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Evolution properties of a radial phased-locked partially coherent Lorentz-Gauss array beam in oceanic turbulence

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ABSTRACT

The concept of a radial phased-locked partially coherent Lorentz-Gauss array (PCLGA) beam generated by a Schell-model source has been introduced. Based on the Huygens-Fresnel principle, the analytical expressions for the cross-spectral density function of a radial phased-locked PCLGA beam propagation in oceanic turbulence has been derived. The average intensity and coherence properties of a radial phased-locked PCLGA beam propagating in oceanic turbulence are illustrated using numerical examples. The influences of coherence length and oceanic turbulence on the evolution properties and spectral degree of coherence are analyzed in details. The results show that radial phased-locked PCLGA beam propagation in stronger oceanic turbulence will evolve into Gaussian-like beam more rapidly as the propagation distance increases.

1. Introduction

In the past decades, the propagation of laser beam in oceanic turbulence has attracted high interest due to the potential application in underwater wireless optical communication and remote sensing. Up to now, based on the extended Huygens-Fresnel principle, the evolution properties of various laser beam propagating in oceanic turbulence have been widely illustrated, such as those of partially coherent beam [1], partially coherent radially polarized doughnut beam [2], Gaussian Schell-model vortex beam [3], stochastic electromagnetic vortex beam [4], partially coherent Hermite-Gaussian linear array beam [5], partially coherent flat-topped laser beam [6], partially coherent flat-topped vortex hollow beam [7], partially coherent cylindrical vector beam [8], multi-mode laser beam [9], Gaussian array beam [10–12], flat-topped vortex hollow beam [13], rectangular multi-Gaussian Schell-model beam [14], chirped Gaussian pulsed beam [15], ultra-short pulse beam [16] and partially coherent four-petal Gaussian vortex beam [17].

On the other hand, the model of Lorentz beam has been provided to describe the output optical field of diode lasers [18], and the propagation properties of Lorentz and Lorentz-Gauss beam have been widely studied [19–28]. At the same time, the phase-locked beam arrays provide a solution to the power limitation associated with single lasers caused by mode control, and which also maintain good beam quality. The researches on the propagation of phase-locked beam arrays in oceanic turbulence are useful to underwater optical communications. Recently, various phased-locked beams have been introduced and studied, including radial phased-locked Lorentz beam array [28], radial phased-locked partially coherent anomalous hollow beam array [29], phase-locked circular dark hollow beams array [30], phase-locked partially coherent flat-topped array laser beams [31], radial phased-locked partially coherent flat-topped vortex beam array [32]. Until now, to the best of our knowledge, radial phased-locked PCLGA beam has not been reported. In this paper, we will introduce a radial phased-locked PCLGA beam, and investigated the average intensity and coherence properties of a radial phased-locked PCLGA beam propagating in oceanic turbulence.

2. Analysis of theory

In this section, we will introduce a radial phased-locked partially coherent Lorentz-Gauss array (PCLGA) beam based on the theory of coherence, and then obtain the cross-spectral density function of a radial phased-locked PCLGA beam propagating in oceanic turbulence.

2.1. Definition of a radial phased-locked PCLGA beam

In the Cartesian coordinate system, considering the definition of phased-locked beam of radial symmetry [21,28–32], the optical

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field of a m-th Lorentz-Gauss beam at the source plane takes the form as:

$$E_m(\mathbf{r}_0,0)=\frac{1}{w_{0x}w_{0y}\left[1+\left(\frac{x_0-r_{mx}}{w_{0x}}\right)^2\right]\left[1+\left(\frac{y_0-r_{my}}{w_{0y}}\right)^2\right]}\times\exp\left[-\frac{(x_0-r_{mx})^2+(y_0-r_{my})^2}{w_0^2}\right]\exp(i\varphi_m) \quad (1)$$

where $\mathbf{r}_0=(x_0,y_0)$ is the position vector at the source plane; w_{0x} and w_{0y} are the parameters related to the beam widths of Lorentz part for Lorentz-Gauss beam in the x and y directions; w_0 is the waist of the Gaussian part for Lorentz-Gauss beam; $r_{mx}=R\cos\varphi_m$ and $r_{my}=R\sin\varphi_m$ are the center of the m-th beamlet located at the source plane, R is the radius of ring; $\varphi_m=m\varphi_0=m\frac{2\pi}{M}, m=1,2,\ldots M$ is the initial phase of the m-th beamlet at the source plane.

