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Propagation of an Airy beam through atmospheric turbulence with optical vortex under fractional Fourier transforms

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ABSTRACT

In this paper, we have theoretically investigated the behaviour of an Airy beam (AiB) with an optical vortex (OV) in atmospheric turbulence. The obtained splitting on each line of the phase pattern indicates the OV. The results show that the OV position changes when the power of the fractional Fourier transforms (FrFT) (p) changes. Moreover, uniformity of the spot beam disappears with the presence of the vortex in the AiB. The characteristics of an AiB, such as the effective beam size, number, width, height, and uniformity of the spot beam, change by changing the value of p. The position of the OV in the beam with scintillation and without scintillation was changed by changing the value of p.

1. Introduction

The FrFT is regarded as a generalization of the conventional Fourier transform, which was taken up for the benefit of theoretical physics [1,2]. In 1993, Mendlovic and Ozaktas [3,4], and Lohmann [5] introduced the FrFT in to optics and designed optical systems to achieve the FrFT. Though the concept of the Airy wave package goes back 30 years, its counterpart in optics was only demonstrated recently. In 2007, a specific type of non-diffracting beam, namely, a self-accelerating Airy beam (AiB) was introduced to optics [6–8]. AiBs are propagated along parabolic trajectories and are endowed with non-diffracting and self-healing properties [9,10]. In past years, AiBs have been studied intensively both in theory and experiment. Various methods of generating AiBs have been developed, such as the phase plate [11,12] and, liquid crystal cell [13]. Cylindrical lenses [14], three-wave mixing processes [15], microchip laser [16], generation of Airy lights bullets [17,18], femtosecond self-healing Airy pulse [19], Generation of electron Airy beams [20], and Airy plasmon [21–23] have also been reported. The applications of AiBs have also been proposed and demonstrated, including clearing optically mediated particles [24], producing curved plasma channels [25], trapping and guiding microparticles [26,27], electron acceleration driven by two crossed Airy beams [28], and so on. The propagation properties [29] of AiBs have been extensively studied. The different optical systems, such as free space [30], water [31], nonlinear medium [32,33], turbulent atmosphere [34–36], and a four-level electromagnetic induced transparency atomic vapour [37], are studied. The interesting features of an AiB have been shown using the geometric optics method and the Wigner distribution function method [38–40]. Most recently, the propagation dynamics of an OV carried by AiBs was theoretically investigated [41]. Finding the position of the OV by simulation is a way to improve the performance of optical systems i.e., the Fourier transforms system (FFT) or the FrFTs system.

2. An AiB with OV in the FFT system and the FrFT system

The field of the beam with the OV at the plane of $z = 0$ in the Cartesian coordinates system is given by the following Eq. [42]

$$E(x,y) = Ai\left(\frac{x}{x_0}\right)exp\left(\frac{ax}{x_0}\right)Ai\left(\frac{y}{y_0}\right)exp\left(\frac{ay}{y_0}\right)((x-x_d)+i(y-y_d))^l \tag{1}$$

Each OV has a topological charge (l) associated with it. In this equation, for simplicity, we set $l = 1$. The parameter x_0 and y_0 is the transverse scale, and a is the exponential truncation factor; these parameters characterize the width and curvature of the beam. So, we suppose that the initial beam is provisional on the plane with position (x_1, y_1, $z = 0$). The initial beam is propagated through space and becomes a new field (FFT system). It is expressed by using the following integral formula, which is called the Fresnel integral.

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$$\begin{aligned}E(x,y,z) &= \frac{ik}{2\pi z}\iint exp\left(i\frac{k}{2z}\left((x_1-x)^2+(y_1-y)^2\right)\right)E(x_1,y_1)dx_1dy_1\\ &= \frac{ik}{2\pi z}\iint exp\left(i\frac{k}{2z}\left((x_1-x)^2+(y_1-y)^2\right)\right)Ai\left(\frac{x_1}{x_0}\right)Ai\left(\frac{y_1}{y_0}\right)\\ &\quad\times exp\left(\frac{ax_1}{x_0}+\frac{ay_1}{y_0}\right)((x_1-x_d)+i(y_1-y_d))dx_1dy_1\end{aligned} \tag{2}$$

Fig. 1 is the general expression of the field of an AiB with the OV passing through an optical system (the FrFT system). The Lohmann I optical system and the Lohmann II optical system is equivalent, and they are described by the following transfer matrix [5]:

$$\begin{aligned}R &= \begin{pmatrix} A & B\\ C & D\end{pmatrix} = \begin{pmatrix} 1 & d\\ 0 & 1\end{pmatrix}\begin{pmatrix} 1 & 0\\ -1/fsin\alpha & 1\end{pmatrix}\begin{pmatrix} 1 & d\\ 0 & 1\end{pmatrix}\\ &= \begin{pmatrix} cos\alpha & fsin\alpha\\ -1/fsin\alpha & cos\alpha\end{pmatrix}\end{aligned} \tag{3}$$

where $\alpha = \pi p/2$, p is the order of fractional FT and f is the focal length. Therefore, A, B, C, and D are proportional to the value of p. In other words, in an optical system with an *ABCD* matrix (3), the characteristic of the AiB is controllable with the value of p. In Fig. 1(a), the distance between the input and output planes is $2d = 2f\tan(\alpha/2)$. In Fig. 1(b), the distance between the input and output planes is $d' = f\sin(\alpha)$.

