Optics and Laser Technology 101 (2018) 433–439

Contents lists available at ScienceDirect

Optics and Laser Technology

journal homepage: www.elsevier.com/locate/optlastec

Full length article

Mode coupling in vortex beams

Halil T. Eyyuboğlu

Çankaya University, Electronics and Communications Department, Mimar Sinan Cad. No: 4, Etimesgut 06790, Ankara, Turkey

A R T I C L E I N F O

Article history:
Received 5 September 2017
Received in revised form 1 November 2017
Accepted 29 November 2017

Keywords:
Vortex beams
Mode coupling
Atmospheric turbulence

A B S T R A C T

We examine the mode coupling in vortex beams. Mode coupling also known as the crosstalk takes place due to turbulent characteristics of the atmospheric communication medium. This way, the transmitted intrinsic mode of the vortex beam leaks power to other extrinsic modes, thus preventing the correct detection of the transmitted symbol which is usually encoded into the mode index or the orbital angular momentum state of the vortex beam. Here we investigate the normalized power mode coupling ratios of several types of vortex beams, namely, Gaussian vortex beam, Bessel Gaussian beam, hypergeometric Gaussian beam and Laguerre Gaussian beam. It is found that smaller mode numbers lead to less mode coupling. The same is partially observed for increasing source sizes. Comparing the vortex beams amongst themselves, it is seen that hypergeometric Gaussian beam is the one retaining the most power in intrinsic mode during propagation, but only at lowest mode index of unity. At higher mode indices this advantage passes over to the Gaussian vortex beam.

1. Introduction

An optical communication link employing vortex beam usually relies on the detection of mode number or the orbital angular momentum contained in the beam mode. In all these cases, conservation of transmitted mode index or the orbital angular momentum is essential. Due to turbulence however, this is not possible, since during propagation, the power of transmitted mode in turbulent atmosphere is coupled into the neighbouring modes. Theoretical and practical aspects of this coupling, also named as crosstalk can be analysed in an attempt to minimize its impact on channel capacity. A sample of such studies are cited below.

In [1], by taking an aperture limited pure vortex beam, mode coupling is formulated and results are summarized in the form of approximated analytic expressions. It is seen that in the presence of strong turbulence, the information of the transmitted mode is lost completely. The effects of non-Kolmogorov turbulence on the spiral spectrum of hypergeometric Gaussian beams is investigated in [2], and it is found that larger values of hollowness parameter, smaller optical angular momentum number and longer wavelengths all yield lower crosstalk. In another study [3], the average capacity of orbital angular momentum multiplexed free space optical communication system based on Laguerre Gaussian beams and operating in non-Kolmogorov turbulence is derived. It is found that capacity increases with smaller structure constant and outer scale, higher transmission height and wavelengths. The radial average power distribution of orbital angular momentum modes of a Gaussian vortex beam propagating in weak to strong turbulence conditions is theoretically formulated and numerically analysed in [4]. It is observed that mode scrambling (coupling) is severer with larger mode index and smaller initial beam radius. In a recent study, the variations in the radial content of OAM photonic states propagating through weak-to-strong atmospheric turbulence are explored by employing numerical simulations. There it is found that two nondimensional parameters are needed to completely determine both the average Laguerre Gaussian and orbital angular momentum mode densities of a field due to a Laguerre Gaussian mode photon propagating through atmospheric turbulence [5]. Based on the coupled mode theory originally proposed for optical fibres, formulation is developed in [6] to analyse the impact of atmospheric turbulence on orbital angular momentum modes in terms of channel efficiency, crosstalk and error probabilities. It is demonstrated in [7] that the use of adaptive optics can dramatically correct the wavefront phase distortions caused by atmospheric turbulence, thus reducing the crosstalk and increasing channel capacity in orbital angular momentum based optical communication links. Modal power correlation coefficients, scintillation index are evaluated in [8], by solving the coupled ordinary differential equations for the higher order statistics of the

E-mail address: h.eyyuboglu@cankaya.edu.tr

https://doi.org/10.1016/j.optlastec.2017.11.050
0030-3992/

orbital angular momentum modal amplitudes. By adopting the pump spectrum model instead of the conventional Kolmogorov type, it is shown in [9] pump model is more accurate in estimating the channel capacity and crosstalk in orbital angular momentum based optical links. For the strong turbulence region, the results of [10] validate that the larger energy difference levels in the orbital angular momentum of Bessel Gaussian beams lead to less crosstalk. Under strong irradiance fluctuations, average probability density and normalized signal and crosstalk powers of orbital angular momentum modes of the fractional order Bessel Gaussian beams are modelled and their variations against quantum number difference, Rytov variance and beam radius examined in [11]. Optimum orbital angular momentum channel sets of Laguerre Gaussian beams are determined in [12] taking into account their aggregate capacity, bit error rate and crosstalk characteristics.

In this work, we analyse mode coupling of four types of vortex beams, namely Gaussian vortex beam, Bessel Gaussian beam, hypergeometric Gaussian beam and Laguerre Gaussian beam. The motivation is to search for the optimum source beam type and its parameters that will minimize mode coupling. The results of our study will be useful for optical communication links employing vortex beams.