The optical field of total radial phased-locked Lorentz-Gauss beam composed of M beamlets at the source plane can be expressed as:

$$E_M(\mathbf{r}_0,0)=\sum_{m=1}^{M}E_m(\mathbf{r}_0,0) \quad (2)$$

Based on the coherence theory, the cross-spectral density function of a radial phased-locked PCLGA beam generated by a Schell-model source can be expressed as [33]:

$$W_M(\mathbf{r}_{10},\mathbf{r}_{20},0)=\langle E_M(\mathbf{r}_{10},0)E_M^*(\mathbf{r}_{20},0)\rangle$$

$$=\frac{1}{w_{0x}^2w_{0y}^2}\sum_m^M\sum_n^M\frac{1}{\left[1+\left(\frac{x_{10}-r_{mx}}{w_{0x}}\right)^2\right]\left[1+\left(\frac{y_{10}-r_{my}}{w_{0y}}\right)^2\right]}\times\frac{1}{\left[1+\left(\frac{x_{20}-r_{nx}}{w_{0x}}\right)^2\right]\left[1+\left(\frac{y_{20}-r_{ny}}{w_{0y}}\right)^2\right]}$$
$$\times\exp\left[-\frac{(x_{10}-r_{mx})^2+(y_{10}-r_{my})^2}{w_0^2}\right]\times\exp\left[-\frac{(x_{20}-r_{nx})^2+(y_{20}-r_{ny})^2}{w_0^2}\right]$$
$$\times\exp\left\{-\frac{[(x_{10}-r_{mx})-(x_{20}-r_{nx})]^2}{2\sigma_x^2}-\frac{[(y_{10}-r_{my})-(y_{20}-r_{ny})]^2}{2\sigma_y^2}\right\}\times\exp[i(\varphi_m-\varphi_n)] \quad (3)$$

where σ_x and σ_y are the transverse coherence length in x and y directions, respectively; the asterisk $*$ denotes the complex conjugate.

By using the relationship of Lorentz distribution and Hermite-Gaussian function [34]:

$$\frac{1}{(x_0^2+w_{0x}^2)(y_0^2+w_{0y}^2)}=\frac{\pi}{2w_{0x}^2w_{0y}^2}\sum_{d=0}^{N}\sum_{h=0}^{N}\sigma_{2d}\sigma_{2h}H_{2d}\left(\frac{x_0}{w_{0x}}\right)H_{2h}\left(\frac{y_0}{w_{0y}}\right)\times\exp\left(-\frac{x_0^2}{2w_{0x}^2}-\frac{y_0^2}{2w_{0y}^2}\right) \quad (4)$$

where N is the number of the expansion. σ_{2d} and σ_{2h} are the expanded coefficients, which can be found in Ref. [34]. As the even numbers $2d$ and $2h$ increase, the values of σ_{2d} and σ_{2h} decrease dramatically; Therefore, N will not be large in the numerical calculations, and N = 5 in this work; H_{2d} and H_{2h} are the 2d and 2h order Hermite polynomial, and $H_{2d}(x)$ can be written as [35]:

$$H_{2d}(x)=\sum_{l=0}^{d}\frac{(-1)^l(2d)!}{l!(2d-2l)!}(2x)^{2d-2l} \quad (5)$$

Therefore, the cross-spectral density function of a radial phased-locked PCLGA beam at the source plane can be rewritten as:

$$W_M(\mathbf{r}_{10},\mathbf{r}_{20},0)=\frac{\pi^2}{4w_{0x}^2w_{0y}^2}\sum_m^M\sum_n^M\sum_{d_1=0}^N\sum_{d_2=0}^N\sum_{h_1=0}^N\sum_{h_2=0}^N\sigma_{2d_1}\sigma_{2d_2}\sigma_{2h_1}\sigma_{2h_2}$$
$$\times H_{2d_1}\left(\frac{x_{10}-r_{mx}}{w_{0x}}\right)H_{2h1}\left(\frac{y_{10}-r_{my}}{w_{0y}}\right)H_{2d_2}\left(\frac{x_{20}-r_{nx}}{w_{0x}}\right)H_{2h_2}\left(\frac{y_{20}-r_{ny}}{w_{0y}}\right)$$
$$\times\exp\left[-\left(\frac{1}{w_0^2}+\frac{1}{2w_{0x}^2}\right)(x_{10}-r_{mx})^2-\left(\frac{1}{w_0^2}+\frac{1}{2w_{0y}^2}\right)(y_{10}-r_{my})^2\right]$$
$$\times\exp\left[-\left(\frac{1}{w_0^2}+\frac{1}{2w_{0x}^2}\right)(x_{20}-r_{nx})^2-\left(\frac{1}{w_0^2}+\frac{1}{2w_{0y}^2}\right)(y_{20}-r_{ny})^2\right]$$
$$\times\exp\left\{-\frac{[(x_{10}-r_{mx})-(x_{20}-r_{nx})]^2}{2\sigma_x^2}-\frac{[(y_{10}-r_{my})-(y_{20}-r_{ny})]^2}{2\sigma_y^2}\right\}\times\exp[i(\varphi_m-\varphi_n)] \quad (6)$$