The paraxial propagation of the OV in an AiB through an optical *ABCD* system can be determined by the Huygens–Fresnel integral to the following form [43]:

$$\begin{aligned}E(x,y,z) &= \frac{exp(ikz)}{i\lambda B}\times exp\left[\frac{ikD(x^2+y^2)}{2B}\right]\int_{-\infty}^{\infty} Ai\left(\frac{x_1}{x_0}\right)\\ &\quad\times exp\left(\frac{ax_1}{x_0}\right)exp\left[\frac{ik}{2B}(Ax_1{}^2-2x_1x)\right]dx_1\\ &\quad\times\int_{-\infty}^{\infty} Ai\left(\frac{y_1}{y_0}\right)exp\left(\frac{ay_1}{y_0}\right)exp\left[\frac{ik}{2B}(Ay_1{}^2-2y_1y)\right]dy_1\end{aligned} \tag{4}$$

The Airy function in terms of the integral can be written as follows [44]:

$$Ai(x) = \frac{1}{\pi}\int_0^\infty exp\left(i\left(\xi x+\tfrac{1}{3}\xi^3\right)\right)dx \tag{5}$$

On substituting Eqs. (1) and (5) with Eq. (4), we can obtain the field distribution of the OV in an AiB with the form of [45]:

$$E(x,y,z) = \frac{B}{k_0A^2}exp[p(x,y,z)](q_1+q_2+q_3) \tag{6}$$

where

$$\begin{aligned}p(x,y,z) &= \frac{a}{A}\left(\frac{x-2x_m}{x_0}+\frac{y-2y_m}{y_0}\right)+\left(\frac{-ikD}{2B}+\frac{ik}{2BA}\right)(x^2+y^2)\\ &\quad+i\left[\frac{B^3}{12A^3k^3}\left(\frac{1}{x_0^6}+\frac{1}{y_0^6}\right)-\frac{a^2B}{2Ak}\left(\frac{1}{x_0^2}+\frac{1}{y_0^2}\right)-\frac{B}{2A^2k}\left(\frac{x}{x_0^3}+\frac{y}{y_0^3}\right)\right]\end{aligned} \tag{7}$$

$$\begin{aligned}q_1 &= \frac{k}{B}Ai\left(\frac{x-x_m}{Ax_0}-\frac{iaB}{Akx_0^2}\right)Ai\left(\frac{y-y_m}{Ay_0}-\frac{iaB}{Aky_0^2}\right)\\ &\quad\times[x-Ax_d-2x_m+i(y-Ay_d-2y_m)]\end{aligned} \tag{8a}$$

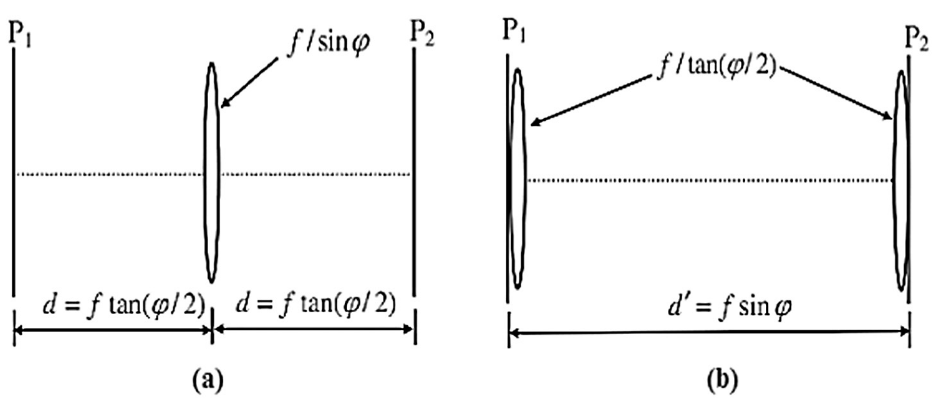

Fig. 1. Optical system for the FrFT. (a) Lohmann I system, (b) Lohmann II system.

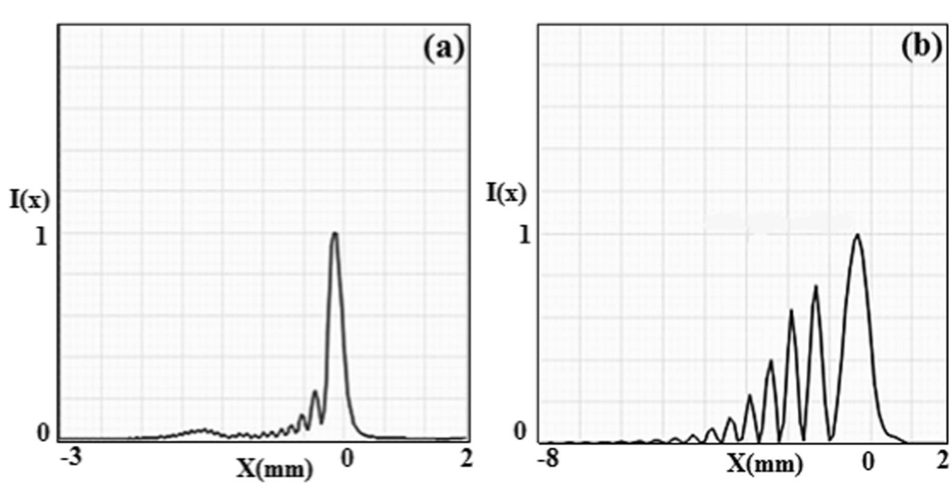

Fig. 2. Intensity normalized in the x- direction of AiB; (a) with OV; (b) without OV.