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Quaternion generalized Chebyshev-Fourier and pseudo-Jacobi-Fourier moments for color object recognition

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ABSTRACT

In this paper, the classical generalized Chebyshev-Fourier moments (G-CHFMs) and generalized pseudo–Jacobi-Fourier moments (G-PJFMs) have been extended to represent color images using quaternion algebra. The proposed quaternion G-CHFMs (QG-CHFMs) and quaternion G-PJFMs (QG-PJFMs) are characterized by a parameter α, called free parameter, which distinguishes them from the conventional Chebyshev-Fourier moments (CHFMs) and pseudo-Jacobi-Fourier moments (PJFMs). All these moments are rotation-invariant and orthogonal. The effect of the parameter α on image reconstruction and object recognition is studied in detail and its optimal values have been obtained for these two image processing tasks. It is shown that the choice of α influences significantly the image reconstruction capability and the object recognition performance of the proposed QG-CHFMs and QG-PJFMs moments. Extensive experiments are conducted to demonstrate the behavior of these moments on image reconstruction and object recognition under normal condition and under rotation, scaling, and noise using COIL-100, SIMPLIcity and Corel datasets of color objects.

1. Introduction

Moments and moment invariants are among the most popular global shape descriptors which have been most widely used in the applications related to digital image processing such as object classification [1], image watermarking [2], face recognition [3], image segmentation [4], image denoising [5], image retrieval [6], image reconstruction [7], image super-resolution [8], etc. It is not an easy task to process an image for a specific application as the image may undergo various geometric and photometric transformations during image acquisition such as the rotation, scaling, translation, partial occlusion, and change in the illumination. Therefore, an image descriptor which is invariant to these changes is of great need. Moment invariants are the powerful and frequently used image descriptors which are robust to these geometric and photometric changes. Moments are broadly classified into orthogonal and non-orthogonal moments. Among these two categories, the orthogonal moments are most widely used in pattern recognition and computer vision applications because of their minimum information redundancy property. The class of orthogonal moments can be further categorized into continuous and discrete moments. Continuous moments provide an infinite number of moment coefficients which are beneficial for describing image details more accurately than their discrete counterparts which have a finite number of moments – as many as the size of the image. Some of the most widely used continuous orthogonal moments are the Zernike moments (ZMs) [9], pseudo-Zernike moments (PZMs) [10], orthogonal Fourier-Mellin moments (OFMMs) [11], Bessel-Fourier moments (BFMs) [12], Chebyshev-Fourier moments (CFMs) [13], pseudo-Jacobi-Fourier moments (PJFMs) [14], etc. The magnitude of continuous orthogonal moments is rotation invariant due to their circular nature. Another class of orthogonal moments is discrete orthogonal moments which are defined in Cartesian coordinate space. Some of the most popular discrete orthogonal moments are Tchebichef moments (TMs) [15], Krawtchouk moments (KMs) [16], Racah moments (RMs) [17], Dual Hahn moments (HMs) [18], etc.

The above-mentioned continuous orthogonal and discrete orthogonal moments were originally introduced for gray-scale images. Nowadays, due to the advancement in technology, almost all the images captured are chromatic. There are two different kinds of approaches to process a color image using conventional moments. The first approach is to convert a color image into gray-scale image and then compute the moments. The second approach is to compute the moments of each color channel (say red, green, and blue) of a color image independently. However, in the recent past, quaternion algebra has been employed to various

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conventional moments to analyze a color image holistically. Guo and Zhu [19] introduced quaternion Fourier-Mellin moments (QFMMs) which are an extension of the conventional Fourier-Mellin moments for the gray-scale image. They also proposed their invariants on rotation, scale, and translation for color object recognition. Chen et al. [20] proposed the quaternion Zernike moments (QZMs) and their invariants for the application of color object recognition and template matching. Li [21] extended the conventional polar harmonic transforms (PHTs) such as polar exponential transform (PET), polar cosine transform (PCT), and polar sine transform (PST) to quaternion PET (QPET), quaternion PCT (QPCT), and quaternion PST (QPST) with the help of quaternion algebra. Guo et al. [22] have developed the quaternion complex moments (QCMs) which is an extension of complex moments (CMs) defined in Cartesian space. The main advantage of CMs over other orthogonal rotation invariant moments (ORIMs) such as ZMs, PZMs, OFMMs, etc., is their computational efficiency. Shao et al. [23] developed the quaternion Bessel-Fourier moments (QBFMs) and investigated their invariants based on magnitude and phase information for color object recognition. The kernel function of polar complex exponential transform (PCETs) is simple and fast to compute and also free from higher order factorial terms. These merits of PCETs led Wang et al. [24] to propose the quaternion PCET (QPCETs) and their invariants for the application of color image retrieval. Li et al. [25] proposed the quaternion generic Fourier descriptor and their invariants to analyze geometric transformations and the effect of noise for color image retrieval and face recognition. Based on the radial harmonic Fourier moments (RHFMs), Wang et al. [26] proposed quaternion RHFMs (QRHFMs) and their invariants to analyze a color image holistically. Also, some of the other moments such as the rotational moments, radial moments, OFMMs, PZMs have been extended to their quaternion versions for color image analysis [27]. Karakasis et al. [28] developed a unified scheme for polar, radial, and discrete moments to accurately compute quaternion and quaternion invariants on translation, scale, and rotation.

