Optics and Laser Technology 105 (2018) 23–34

Contents lists available at ScienceDirect

Optics and Laser Technology

journal homepage: www.elsevier.com/locate/optlastec

Full length article

The Poynting vector and angular momentum density of the autofocusing Butterfly-Gauss beams

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ARTICLE INFO

Article history:
Received 1 July 2017
Received in revised form 23 November 2017
Accepted 15 February 2018

Keywords:
Butterfly-Gauss beam
Autofocusing
Poynting vector
Angular momentum density
Chiral medium

ABSTRACT

Three different Butterfly-Gauss beams are proposed by introducing higher order Butterfly catastrophe to the field of optics, where an autofocusing Butterfly-Gauss beam with a special profile is emphatically studied and it can be regarded as the sum of two Half-Butterfly-Gauss beams. Based on the Collins integral formula, the autofocusing behavior, Poynting vector and angular momentum density of the corresponding Butterfly-Gauss beams during propagation in free space, focus system and chiral medium are investigated, respectively. The results show that the Butterfly-Gauss beam not only exhibits autofocusing behavior similar to the Pearcey beams or circular Airy beams, but also presents rotation in analogy with the Swallowtail beam during propagation in free space. In the focus system, the tail directions of special patterns in the focal plane can be controlled by scaling lengths. In the chiral medium, the greater chirality parameters correspond to the increasing of phase velocity of beam, which results in the fact that the distance of autofocusing plane becomes smaller, and it is easier to autofocus the beam. The proposed Butterfly-Gauss beams have the application possibility in the field of manipulating microparticles along intensity channels due to their special spatial structures in focal plane, where the Poynting vectors flow from center spot to the controlled tail.

1. Introduction

Much interest has been exhibited recently in some laser beams with the catastrophe function from both theoretical aspects and applications [1–7]. In general, the catastrophe function can be described by seven elementary catastrophes, i.e. fold, cusp, swallowtail, butterfly, hyperbolic umbilic, elliptic umbilic and parabolic umbilic catastrophes, which obey the following equation [6,8]

$$C_n(a_1, a_2, \cdots, a_{n-1}) = \int_{-\infty}^{\infty} \exp[iP_n(a_1, a_2, \cdots, a_{n-1}; u)]du. \tag{1}$$

Here the canonical potential function

$$P_n(a_1, a_2, \cdots, a_{n-1}; u) = u^{n+1} + \sum_{j=1}^{n-1} a_j u^j \tag{2}$$

with the integer $n \geq 2$. The canonical potential function $P_n(a, u)$ plays important role in dynamics propagation of the corresponding beams. For example, for the case of $n = 2$ Eq. (1) corresponds to the fold catastrophe, which is proportional to the well-known Airy function. By introducing the Airy function into the field of optics [1,2], the finite-energy Airy beam truncated by exponential decaying presents many intriguing properties such as parabolic trajectory, self-healing, nondiffraction, sorting microparticles and electron acceleration [9–12]. For the cusp catastrophe of $n = 3$, the Pearcey-Gauss beams exhibits the autofocusing, self-healing and form-invariance during propagation [3]. Moreover, the half Pearcey beams [4] and dual Pearcey beams [5] are studied by Kovalev et al. and Ren et al., respectively. For $n = 4$, the Swallowtail-Gauss beam shows the rotation of main-lobe and the potential applications in manipulating microparticles, but which does not display autofocusing behavior during propagation [7].

When $n = 5$, the Eq. (1) becomes the Butterfly catastrophe, where we shall henceforth write it in the following form

$$Bu(a_1, a_2, a_3, a_4) = \int_{-\infty}^{\infty} \exp[i(u^6 + a_4u^4 + a_3u^3 + a_2u^2 + a_1u)]du. \tag{3}$$

Here the higher hierarchy Butterfly catastrophe contains four control parameters, i.e., a_1, a_2, a_3 and a_4, and any two parameters can be set as transverse spatial coordinates (x, y). Can the higher optical catastrophe beams present the similar propagation dynamics of the lower catastrophe beams such as the Airy, Pearcey and Swallowtail beams? The motivation of the present paper is to