2.2. Propagation of a radial phased-locked PCLGA beam in oceanic turbulence

Using the extended Huygens-Fresnel principle, the cross-spectral density function of a radial phased-locked PCLGA beam propagating in oceanic turbulence at the receiver plane (z > 0) is written as [1–15]:

$$W(\mathbf{r}_1,\mathbf{r}_2,z)=\frac{k^2}{4\pi^2z^2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}W_M(\mathbf{r}_{10},\mathbf{r}_{20},0)\times\exp\left[-\frac{ik}{2z}(\mathbf{r}_1-\mathbf{r}_{10})^2+\frac{ik}{2z}(\mathbf{r}_2-\mathbf{r}_{20})^2\right]\times\langle\exp[\psi(\mathbf{r}_{10},\mathbf{r}_1)+\psi^*(\mathbf{r}_{20},\mathbf{r}_2)]\rangle d\mathbf{r}_{10}d\mathbf{r}_{20} \quad (7)$$

where $k=2\pi/\lambda$ is the wave number with λ being the wavelength of optical field; $\mathbf{r}_1=(x_1,y_1)$ and $\mathbf{r}_2=(x_2,y_2)$ are the two-dimensional position vectors at the receiver plane z; $\psi(\mathbf{r}_0,\mathbf{r})$ is the complex phase perturbation due to the random media; the last term in sharp brackets of Eq. (7) for a radial phased-locked PCLGA beam propagating in oceanic turbulence can expressed as [1–8]:

$$\left\langle\exp\left[\psi(\mathbf{r}_{10},\mathbf{r}_1)+\psi^*(\mathbf{r}_{20},\mathbf{r}_2)\right]\right\rangle=\exp\left[-\frac{(\mathbf{r}_{10}-\mathbf{r}_{20})^2+(\mathbf{r}_{10}-\mathbf{r}_{20})(\mathbf{r}_1-\mathbf{r}_2)+(\mathbf{r}_1-\mathbf{r}_2)^2}{\rho_0^2}\right] \quad (8)$$

where ρ_0 is the coherence length of a spherical wave propagating in oceanic turbulence and $\rho_0^2=3/\pi^2k^2z\int_0^\infty d\kappa\kappa\Phi(\kappa)$, κ is spatial frequency; And $\Phi(\kappa)$ is the one-dimensional spatial power spectrum of oceanic turbulence, which can be written as:

$$\Phi(\kappa)=0.388\times10^{-8}\varepsilon^{-11/3}[1+2.35(\kappa\eta)^{2/3}]f(\kappa,\varsigma,\chi_T) \quad (9)$$

where ε is the rate of dissipation of turbulent kinetic energy per unit mass of fluid, which may vary in the range from 10^{-1} m^2 s^{-3} to 10^{-10} m^2 s^{-3}, $\eta=10^{-3}$ being the Kolmogorov microscale (inner scale), and

$$f(\kappa,\varsigma,\chi_T)=\frac{\chi_T}{\varsigma^2}[\varsigma^2\exp(-A_T\delta)+\exp(-A_S\delta)-2\varsigma\exp(-A_{TS}\delta)] \quad (10)$$

with χ_T is the rate of dissipation of mean square temperature taking value in the range from 10^{-4} K^2 s^{-1} to 10^{-10} K^2 s^{-1}, $A_T=1.863\times10^{-2}$, $A_S=1.9\times10^{-4}$, $A_{TS}=9.41\times10^{-3}$, $\delta=8.284(\kappa\eta)^{4/3}+12.978(\kappa\eta)^2$, ς is the relative strength of temperature and salinity fluctuations, which in the ocean waters can vary in the range from -5 to 0, 0 value corresponding to the case when salinity-driven