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Far-field diffraction of linear chirped gratings

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ABSTRACT

We analyze the far-field diffraction pattern produced by linear spatial chirped gratings. An intuitive analytical interpretation of the generated diffraction orders is proposed for gratings with linear variation of the period and linear variation of the spatial frequency. Also, experiments using Gaussian beams and plane wave illumination are performed. The analytical expressions are compared to numerical and experimental results, showing a high agreement. Chirped gratings can be implemented in interesting applications: we analyze how they can be used as a deflector, since tunable direction of diffracted orders can be achieved by displacing laterally the grating with respect to the incident light beam. Also the angular width of diffraction orders can be controlled and chirped gratings can be used to generate uniform illumination over a controlled angular range. These two applications have also been experimentally shown.

1. Introduction

Diffraction gratings are one of the most common optical elements, which consist of a periodical pattern that modulates the incident light beam. As it is well known, a diffraction grating produces diffraction orders at the far field that propagate along directions θ_n given by the grating equation, $p \sin\theta_n = n\lambda$, where λ is the incident wavelength, n is an integer which represents the diffraction order, and p is the period of the grating, which typically is constant. In the recent years, quasi-periodic structures have aroused interest in the optical community [1–10]. For example, in the temporal range, chirped fiber gratings are used as a solution for dispersion compensation [11]. In the spatial range, chirped gratings (CGs) have also been applied to produce curved diffraction orders [12], to extend the bandwidth in surface plasmon applications [13], as spectral selective elements in optical spectrometers and monochromators [8,14], in external cavity semiconductor laser diodes [15,16], and as a nanometer gap measurement device [2]. Their advantages as focusing elements have been applied [4], and they have also been used as reference marks in position optical encoders [17].

From a theoretical point of view, CGs have been analyzed using a geometrical scheme [8], the ABCD matrix formalism [3], and also Fresnel approach for the near field (pseudo-self-imaging formation) [7]. Nevertheless, the far-field optical properties of CGs have not been investigated yet. Since CGs do not present a periodic structure, an analysis based on Fourier series and diffraction orders can only be performed in some particular cases. In this work, we analyze the far-field diffraction pattern produced by CGs of two kinds: p-chirped and q-chirped which correspond to linear variation of the period of the grating and linear variation of the spatial frequency of the grating respectively. Analytical expressions to explain the diffraction orders behavior produced by this kind of non-periodical grating are obtained, which are compared to numerical simulations and experimental results. Simulations based on Fast Fourier Transform and experimental results for q-chirped diffraction gratings are obtained, which corroborates the theoretical approach. Due to the structure of the far field diffraction pattern, CGs can be used in interesting applications, such as a deflector which can easily change the angle of the diffraction orders just moving the grating perpendicularly to the beam, or as a line generator since, for highly chirped gratings, the width of the diffraction orders can be controlled with the initial and final period of the grating.

2. Theoretical analysis

A CG of length L is defined in the real space by its initial and final periods, p_0 and p_1, and its variation rate of the lattice, p_a. Alternatively, in the reciprocal space the CG its defined by its initial and final spatial frequencies, $q_0 = 2\pi/p_0$ and $q_1 = 2\pi/p_1$ and its variation rate, q_a. We define two illustrative cases with linear variation in the real and reciprocal space, the p-chirped and q-chirped gratings

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$$p(x) = p_c + p_a x, \qquad q(x) = q_c + q_a x, \tag{1}$$

where $p_c = (p_0 + p_1)/2, p_a = (p_1 - p_0)/L, q_c = (q_0 + q_1)/2,$ and $q_a = (q_1 - q_0)/L$, respectively. The center of the grating is located, without loss of generality, at $x = 0$. Both gratings are generated by binarizing the sign of

$$t(x) = \text{bin}\left\{\cos\left[\int^x g(x')dx'\right]\right\}, \tag{2}$$

where $g(x')$ is replaced by the frequency for p- or q-CGs. Therefore, the q-CG is defined as

$$t_q(x) = bin\left[\cos\left(\int^x q(x')dx'\right)\right] = bin\left[\cos\left(q_c x + \frac{1}{2}q_a x^2\right)\right] \tag{3}$$

and the p-CG grating is defined as

$$t_p(x) = bin\left[\cos\left(\int^x \frac{2\pi}{p(x')}dx'\right)\right] = bin\left[\cos\left(\frac{2\pi}{p_a}\log(p_c + p_a x)\right)\right]. \tag{4}$$

In Fig. 1 we can see an example of both gratings.

The next step is to analyze how these CGs behave in the far field when they are illuminated with a monochromatic Gaussian light beam whose beam width, ω_0, is placed at the plane of the CG, $z = 0$, and it is centered at $x_g : u_0(x) \propto \exp[-(x - x_g)^2/\omega_0^2]$. Since the proposed p-CGs and q-CGs are not periodical, a simple analysis of the far field diffraction pattern is not possible. Nevertheless, after binarization, both p-chirped and q-chirped diffraction gratings can be described as

$$t_\alpha(x) = \sum_n a_n e^{i n f_\alpha(x)}, \tag{5}$$

where $\alpha = p, q, \quad f_p(x) = \frac{2\pi}{p_a}\log(p_c + p_a x), f_q(x) = q_c x + \frac{1}{2}q_a x^2, a_n$ are the Fourier coefficients of the grating, and n are entire numbers.