Recently, Zhu et al. [29] have developed two continuous generalized moments which are orthogonal over the unit disk - the generalized Chebyshev-Fourier moments (G-CHFMs) and generalized pseudo-Jacobi-Fourier moments (G-PJFMs). The generalization has been achieved by introducing a free real parameter $\alpha > -1$. The classical CHFMs and PJFMs are obtained as special cases of these moments by letting $\alpha = 0$. The parameter α controls the distribution of zeros of the radial polynomials of the moments about the r - *axis*. This property of these polynomials enhances the information packing capacity of the moments in the low order moment coefficients which is reflected in low mean-square error (MSE) between the original image and reconstructed image when only a few moments of low order are used. There is significant enhancement in the performance of G-CHFMs and G-PJFMs for high values of the parameter α as compared to several known orthogonal moments belonging to families of these moments such as the ZMs, PZMs, OFMMs, CHFMs, JFMs, etc.

In this paper, we introduce quaternion forms of the G-CHFMs and G-PJFMs, namely QG-CHFMs and QG-PJFMs, to represent color images using these two families of moments and analyze the effect of the free parameter α on the performance of the quaternion moments. As discussed above, a quaternion representation of a vector quantity enables the interaction among the various components of a vector, the behavior of the free parameter α will determine how one can achieve the best performance of QG-CHFMs and QG-PJFMs in the task of color object recognition.

The rest of the paper is organized as follows. An overview of G-CHFMs and G-PJFMs is discussed in Section 2. In Section 3, we present the definition of quaternion generalized Chebyshev-Fourier and quaternion pseudo-Jacobi-Fourier moments. The invariants on translation, rotation and scale are given in Section 4. In Section 5, a detailed experimental analysis is performed on COIL-100, SIMPLIcity, and Corel datasets under normal, rotation, scale and noisy conditions. Section 6 concludes the paper.

2. Generalized Chebyshev-Fourier and generalized pseudo-Jacobi-Fourier polynomials and moments

2.1. Generalized radial polynomial in shifted Jacobi form

The generalized Chebyshev-Fourier polynomials (G-CHFPs) $C_n^\alpha(r)$ and generalized pseudo-Jacobi-Fourier polynomials (G-PJFPs) $J_n^\alpha(r)$ of order n and with free parameter α, in the shifted form of Jacobi polynomials are defined as follows [29]:

$$C_n^\alpha(r) = (-1)^n\left[\frac{64(1-r)}{\pi^2 r}\right]^{\frac{1}{4}}(n+1) \frac{n!\Gamma(3/2)}{(\alpha+n+1)!}\sum_{s=0}^{n}\frac{(-1)^s(\alpha+n+s+1)!}{(n-s)!s!\Gamma(s+3/2)}r^s, \tag{1}$$

and

$$J_n^\alpha(r) = (-1)^n\left[\frac{(n+1)(n+2)^3(n+3)(r-r^2)}{2}\right]^{1/2} \frac{n!2!}{(\alpha+n+3)!}\sum_{s=0}^{n}\frac{(-1)^s(\alpha+n+s+3)!}{(n-s)!s!(s+2)!}r^s, \tag{2}$$

respectively.

The normalized G-CHFP and G-PJFP are given as:

$$\widetilde{C}_n^\alpha(r) = C_n^\alpha(r)\sqrt{\frac{(2n+\alpha+2)n!(n+\alpha+1)!\Gamma(n+3/2)(1-r)^\alpha}{4\pi(n+\alpha+\frac{1}{2})!\Gamma^2(n+2)}}, \tag{3}$$

and

$$\widetilde{J}_n^\alpha(r) = J_n^\alpha(r)\sqrt{\frac{(2n+\alpha+4)(n+\alpha+3)(n+\alpha+2)(1-r)^\alpha}{4\pi(n+3)(n+2)^2}}, \tag{4}$$

respectively. Figs. 1 and 2 show the distribution of zeros up to order 5 for $\widetilde{C}_n^\alpha(r)$ given by Eq. (3) and $\widetilde{J}_n^\alpha(r)/2$ given by Eq. (4) with different choices of free parameter α.

2.2. Generalized Chebyshev-Fourier moments (G-CHFMs) and pseudo-Jacobi-Fourier moments (G-PJFMs)

The general expression for continuous orthogonal moments of order n and repetition m over a unit disk for an image function $f(r,\theta)$ in polar form is given as follows:

$$Z_{nm}^\alpha(f) = \int_0^{2\pi}\int_0^1 f(r,\theta)[\overline{D}_{nm}^\alpha(r,\theta)]^* r dr d\theta, \tag{5}$$

where $n \geqslant 0,\ |m| \geqslant 0,\ j = \sqrt{-1}$, and $[\overline{D}_{nm}^\alpha(r,\theta)]^*$ is the complex conjugate of $[\overline{D}_{nm}^\alpha(r,\theta)]$.

The function $\overline{D}_{nm}^\alpha(r,\theta)$ is replaced by $\overline{C}_{nm}^\alpha(r,\theta)$ and $\overline{J}_{nm}^\alpha(r,\theta)$ for G-CHFMs and G-PJFMs, respectively. The function $\overline{D}_{nm}^\alpha(r,\theta)$ is a separable function of r and θ which can be expressed as:

$$[\overline{D}_{nm}^\alpha(r,\theta)] = \widetilde{D}_n^\alpha(r)\exp(jm\theta), \tag{6}$$

where $r = \sqrt{x^2+y^2},\ \theta = \tan^{-1}(y/x),\ \theta\epsilon[0,2\pi]$.

The G-CHFPs $\overline{C}_{nm}^\alpha(r,\theta)$ and G-PJFPs $\overline{J}_{nm}^\alpha(r,\theta)$ are orthogonal over the unit disk: