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A shape-preserving oriented partial differential equation based on a new fidelity term for electronic speckle pattern interferometry fringe patterns denoising

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ABSTRACT

Oriented partial differential equations (OPDEs) have been demonstrated to be a powerful tool for preserving the integrity of fringes while filtering electronic speckle pattern interferometry (ESPI) fringe patterns. However, the main drawback of OPDEs-based methods is that many iterations are often needed, which causes the change in the shape of fringes. Change in the shape of fringes will affect the accuracy of subsequent fringe analysis. In this paper, we focus on preserving the shape of fringes while filtering, suggested here for the first time. We propose a shape-preserving OPDE for ESPI fringe patterns denoising by introducing a new fidelity term to the previous second-order single oriented PDE (SOOPDE). In our proposed fidelity term, the evolution image is subtracted from the shrinkage result of original noisy image by shearlet transform. Our proposed shape-preserving OPDE is capable of eliminating noise effectively, keeping the integrity of fringes, and more importantly, preserving the shape of fringes. We test the proposed shape-preserving OPDE on three computer-simulated and three experimentally obtained ESPI fringe patterns with poor quality. Furthermore, we compare our model with three representative filtering methods, including the widely used SOOPDE, shearlet transform and coherence-enhancing diffusion (CED). We also compare our proposed fidelity term with the traditional fidelity term. Experimental results show that the proposed shape-preserving OPDE not only yields filtered images with visual quality on par with those by CED which is the state-of-the-art method for ESPI fringe patterns denoising, but also keeps the shape of ESPI fringe patterns.

1. Introduction

Electronic speckle pattern interferometry (ESPI) has been well known as a useful optical measurement technology for its simple optical devices, as well as the capability of providing high resolution, full field measurement in a non-contact mode. With the rapid development of modern science and technology, ESPI has been extensively researched and widely applied in a variety of fields such as vibration measurement, displacement measurement and its derivative measurement, as well as 3D object reconstruction, which often lead to more and more complex ESPI fringe patterns [1–3]. Since the phase carries the information about the physical quantities that need to be measured, accurate phase extraction is crucial to the successful application of ESPI techniques. In general, there are two ways for phase extraction: one is based on single ESPI fringe patterns, and the other is based upon ESPI wrapped phase patterns yielded by phase-shifting technique [4]. Both ESPI fringe patterns and wrapped phase patterns, however, contain a large quantity of speckle noise, so the removal of speckle noise is of critical importance.

Over the past few years, various filtering methods have been proposed for ESPI fringe patterns and wrapped phase patterns denoising, such as the wavelet method [5], the windowed Fourier transform method (WFF) [6], the localized Fourier transform filter (LFF) [7], the oriented regularized quadratic cost function (ORQCF) method [8], the oriented spatial filter masks (OSFM) method [9], image decomposition methods [10–12], and the partial differential equations (PDEs) based filtering methods. It is worth mentioning that PDEs based filtering methods have been widely used in ESPI fringe pattern denoising. Generally speaking, the filtering methods based on PDE can be divided into two categories: the non-oriented PDEs [4,13] and the oriented PDEs [14–20]. Additionally, numerous possible oriented PDE filtering models can be found in [21].

The previous filtering methods mainly focused on removing noise and preserving all fringes perfectly. For example, in the

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second-order single oriented PDE (SOOPDE) [14], the energy functional took the following form,

$$E(u) = \iint_{\Omega} \frac{1}{2}\left|\frac{\partial u}{\partial \rho}\right|^2 dxdy = \iint_{\Omega} \frac{1}{2}\left(u_x \cos\theta + u_y \sin\theta\right)^2 dxdy, \tag{1}$$

where θ denotes the angle between the fringe orientation ρ and the x coordinate. Using variational method and gradient descent procedure, the SOOPDE was constructed,

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial \rho^2} = u_{xx}\cos^2\theta + 2u_{xy}\sin\theta\cos\theta + u_{yy}\sin^2\theta. \tag{2}$$

Since SOOPDE allows diffusion only along the fringe orientation, it can keep the fringes intact.

If the energy functional was

$$\begin{aligned} E(u) &= \frac{1}{2}\iint_{\Omega}\left(\lambda_1\left|\frac{\partial u}{\partial \rho_\perp}\right|^2 + \left|\frac{\partial u}{\partial \rho}\right|^2\right) dxdy \\ &= \frac{1}{2}\iint_{\Omega}\left(\lambda_1\left(u_x\sin\theta - u_y\cos\theta\right)^2 + \left(u_x\cos\theta + u_y\sin\theta\right)^2\right) dxdy, \end{aligned} \tag{3}$$

then the coherence-enhancing diffusion (CED) [17] was constructed,

$$\begin{aligned} \frac{\partial u}{\partial t} &= \lambda_1 \frac{\partial^2 u}{\partial \rho_\perp^2} + \frac{\partial^2 u}{\partial \rho^2} \\ &= \lambda_1\left(u_{xx}\sin^2\theta - 2u_{xy}\sin\theta\cos\theta + u_{yy}\cos^2\theta\right) \\ &\quad + \left(u_{xx}\cos^2\theta + 2u_{xy}\sin\theta\cos\theta + u_{yy}\sin^2\theta\right) \end{aligned} \tag{4}$$

By dividing the directions of diffusion into the ones parallel and perpendicular to fringe orientation, and using different coefficients to control the rate of diffusion in both directions, CED can keep the low fringe density areas sufficiently smooth and high density fringes not blurred.

