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Diffraction by a radial phase modulated spiral zone plate of abruptly autofocusing beams generated with multiple Bessel-like beams

Faroq Saad, El Mostafa El Halba, Abdelmajid Belafhal *

Laboratory of Nuclear, Atomic and Molecular Physics, Department of Physics, Faculty of Sciences, Chouaïb Doukkali University, P.B. 20, 24000 El Jadida, Morocco

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ABSTRACT

In this paper, we present the generation of optical vortices using an abruptly autofocusing beam that is combined from multiple Bessel-like beams, diffracted by a radial phase shift modulated spiral zone plate (RSSZP). The analytical formula for the diffracted wave field amplitude of the produced beam is derived based on the Fresnel diffraction integral formula. The results show that the intensity distributions of the produced output beam in radial direction are determined by the initial beam parameters and the RSSZP parameters, such as shifting parameter and topological charge. The present work gives also the diffraction by a RSSZP of a fundamental Gaussian beam which is deduced as a particular case in this investigation.

1. Introduction

In the last two decades, optical vortex laser beams with a ring-shaped intensity profile have attracted much attention [1–5], because they can possess an orbital angular momentum of $\pm l\hbar$ for each photon corresponding to the azimuthal phase factor, $exp(\pm il\varphi)$, where l denotes the topological charge. Due to of this unique property, these beams have a lot of potential applications in a number of specialized areas such as optical communications, optical tweezers, optical trapping and in the manipulation of small particles [6–8]. Therefore, many efforts have been devoted to generate the optical vortex laser beam [9–20]. Khonina et al., have studied a possibility to produce a vortex laser beam using phase spiral plate (SPP) and helical axicon [9,10]. The conversion of the Gaussian laser beam by a fork-shaped hologram into optical vortex beam has been investigated [11,12]. Recently, much work have been reported to study the generation of optical vortices and generalized spiraling Bessel beams by use of the helical axicon and of the curved fork shaped hologram [13–20].

On the other hand, a laser beam with phase singularities can be possessed in the simplest case the form of spiral Fresnel zone plate which is able to generate a phase front containing screw dislocations known as optical vortex [21]. In 2014, Sabatyan and Meshginqalam [22] have introduced a type of Fresnel zone plate in which its phase is shifted radially and results in an annular beam at the focal plane. The produced element is called a RSSZP that created by modulating of both a radial phase shift and a spiral zone plate phase. Other type of square array of optical vortices can be generated by multiregion spiral square zone plate (MRSSZP), has been presented [23]. More recently, Sabatyan and Behjat [24] have proposed a novel diffractive element to produce a perfect vortex beam by using a RSSZP.

In the present work, we will explore and investigate the generation of optical vortices which created by illuminating a RSSZP with abruptly autofocusing beams (AABs). However, to the best of our knowledge, the diffraction by a RSSZP of AABs hasn't been investigated yet. The rest parts of the paper are organized as follows: In Section 2, we present the definition of an input field distribution and intensity profile of AABs. The intensity distribution of AABs diffracted by a RSSZP, expressed in an analytical formula by using the Fresnel diffraction integral formula, will be established in Section 3. In order to illustrate the obtained results and analyzing the effect of some parameters on propagation properties of produced output beam, some numerical calculations are presented in Section 4. Finally, conclusions of the present work are presented in Section 5.

* Corresponding author.
E-mail address: belafhal@gmail.com (A. Belafhal).

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2. Incident field distribution of AABs

Based on Ref. [25], an abruptly autofocusing beam (AABs) that is combined from multiple Bessel-like beams can be described mathematically as

$$U(x,y,0) = A_1 exp(iQ_1(x,y)) + A_2 exp(iQ_2(x,y)), \quad (1)$$

where

$A_1 = exp\left(-\left(x^2 + (y+a)^2\right)/w\right), A_2 = exp\left(-\left(x^2 + (y-a)^2\right)/w\right)$ is the Gaussian beam with a size w and a beam center $(0, -a)$ and $(0, a)$. Q_1 is the corresponding phase for A_1. To increase the local intensity density of the focal point, the combined autofocusing beam with number n (1000 or even more) component Bessel-like beams can be described as follows

$$\begin{aligned} U(r,0) &= A_1 exp(iQ_1(x,y)) + A_2 exp(iQ_2(x,y)) + A_3 exp(iQ_3(x,y)) \\ &\quad + \ldots + A_n exp(iQ_n(x,y)) \\ &= \sum_{\theta=2\pi/n}^{2\pi} exp\left(-\frac{(x-acos\theta)^2+(y-asin\theta)^2}{w}\right) exp(iQ_\theta(x,y)). \end{aligned} \quad (2)$$

If $n \to \infty$, and by introducing polar coordinates as $x = rcos\theta$ and $y = rsin\theta$, the initial beam becomes [25]

$$U(r,0) = exp\left(-\frac{(r-a)^2}{w}\right) exp(iQ(r-a)), \quad (3)$$

Fig. 1, displays the incident intensity distribution of AABs for different values of the initial coordinate parameter a (=0, 0.75 mm, 1.25 mm, and 2.27 mm), which simulated from Eq. (3).

It can be seen from this figure that, if $a = 0$, the intensity profile reduces to the case of a fundamental Gaussian beam (Fig. 1a). While, for the case of nonzero of parameter a, the radius of dark ring at the center appears and increases with a increases (see Fig. 1(b–d)).

3. Mathematical description of AABs diffracted by RSSZP

RSSZP is a well-known diffracted element that is produced by the modulation of the spiral Fresnel zone plate and the radial phase shift. The RSSZP's transmittance function $t(r,\phi)$is defined by the following expression [24].

$$t(r, \varphi) = exp\left(-i\frac{\pi(r-\alpha R)^2}{\lambda f} + ip\varphi\right), \quad (4)$$

where (r, φ) are polar coordinates, λ is the wavelength of the incident beam, R and f are the radius and the focal length of the element. p is the topological charge and α is the shifting parameter. Figs. 2 and 3 show the geometry of RSSZP. In Fig. 2, the parameters used in the simulation are: $p = 2$ and three values of $\alpha = 0.05$, 0.1 and 0.2. The other parameters are set as $\lambda = 632.8$ nm, $f = 500$ mm and R = 6 mm. From Fig. 2, when the increase of shifting parameter α,the phase distribution of the RSSZP changes and becomes more observable. Fig. 3 shows the influence of the topological charge on the phase structure of the RSSZP. However, from this figure, we can see that, when the topological charge is equal to zero, the produced element corresponds to a simple radially phase shifted zone plate. Moreover, the number of spiral arms increases at the center with increasing p (see Fig. 3(b-d)).

On the other hand and in order to study the generation of optical vortices created by the illumination of AABs with the RSSZP, the field distribution situated at a distance z from the RSSZP plane, can be described by the Fresnel-Kirchhoff integral as follows [26]

$$\begin{aligned} U(\rho,\varphi,z) &= \frac{ik}{2\pi z} exp\left[-ik\left(z + \frac{\rho^2}{2z}\right)\right] \int_0^\infty \int_0^{2\pi} t(r,\varphi)\, U(r,0) \\ &\quad \times exp\left[-i\frac{k}{2}\left(\frac{r^2}{z} - \frac{2r\rho cos(\varphi-\theta)}{z}\right)\right] rdrd\varphi, \end{aligned} \quad (5)$$

By substituting Eqs. (3) and (4) into Eq. (5), the field distribution can be written as

$$\begin{aligned} U(\rho,\theta,z) &= \frac{ik}{2\pi z} exp\left[-ik\left(z + \frac{\rho^2}{2z}\right)\right] \int_0^\infty \int_0^{2\pi} exp\left(-i\frac{\pi(r-\alpha R)^2}{\lambda f} + ip\varphi\right) \\ &\quad \times exp\left(\frac{-(r-a)^2}{w}\right) exp(iQ(r-a)) \\ &\quad \times exp\left[-i\frac{k}{2}\left(\frac{r^2}{z} - \frac{2r\rho cos(\varphi-\theta)}{z}\right)\right] rdrd\varphi. \end{aligned} \quad (6)$$

We recall the following integral formulae [27]

$$\int_0^{2\pi} exp[ip\varphi + ixcos(\varphi-\theta)]d\varphi = 2\pi\, i^p exp(ip\theta) J_p(x), \quad (7)$$

where J_m is the m-th order Bessel function of the first kind and after integration over the azimuthal variable φ, Eq. (6) becomes

$$\begin{aligned} U(\rho,\theta,z) &= \frac{i^{p+1}k}{z} exp\left[-ik\left(z + \frac{\rho^2}{2z} + \frac{\alpha^2R^2}{2f}\right)\right] exp\left[-\left(\frac{a^2}{w} + iQa\right)\right] e^{ip\theta} \\ &\quad \times \int_0^\infty exp(-\beta r^2) J_p\left(\frac{k\rho}{z} r\right) exp(\gamma r) rdr, \end{aligned} \quad (8)$$

where

$\beta = \frac{1}{w} + \frac{ik}{2}\left(\frac{1}{f} + \frac{1}{z}\right)$ and $\gamma = \frac{2a}{w} + iQ + \frac{ik\alpha R}{f}$.

The radial integral of Eq. (8) can be solved analytically by using Taylor series as [27]

$$exp(\gamma r) = \sum_{n=0}^\infty \frac{(\gamma r)^n}{n!}. \quad (9)$$

Then substituting this later relation into Eq. (8), this last equation reduces to

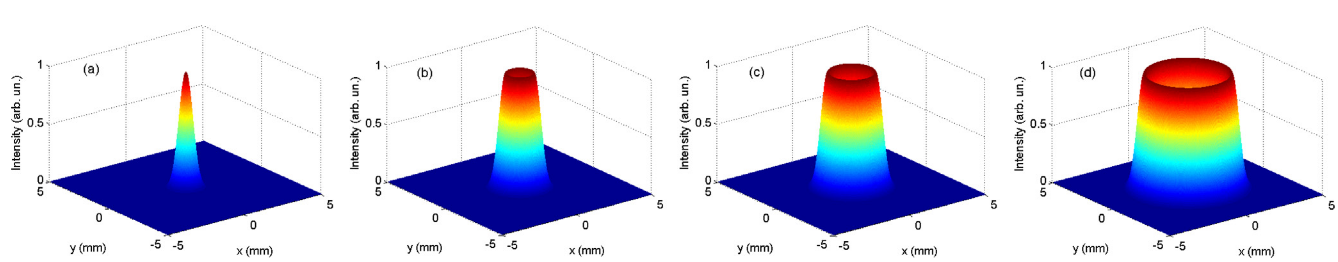

Fig. 1. 3D intensity distribution of AABs at the source plane $z = 0$, for $w = 0.5$ mm, with various initial coordinate: (a) $a = 0$, (b) $a = 0.75$ mm, (c) $a = 1.25$ mm and (d) $a = 2.27$ mm.