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Determination of higher order stress terms in cracked Brazilian disc specimen under mode I loading using digital image correlation technique

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The digital image correlation (DIC) technique is used to calculate the coefficients of higher order terms of the Williams' expansion in the centrally-cracked Brazilian disc specimens of different crack lengths under pure mode I loading. The specimens are subjected to diametral-compression loading and the displacement field is obtained for the cracked Brazilian disc by a correlation between the undeformed and deformed images captured before and after loading. The rigid body motion and rotation of each specimen are detected and eliminated from the displacement field by using a code developed based on the Williams' series solution. Then, by employing an over-deterministic system of equations, the coefficients of higher order terms of the Williams expansion are calculated. The same specimens are then simulated using finite element method. It is shown that there is good agreement between the DIC and the finite element results. Therefore, the DIC technique can be proposed as a reliable method to experimentally obtain the mode I stress intensity factor K_I, the T-stress and the coefficients of higher order terms in the Williams' series expansion.

1. Introduction

Brittle fracture is a frequent mode of failure in mechanical components particularly in the presence of sharp cracks. The inelastic zone around the crack tip in brittle fracture is relatively small and the concept of linear elastic fracture mechanics (LEFM) can be applied to investigate the mechanical behavior of the specimens [1,2]. For a cracked specimen subjected to an arbitrary in-plane loading, the Williams' series expansion expresses the elastic stresses σ_x, σ_y and τ_{xy} around the crack tip as [3]:

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix}=\sum_{n=1}^{\infty}\left(\frac{n}{2}A_n r^{\left(\frac{n}{2}-1\right)}\begin{Bmatrix}\left(2+\frac{n}{2}+(-1)^n\right)\cos\left(\frac{n}{2}-1\right)\theta-\left(\frac{n}{2}-1\right)\cos\left(\frac{n}{2}-3\right)\theta\\ \left(2-\frac{n}{2}-(-1)^n\right)\cos\left(\frac{n}{2}-1\right)\theta+\left(\frac{n}{2}-1\right)\cos\left(\frac{n}{2}-3\right)\theta\\ \left(\frac{n}{2}-1\right)\sin\left(\frac{n}{2}-3\right)\theta-\left(\frac{n}{2}+(-1)^n\right)\sin\left(\frac{n}{2}-1\right)\theta\end{Bmatrix}\right)$$
$$-\sum_{n=1}^{\infty}\left(\frac{n}{2}B_n r^{\left(\frac{n}{2}-1\right)}\begin{Bmatrix}\left(2+\frac{n}{2}-(-1)^n\right)\sin\left(\frac{n}{2}-1\right)\theta-\left(\frac{n}{2}-1\right)\sin\left(\frac{n}{2}-3\right)\theta\\ \left(2-\frac{n}{2}+(-1)^n\right)\sin\left(\frac{n}{2}-1\right)\theta+\left(\frac{n}{2}-1\right)\sin\left(\frac{n}{2}-3\right)\theta\\ -\left(\frac{n}{2}-1\right)\cos\left(\frac{n}{2}-3\right)\theta+\left(\frac{n}{2}-(-1)^n\right)\cos\left(\frac{n}{2}-1\right)\theta\end{Bmatrix}\right) \quad (1)$$

where σ_x and σ_y are the components of stress in x and y directions, τ_{xy} is the shear stress, r and θ are the polar coordinates shown in Fig. 1, n is the order of coefficients, and A_n and B_n are the constant coefficients corresponding to mode I and mode II loading, respectively. The mode I and mode II stress intensity factors (K_I and K_{II}) are related to the coefficients of first (or singular) term (i.e. $n = 1$), and the T-stress is related to the second term (A_2) as

$$K_I=\sqrt{2\pi}A_1,\;\; K_{II}=\sqrt{2\pi}B_1,\;\; T=4A_2 \quad (2)$$

Although the stress intensity factors (SIFs) are the most important crack parameters [4–7], some investigations have revealed that the higher-order terms of Williams' expansion can also be of significant contribution in the stress and strain fields near the crack tip. For example, as shown in [8–13] the second term (or the T-stress) affects the angle of fracture initiation and the size and shape of the plastic zone near the crack significantly. It has also been shown that the sign and magnitude of *T*-stress are affected by the material non-homogeneity and anisotropy [14]. There are also some investigations indicating that the T-stress influences the crack tip constraint and crack path stability [15,16]. In addition, Soediono et al. [17], Dyskin [18] and Ayatollahi and Akbardoost [5] demonstrated the importance of the higher-order terms of

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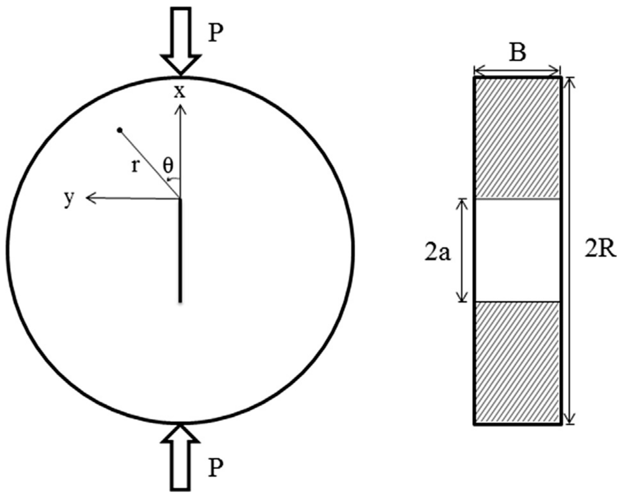

Fig. 1. The cracked Brazilian disc and crack tip polar coordinate system.

Williams' series expansion when exploring size effects on sudden fracture in brittle and quasi-brittle materials. Ayatollahi and Akbardoost [19] have shown that by considering the contribution of higher order terms in the Williams' series expansion, more accurate predictions can be provided for the mode II fracture resistance of rock materials which are known to have large process zone sizes. They suggested a modified form of the maximum tangential stress (MMTS) criterion which took into account the higher order terms of Williams' series expansion in addition to the singular terms. In a different investigation, the same researchers [8] studied the fracture process zone around the crack tip and showed that the MMTS criterion could provide very good prediction for the experimentally obtained fracture data when the higher order terms are used in calculations of the fracture process zone length. Therefore, obtaining the coefficients of higher order terms is important for evaluating the initiation and propagation of cracks in brittle components.

There are different numerical techniques for investigating the mechanical failure in engineering structures and components [20–23]. In particular, finite element (FE) method has been used extensively for calculating the stress intensity factors and the T-stress (e.g. [10,24–27]). For instance, Karihaloo and Xiao [28] made use of higher-order hybrid crack elements and determined the coefficients of first few terms of elastic stresses around the crack tip. Recently, Ayatollahi and Nejati [29] proposed a numerical procedure called finite element over-deterministic (FEOD) method to calculate the coefficients of singular and higher-order terms using the displacement fields obtained from the FE simulation of cracked specimens.

A large number of test specimens have been used in the past for investigating brittle fracture in cracked specimens experimentally. One of these specimens which has simple geometry and loading conditions is the centrally-cracked Brazilian disc. Fig. 1 shows a schematic of the cracked Brazilian disc under pure mode I loading. This specimen has been widely used to study crack extension in brittle and quasi-brittle materials under pure mode I and mixed mode loading, see for example [12,30–32].

In addition to the numerical and analytical methods, some experimental techniques such as strain gauging and optical methods are also available for determining the stress intensity factors and other coefficients of Williams' series expansion [33–35]. Since the strain gauge testing provides very limited test data, this method is not as popular as full-field optical approaches. Meanwhile, within the optical methods, the digital image correlation (DIC) has certain advantages relative to other methods such as the photo-elasticity and Moire interferometry. For example, it has simple setup configuration, is applicable to both transparent and opaque materials, and possesses very good accuracy. As a result, the use of DIC technique has been recently extended in order to obtain displacement field near the crack tip for different geometry and loading conditions [36–42].

In the present paper, the DIC method is used to calculate the coefficients of the first few terms in the Williams' series expansion for the centrally-cracked Brazilian discs of different crack lengths under pure mode I loading. To this end, the full displacement field around the crack tip is first calculated using a 2D-DIC method. Then, the results related to the displacements are employed to determine the coefficients of higher-order terms in the Williams' series expansion using an over-deterministic system of equations and based on the least-squares method. In order to evaluate the accuracy of the experimental results, the same specimens are simulated by the finite element method and the results of the DIC technique are compared with the FE results for Brazilian discs of different crack lengths.

2. DIC technique for determination of coefficients of Williams expansion

As an experimental procedure to determine the displacement field, the DIC technique is based on the difference between the digital images captured before and after the surface deformation. It is generally categorized into the 2D and 3D DIC methods. In 2D-DIC, the displacement field in the plane perpendicular to the optical axis of the camera can be obtained using a single camera. The 3D-DIC method is more complex than the 2D-DIC system as it uses stereo vision systems with additional geometric complexities associated with multi-camera configurations. The use of 3D-DIC method is often recommended for the particular cases where the specimen surface is not flat or there is significant out-of-plane deformation during the loading procedure. The three main sequential steps of this procedure are briefly explained in this section. The first step is the preparation of the specimen. In the correlation procedure, the captured digital images of specimen should have enough contrast. Thus, the surface of specimen is often covered by a white layer and then is sprayed by a black dots pattern. In the next step, the specimen is positioned in the loading fixture in a way its surface be oriented perpendicular to the optical axis of the camera. Then, the reference and deformed images of the specimen are captured before and during the loading process. In this process, the displacement field is determined by comparing the captured images and calculating the amount of motion of subsets, where a subset is an adjustable square region which consists of a number of pixels [43]. In addition to the size of subsets, the step size which refers to the distance between subsets has an important effect on the results of the digital image correlation. To investigate the efficiency of the correlation process, a constant parameter C has been defined in previous research studies (e.g. [41]) based on the contrast of the coordinate of subsets in two images. Eq. (3) expresses this constant as follows:

$$C(x,y,u,v) = \frac{\sum_{i=-n}^{i=n}\sum_{j=-n}^{j=n}\left(I_R(x_p+i,y_p+j) - I_D(x_p+i+u_p,y_p+j+v_p)\right)^2}{\sum_{i=-n}^{i=n}\sum_{j=-n}^{j=n}\left(I_R(x_p+i,y_p+j)\right)^2} \tag{3}$$

where, x_p and y_p are the coordinates of each pixel in the subset in the reference image, and u and v are the displacement components. I_R and I_D refer to the light intensity functions for the undeformed and deformed images, respectively. In Eq. (3), $n = 0.5 \times (m-1)$, where m is the subset size. In general, the coefficient C should be optimized to have a displacement field with a high accuracy.