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https://doi.org/10.1016/j.optlastec.2018.02.029
0030-3992/

study the autofocusing behavior, Poynting vector and angular momentum density of the finite-energy Butterfly-Gauss (BuG) beam passing through free space, focal system and chiral medium, where two Half-Butterfly-Gauss beams are also dealt with. The results obtained in this paper are different to the propagation-invariance of Pearcey beams, which are also useful for studying the autofocusing behavior and microparticles manipulation in special spatial structures. The paper is organized as follows. Sections 2 gives analytical propagation expressions of three different BuG beams, respectively, where an autofocusing BuG beam in the case of $(a_1, a_3) = (x/x_0, y/y_0)$ is emphatically studied. The evolution of the Poynting vector and angular momentum density for the autofocusing BuG beam propagation in different media is analyzed in Sections 3 and 4, respectively, and the conclusions follow in Section 5.

2. Three different Butterfly-Gauss beams

The Butterfly catastrophe possesses four control parameters and any two parameters can be selected as the transverse spatial coordinates in optics. In this section we mainly study the following three different cases, i.e. $(a_1 = x/x_0,\ a_3 = y/y_0)$, $(a_1 = x/x_0,\ a_2 = y/y_0)$ and $(a_2 = x/x_0,\ a_3 = y/y_0)$.

2.1. A special type of autofocusing Butterfly-Gauss beams

Assume that the control parameters of the Butterfly catastrophe in Eq. (3) are set as $a_1 = x/x_0$, $a_2 = 0$, $a_3 = y/y_0$ and $a_4 = 0$, respectively. By introducing this catastrophe to optics, the initial field of the Butterfly beam in the Cartesian coordinate system can be written as [6,8]

$$Bu(x/x_0,0,y/y_0,0) = \int_{-\infty}^{\infty} \exp\left\{i\left[u^6 + \left(\frac{y}{y_0}\right)u^3 + \left(\frac{x}{x_0}\right)u\right]\right\}du = HBu_-(x/x_0,0,y/y_0,0) + HBu_+(x/x_0,0,y/y_0,0), \quad (4)$$

where x_0 and y_0 are scaling lengths along x- and y-axes, respectively. The two Half-Butterfly catastrophe in Eq. (4) are given by

$$HBu_-(x/x_0,0,y/y_0,0) = \int_{-\infty}^{0} \exp\left\{i\left[u^6 + \left(\frac{y}{y_0}\right)u^3 + \left(\frac{x}{x_0}\right)u\right]\right\}du, \quad (5)$$

$$HBu_+(x/x_0,0,y/y_0,0) = \int_{0}^{\infty} \exp\left\{i\left[u^6 + \left(\frac{y}{y_0}\right)u^3 + \left(\frac{x}{x_0}\right)u\right]\right\}du, \quad (6)$$

respectively. The integrals in Eqs. (4)–(6) can be calculated numerically by using the contour integral method in complex u plane [13], which make the integrand along the real axis rapid convergence with no violent oscillation by transforming the variable u into $u'\exp(i\pi/12)$ (see Figs. S1 and S2 of Supplemental materials).

The Fourier transform of the Butterfly beam in Eq. (4) can be expressed as

$$\tilde{B}u(k_x,k_y) = \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} Bu(x/x_0,0,y/y_0,0)\exp[-i(k_x x + k_y y)]dxdy = x_0 y_0 \delta(k_x^3 x_0^3 - k_y y_0)\exp(ik_x^6 x_0^6), \quad (7)$$

where k_x and k_y are the Fourier transform pairs, δ denotes the Dirac delta-function. Eq. (7) indicates that the amplitude of Fourier spectrum of the Butterfly beam is concentrated by δ-lines with power function $k_y y_0 = k_x{}^3 x_0^3$, which is different to Gaussian distributions of Airy beams [1] and parabolic curves of Pearcey beams [3].

Based on the Huygens-Fresnel diffraction integrals, the propagating expression of the Butterfly beam in free space can be obtained by

$$Bu(x,y,z) = \frac{k}{i2\pi z}\iint_{\infty} Bu(x_1/x_0,0,y_1/y_0,0) \exp\left\{\frac{ik}{2z}[(x-x_1)^2 + (y-y_1)^2]\right\}dx_1 dy_1 = (1 - z/z_e)^{-\frac{1}{6}} Bu\left(\frac{x}{x_0}(1-z/z_e)^{-\frac{1}{6}}, -\frac{z}{2kx_0^2}(1-z/z_e)^{-\frac{1}{3}}, \frac{y}{y_0}(1-z/z_e)^{-\frac{1}{2}}, 0\right), \quad (8)$$

where $z_e \equiv 2ky_0^2$ and the wave number $k = 2\pi/\lambda$ with wavelength λ. Eq. (8) indicates that the beam's shape changes with the evolution of z due to the increasing of control parameters from two to three. Furthermore, from Eq. (8) one can find that there is a singularity at $z = z_e$ by view of mathematics, which leads to the fact that the corresponding Butterfly beam gradually approaches infinity with $z \to z_e$. Similar to the Pearcey beam [3], the phenomenon can be considered as the autofocusing behavior owing to the existence of a real-valued singularity in the z-axis. Fig. 1 shows the intensity evolution in x- and y-directions of the Butterfly beam during propagation, respectively. There is the autofocusing behavior with the autofocusing plane of $z = z_e$ in y-direction as z increases, although the autofocusing in the x-direction is not found.

On the other hand, the amplitude of Fourier spectrum of Butterfly beam is determined by Dirac δ-function, which results in the infinite energy of the pure Butterfly beam. In order to ensure the finite energy in physical realization, a Butterfly-Gauss (BuG) beam is proposed, namely, the initial field of Butterfly beam at $z = 0$ modulated by Gaussian factors, which retains the autofocusing behavior while limiting its energy.

For the case of $a_1 = x/x_0$, $a_3 = y/y_0$, in a Cartesian coordinate system the initial field of the BuG beam at $z = 0$ is expressed by

$$BuG_1(x,y,z=0) = \exp\left[-\frac{x^2+y^2}{w_0^2}\right]Bu(x/x_0,0,y/y_0,0), \quad (9)$$

where w_0 is the waist width of Gaussian factor and $Bu(x/x_0, 0, y/y_0, 0)$ is defined by Eq. (4).

The Fourier spectrum of the BuG beams at $z = 0$ is also given by

$$\tilde{B}uG_1(k_x,k_y) = \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} BuG_1(x,y,z=0) \exp[-i(k_x x + k_y y)]dxdy = H\tilde{B}uG_-(k_x,k_y) + H\tilde{B}uG_+(k_x,k_y). \quad (10)$$

Here the Fourier spectrums of two Half-Butterfly-Gauss beams are given by

$$H\tilde{B}uG_\pm(k_x,k_y) = \frac{w_0^4}{4\pi}\exp[-w_0^2(k_x^2+k_y^2)/4]\left(1+\frac{iw_0^2}{4y_0^2}\right)^{-\frac{1}{6}} HBu_\pm(\alpha,\beta,\gamma,0), \quad (11)$$

where the term of $HBu_\pm(\alpha, \beta, \gamma, 0)$ are determined by Eqs. (5) and (6) with

$$\alpha = \frac{-iw_0^2 k_x}{2x_0}\left(1+\frac{iw_0^2}{4y_0^2}\right)^{-\frac{1}{6}}, \beta = \frac{iw_0^2}{4x_0^2}\left(1+\frac{iw_0^2}{4y_0^2}\right)^{-\frac{1}{3}}, \gamma = \frac{-iw_0^2 k_y}{2y_0}\left(1+\frac{iw_0^2}{4y_0^2}\right)^{-\frac{1}{2}}. \quad (12)$$

From Eq. (11), one can see that the energy of the BuG beams is obviously constrained by the Gaussian spectrum of $\exp[-w_0^2(k_x^2+k_y^2)/4]$, and the term of $HBu_\pm(\alpha, \beta, \gamma, 0)$ modulate both the amplitude and phase of the Fourier spectrums whilst