACT

Some geometric parameters of a metamaterial unit cell with single-gap Split Ring Resonator (SRR) are modified and the effect of these changes on its electromagnetic properties is examined. In particular, the relative magnetic permeability has been investigated. For unit cells with various geometric parameters the frequency $f_p$ in which the relative magnetic permeability is close to zero as well as the resonance frequency $f_0$ are obtained. Finally, as an example, for each of the unit cells, the relative magnetic permeability at 9.5 GHz is obtained and by using these results an electromagnetic invisibility cloak is designed operating at 9.5 GHz which can has three different unit cells with different geometric parameters for each of its layers.



## 1. Introduction

A metamaterial is usually designed at scales that are smaller than the applied wavelength in desired applications [1]. The electromagnetic properties of a metamaterial derive from its new designed structure. Their precise size, shape, geometry and arrangement gives them their smart properties capable of manipulating electromagnetic waves: by enhancing, blocking, absorbing, or bending waves, to achieve benefits that go beyond what is possible with conventional materials. Some metamaterials that exhibit a negative refractive index for particular electromagnetic wavelengths, are known as negative-index metamaterials [2–4].

In a theoretical study, Veselago presented metamaterials in which the sign of the magnetic permeability ($\mu$) and electric permittivity ($\varepsilon$), were simultaneously negative [4]. At first, John Pendry introduced a practical way to fabricate a negative index metamaterial [5,6].

John Pendry predicted that metallic wires aligned along the direction of a wave, could exhibit negative permittivity ($\varepsilon < 0$). Some natural materials display negative permittivity, the problem was achieving negative permeability ($\mu < 0$). In 1999, Pendry introduced a C shape split ring to achieve negative permeability and presented a structure made of wires and rings to obtain negative refractive index [7]. The experimental presentation of operational electromagnetic metamaterials was reported by Smith et al., in 2000 [8]. In 2007, many groups presented experimental researches on the negative refractive index [9,10]. In 2015, a cylindrical waveguide with negative index cladding was proposed by Xue et al. [11]. The first imperfect invisibility cloak at microwave frequencies was realized in 2006 [12]. During the last few years, designing various electromagnetic invisibility cloaks based on localized resonance [13–15], dipolar scattering elimination [16,17], tunneling light transmittance [18], active sources [19], and transformation optics [20–22] attracted significant attentions. Recently, some researchers worked on using nanocomposites as metamaterials [23] as well as design, simulation, and measurement of metamaterial absorber [24,25]. Also, a metal–dielectric metamaterial refractive index sensor has been designed by Li et al., in 2018 [26].

As an artificial structure, the split ring resonator (SRR) has been commonly used in metamaterials to produce the required magnetic response at desired frequencies. Also, the SRRs prepare significant magnetic coupling to a practical electromagnetic field. For example, a set of split ring resonators produces an effect such as negative permeability [27]. As another application, the SRRs are used to design invisibility cloak in microwave regime [12].

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https://doi.org/10.1016/j.optlastec.2018.07.025
0030-3992/

This paper includes two main sections. In the first section, we design and simulate a metamaterial unit cell based on a single-gap SRR. Then, we examine the effect of various geometric parameters of the unit cell on its electromagnetic properties. Specially, for each unit cell, we obtain the effective permeability (at 9.5 GHz), the magnetic resonance frequency and the magnetic plasma frequency. In the second section, we explain an invisible cloak using the transformation optics theory in brief. As an application of the SRR unit cells which were designed in the first section, we introduce a ten-layer cylindrical cloak. It is an interior cloak for TE polarization in microwave regime [12]. According to the mentioned theory, we determine the effective permeability of each layer. So, for each layer of the cloak, we choose suitable unit cells from Section 1.

## 2. Section 1: Effective permeability of SRR

In this section, we design a metamaterial unit cell including a single-gap SRR. For this unit cell, some geometric parameters are defined. Effective permeability of the unit cells with various geometric parameters is obtained. Also, the effect of geometric parameters of the unit cell on its electromagnetic properties is investigated.

### 2.1. Designing

The metamaterial unit cell is composed of a teflon slab with a positive refraction index ($\varepsilon = 2.1 + 0.001i$) and a single SRR printed on Rogers 5880 substrate. We define $t_1$ and $t_2$ as the thickness of the teflon slab and the thickness of the Rogers substrate, respectively. So, we have a unit cell with dimensions of 1.354 mm × 5 mm × 5 mm. Fig. 1 shows the composition of the unit cell and the size of the SRR. Teflon slab and SRR are arranged along the x-axis.