Recently, we have improved the SOOPDE and CED from computational efficiency (seeing Ref. [20]) and filtering results (seeing Ref. [22]). The main shortcoming of SOOPDE and CED is that many iteration times are generally needed for obtaining the desired results. For solving this problem, in our previous work [20], we put forward a parabolic–hyperbolic SOOPDE and a parabolic–hyperbolic CED. The parabolic–hyperbolic SOOPDE and the parabolic–hyperbolic CED yielded filtered images with visual quality comparable to those by SOOPDE and CED, and had significantly better performance in computational efficiency compared to the SOOPDE and CED models. SOOPDE can keep the integrity of fringes, but in the regions with low fringe density, the filtered results produced by SOOPDE are not sufficient [22]. For solving this problem, in our previous work [22], we proposed a combination of SOOPDE and shearlet transform and this combination gave satisfactory filtering results.

It has been shown that oriented PDEs are powerful tools for filtering and preserving all fringes. Although the oriented PDEs can keep the integrity of fringes, the sizes and positions of the filtered fringes have changed. What's more, along with the increasing of iteration times, this phenomenon will be more and more serious. Change in the shape of fringes will affect the accuracy of subsequent fringe analysis. For example, if the skeletons are extracted from the deformed fringe patterns, the positions of the skeletons will shift, and there will be a large error in the evaluated three-dimensional phase based on such skeletons. Therefore, it is very important to preserve the shape of fringes while filtering.

Unfortunately, almost all previous OPDE filtering methods have neglected this problem. The energy functionals corresponding to SOOPDE and CED only contain regularization terms. Minimizing the energy functional (1) is substantially equivalent to smoothing the image along fringe orientation. And the minimization of energy functional (3) is equivalent to smoothing the image along directions parallel and perpendicular to fringe orientation. In this theoretical framework, as the number of iterations increases, the shape of fringes in the filtered images may deviate from the original fringe patterns. To the best of our knowledge, there has been no report on the shape preservation of fringes in the literature. In this paper, we focus on preserving the shape of fringes while filtering, suggested here for the first time.

In this paper, by introducing a new fidelity term to the previous SOOPDE, we propose a shape-preserving OPDE for ESPI fringe patterns denoising. The function of fidelity term is to avoid the evolution image u being too different from the original noisy image u_0. However, due to the high noise level, the traditional fidelity term $(u_0 - u)$ cannot strike a good balance between noise elimination and shape preservation when filtering ESPI fringe patterns. Here, we propose a new fidelity term. In our proposed fidelity term, the evolution image is subtracted from the shrinkage result of original noisy image by shearlet transform. The motivation to use shearlet transform is that it has superior directional sensitivity, so it can preserve the wanted oriented fringes and then preserve the shape of fringe patterns. Thus our proposed shape-preserving OPDE, which includes both the regularization term SOOPDE and the proposed fidelity term, not only preserves the integrity of the fringes, but more importantly, keeps the shape of fringes while filtering. We would expect to see something that shows the uniqueness of our model. In fact, the subsequent experimental results do demonstrate the good performance of our proposed model.

One can find that this paper differs from the Refs. [20,22] in two points. The first is that the motivations are different. In Ref. [20], we focused on computational efficiency. And in Ref. [22], we focused on improving the filtering results. In this paper, we focus on preserving the shape of fringes while filtering. The second point is that, as described above, the methods are different.

The organization of this paper is as follows. In Section 2, our model is introduced in detail and the corresponding discrete scheme is developed. Section 3 is devoted to experimental results. We test the proposed model on three computer-simulated and three experimentally obtained ESPI fringe patterns and compare our model with three representative filtering methods, including the SOOPDE, shearlet transform and CED. We also compare our proposed fidelity term with the traditional fidelity term. In all cases, the proposed model produces filtered images with visual quality on par with those by CED, and preserves the shape of ESPI fringe patterns. A conclusion is subsequently made in Section 4.

2. The description of our method

Generally, the energy functional of PDE denoising model can be expressed as

$$E(u) = E_{regularization}(u) + \lambda E_{fidelity}(u, u_0), \tag{5}$$

where $E_{regularization}$ is the energy functional of regularization term which rewards smooth images and penalizes oscillatory ones, and $E_{fidelity}$ is the energy functional of fidelity term which avoids the evolution image u being too different from the original noisy image u_0. The controlling fidelity coefficient λ is used to govern the trade-off between smoothing and shape preservation [23].

The energy functional of traditional fidelity term is of the form

$$E_{fidelity}(u, u_0) = \|u_0 - u\|_{L^2}^2 = \iint_{\Omega}(u_0 - u)^2 dxdy \tag{6}$$

Then, the energy functional of SOOPDE based on traditional fidelity term is