On the other hand, the field after the q-CG can be described as

$$u_{1,q}(x) = u_0(x)t_q(x) \propto e^{-\frac{(x-x_g)^2}{\omega_0^2}} \sum_n a_n e^{i n \left(q_c x + \frac{1}{2}q_a x^2\right)}; \tag{6}$$

The far field intensity distribution is obtained using Fraunhofer approximation by solving

$$u_q(\theta) \propto \int_{-\infty}^{+\infty} u_{1,q}(x) e^{-i k x \sin\theta} dx, \tag{7}$$

where $k = 2\pi/\lambda$ is the wavenumber. This integral for q-CGs is easily solved, since there are only linear and quadratic terms in the exponential, resulting

$$u_q(\theta) \propto \sum_n \frac{a_n}{\sqrt{2 - i n q_a \omega_0^2}} e^{-\left(\frac{x_g}{\omega_0}\right)^2} e^{-\frac{\left(k \sin\theta - n q_c + 2i x_g/\omega_0^2\right)^2}{4/\omega_0^2 - 2i n q_a}}; \tag{8}$$

When there is not interference between orders, the intensity distribution at the far field results in $I_\alpha(\theta) = \sum_n u_n(\theta) u_n^*(\theta)$, being u_n the amplitude for each diffraction order. Considering Eq. (8), we obtain

$$I_q(\theta) \propto \sum_n \frac{|a_n|^2}{\omega_q} e^{-\frac{[k \sin\theta - n(q_c + q_a x_g)]^2}{\omega_{n,q}^2}}, \tag{9}$$

where $\omega_{n,q} = \sqrt{2}\sqrt{(1/\omega_0)^2 + (n q_a \omega_0/2)^2}$ is the angular width of the diffraction order n. For the limit $q_a \to 0$ we recover the far field diffraction pattern produced by a periodical grating. Now, let us analyze which are the differences between standard periodic gratings and q-CGs. In the first place, we can see that q-CGs produce diffraction orders which propagate following the grating equation, where the frequency for determining the angular separation of orders is that at the center of the Gaussian beam:

$$k \sin\theta = n(q_c + q_a x_g); \tag{10}$$

These diffraction orders present a total power proportional to $|a_n|^2$. On the other hand, the angular width of diffraction orders, $\omega_{n,q}$, is not constant but it increases with the diffraction order n and the chirping parameter q_a. In Fig. 2a we can see an example of far field diffraction pattern obtained with Eq. (9). It is clear that the width of the diffraction orders is not equal, as for the case of constant period diffraction gratings, but depends on the order. This far field diffraction pattern is compared to that obtained numerically with the Fast Fourier Transform of the field after the grating $|FFT[u_{1,q}(x)]|$. There is an excellent agreement between analytical and numerical approaches.

For the case of p-CGs, that is, gratings with a linear variation in the period, the integral required to determine the far field diffraction pattern

$$u_p(\theta) \propto \sum_n a_n \int_{-\infty}^{+\infty} e^{-\left(x - x_g\right)^2/\omega_0^2} e^{i n \frac{2\pi}{p_a}\log(p_c + p_a x)} e^{-i k x \sin\theta} dx, \tag{11}$$

cannot be solved analytically, and an approximation is required. For this, we have performed a quadratic series expansion of logarithm in second exponential resulting in

$$e^{i n \frac{2\pi}{p_a}\log(p_c + p_a x)} \approx e^{i\frac{2\pi n}{p_a}\left[\log(p_c) + \frac{p_a}{p_c}x - \frac{p_a^2}{2p_c^2}x^2\right]}.$$

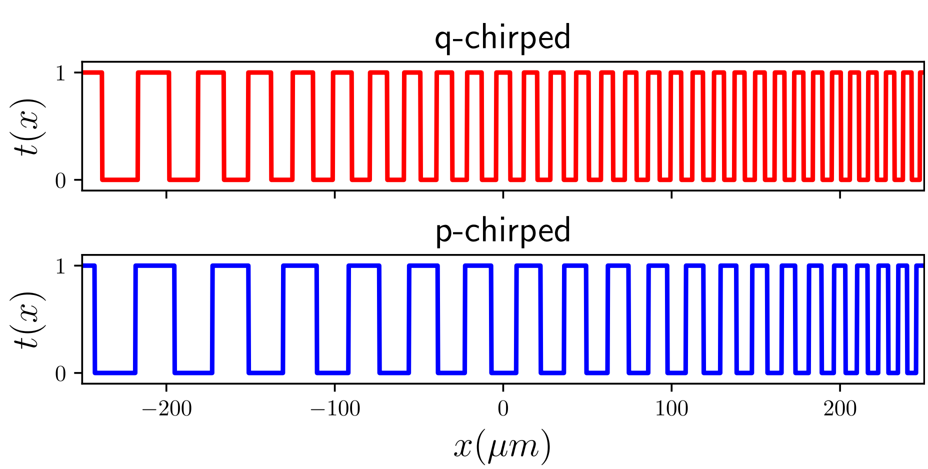

Fig. 1. q- and p-chirped binary amplitude diffraction gratings with starting period $p_0 = 50$ μm, final period $p_1 = 10$ μm, and length $L = 500$ μm.