The thickness of the SRR copper wires, the size of the gap and the width of the SRR copper wires are $t$, $s$ and $w$, respectively. $l_y$ and $l_z$ are the length of the SRR copper wire at y and z directions, respectively. We examine the effect of geometric parameters ($s$, $w$, $l_y$, $l_z$ and $t$) on the relative magnetic permeability, separately. To find the effect of each parameter, we consider other parameters to be constant.

### 2.2. Theory

Single SRRs can be analyzed from the view of equivalent circuit [28]. The presented SRR in Fig. 1(b) can be considered as an LC resonator, so that the inductance and capacitance are related to the ring and gap, respectively. The equivalent circuit of a single SRR can be seen in Fig. 1(c). In this oscillator, the magnetic field vector must penetrate into the ring (as the inductor) to excite resonance. The electric field vector aligned normal to the surfaces of the cut (as the capacitor) can also excite resonance.

The magnetic resonance frequency of a single SRR as an LC oscillator is presented as:

$$\omega = 1/\sqrt{LC} \tag{1}$$

Using the quasi-static formulas for a parallel plate capacitor, the capacitance of a single SRR is obtained as $C = k\varepsilon_0 wt/s$ and using the quasi-static formulas for a solenoid, the inductance of the SRR is estimated as $L = \mu_0 l_y l_z/t$ [29].

So that $k$ is the dielectric constant of Rogers. Using above expressions of capacitance and inductance, the resonance frequency of the SRR is obtained as:

$$f_0 = 1/\left(2\pi\sqrt{LC}\right) = (c_0/2\pi)\sqrt{(s/kwl_y l_z)} \tag{2}$$

Speed of light in vacuum is denoted by $c_0$. Although this formula created for perfect metal with infinite carrier density, it presents a simple relation between the resonance frequency and some geometric parameters of the SRR. Calculation of the exact value of $f_0$ in theory is out of our work.

The estimated Eq. (2) is not suitable for a discussion of the thickness ($t$) dependence of resonance frequency ($f_0$). Eq. (2) predicts no thickness dependence of $f_0$. However, the thickness dependence of the resonances frequency of SRRs in the optical regime has been studied in 2007 [30]. The resonance frequency of the SRR increases with its thickness in the optical regime [30]. In 2013, however, another study showed that the resonance frequency reduces with increase the thickness of the SRR [31].

### 2.3. Simulating

First, we extract relative magnetic permeability of a unit cell with typical geometric parameters: $s = 0.3$ mm, $w = 0.3$ mm, $t = 0.06$ mm and $l_y = l_z = 3$ mm.

The real part of relative magnetic permeability is denoted as $\mu_{eff}$. Also, $f_p$ is a frequency in which the effective permeability $\mu_{eff}$ is close to zero. In other words, $f_p$ is the magnetic plasma frequency [32]. The resonance region is narrow with a very large imaginary part, which may result in a high loss.

The S-parameters retrieval method [33] has been used to compute $\mu_{eff}$ around the resonance band as shown in Fig. 2. According to this figure, N band is a frequency range in which the real part of the permeability is negative and the imaginary part of the permeability is rather small. These conditions produce a frequency range with low loss [32]. So, loss in range N is very low, which can be applied to realize negative refraction effectively. There is a band-stop around the magnetic plasma frequency $f_p$ induced by the total reflection. As shown in Fig. 2, the center frequency of the F band has been obtained as $f_p = 9.5$ GHz. The absolute value of permeabil-

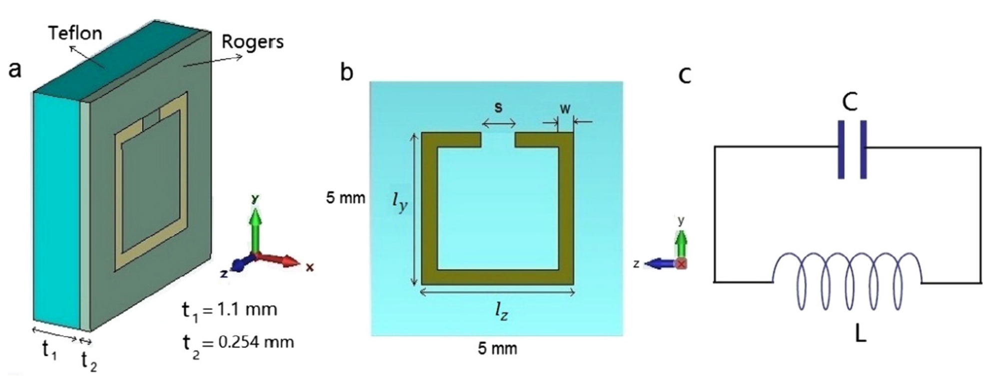


**Fig. 1.** (a) The composition and typical size of the anisotropic bulk metamaterial, (b) Geometric parameters of SRR, (c) equivalent LC circuit.