2. Formulation of power mode coupling

The source field expressions of the vortex beams that are to be examined can be listed as follows

$$\begin{aligned}
e_{GV}(s,\phi_s) &= \left(\frac{s}{\alpha_s}\right)^m \exp\left(-\frac{s^2}{\alpha_s^2}\right)\exp(jm\phi_s)\\
e_{BG}(s,\phi_s) &= \exp\left(-\frac{s^2}{\alpha_s^2}\right)J_m(a_B s)\exp(jm\phi_s)\\
e_{HG}(s,\phi_s) &= \left(\frac{s}{\alpha_s}\right)^t \exp\left(-\frac{s^2}{\alpha_s^2}\right)\exp(jm\phi_s)\\
e_{LG}(s,\phi_s) &= \left(\sqrt{2}\frac{s}{\alpha_s}\right)^m \exp\left(-\frac{s^2}{\alpha_s^2}\right)L_n^m\left(2\frac{s^2}{\alpha_s^2}\right)\exp(jm\phi_s)
\end{aligned} \tag{1}$$

where the lower abbreviations, GV, BG, HG, LG refer to Gaussian vortex, Bessel Gaussian, hypergeometric Gaussian and Laguerre Gaussian beams respectively. (s,ϕ_s) are the radial source plane coordinates, α_s is the source size, m is the mode number (or mode index), $J_m()$ is the m the order Bessel function of first kind, a_B is the width parameter, t is the hollowness parameter, $L_n^m()$ is the m th order Laguerre polynomial with degree n. From Eq. (1), it is possible to arrive at the free space receiver field expression by using the Huygens Fresnel integral. Excluding the common phase factor, this integral is

$$e_i(r,\phi_r,L) = \frac{-jk}{2\pi L}\int_0^\infty\int_0^{2\pi} ds s d\phi_s e_i(s,\phi_s) \times \exp\left\{\frac{jk}{2L}\left[-2rs\cos(\phi_r-\phi_s)+s^2+r^2\right]\right\} \tag{2}$$

where $e_i(r,\phi_r,L)$ and $e_i(s,\phi_s)$ stand for any of the receiver and source plane field expressions of those beams cited in Eq. (1) with (r,ϕ_r) representing receiver plane radial coordinates and L being the propagation distance, k being the wave number. After carrying out the double integration in Eq. (2), we obtain the following results for the various beams of interest

$$\begin{aligned}
e_{GV}(r,\phi_r,L) &= (-jk)^{m+1} r^m \alpha_s^{m+2}\left(\frac{1}{2L-jk\alpha_s^2}\right)^{m+1}\exp\left(-\frac{jkr^2}{jk\alpha_s^2-2L}\right)\exp(jm\phi_r)\\
e_{BG}(r,\phi_r,L) &= \frac{k\alpha_s^2}{k\alpha_s^2+2jL}\exp\left[-\frac{j\alpha_s^2 a_B^2 L+2kr^2}{2(k\alpha_s^2+2jL)}\right]J_m\left(\frac{k\alpha_s^2 a_B r}{k\alpha_s^2+2jL}\right)\exp(jm\phi_r)\\
e_{HG}(r,\phi_r,L) &= (-0.5jk)^{m+1}\frac{r^m}{m!\alpha_s^t L^{0.5m-0.5t}}\left(\frac{2\alpha_s^2}{2L-jk\alpha_s^2}\right)^{0.5m+0.5t+1}\\
&\Gamma(0.5m+0.5t+1)\exp\left(\frac{jkr^2}{2L}\right)_1\\
&F_1\left[0.5m+0.5t+1, m+1, -\frac{k^2\alpha_s^2 r^2}{2L(2L-jk\alpha_s^2)}\right]\exp(jm\phi_r)\\
e_{LG}(r,\phi_r,L) &= (k\alpha_s^2)^{m+1}\left(\frac{\sqrt{2}r}{\alpha_s}\right)^m\frac{(k\alpha_s^2-2jL)^n}{(k\alpha_s^2+2jL)^{m+n+1}}\\
&\times\exp\left(\frac{-kr^2}{k\alpha_s^2+2jL}\right)L_n^m\left(\frac{2k^2\alpha_s^2 r^2}{k^2\alpha_s^4+4L^2}\right)\exp(jm\phi_r)
\end{aligned} \tag{3}$$

where $\Gamma()$ is the Gamma function, while ${}_1F_1()$ is the confluent hypergeometric function. According to Rytov theory, turbulence can be incorporated into the expressions of Eq. (3) in the following manner

$$e_{it}(r,\phi_r,L) = e_i(r,\phi_r,L)\exp[\psi(r,\phi_r,L)] \tag{4}$$

where again $e_i(r,\phi_r,L)$ is the generalized free space receiver field of all beams, while the $e_{it}(r,\phi_r,L)$ corresponds to their representations in turbulence. $\psi(r,\phi_r,L)$ is the complex phase perturbation arising from turbulence.

We expand $\exp[\psi(r,\phi_r,L)]$ in a Fourier series as shown below

$$\exp[\psi(r,\phi_r,L)] = \sum_{l=-\infty}^{\infty} g_l(r,L)\exp(jl\phi_r) \tag{5}$$

where the coefficients $g_l(r,L)$ are related to $\exp[\psi(r,\phi_r,L)]$ via the following integral

$$g_l(r,L) = \frac{1}{2\pi}\int_0^{2\pi} d\phi_r \exp[\psi(r,\phi_r,L)]\exp(-jl\phi_r) \tag{6}$$

Similarly the received field $e_{it}(r,\phi_r,L)$ can also be expanded as

$$e_{it}(r,\phi_r,L) = \sum_{p=-\infty}^{\infty} e_p(r,L)\exp(jp\phi_r) \tag{7}$$

with

$$e_p(r,L) = \frac{1}{2\pi}\int_0^{2\pi} d\phi_r e_{it}(r,\phi_r,L)\exp(-jp\phi_r) \tag{8}$$

Inserting in Eq. (8) for $e_{it}(r,\phi_r,L)$ from Eq. (4) and subsequently from Eq. (5), we get

$$e_p(r,L) = \frac{e_i(r,L)}{2\pi}\sum_{l=-\infty}^{\infty} g_l(r,L)\int_0^{2\pi} d\phi_r \exp(jl\phi_r)\exp(-jp\phi_r) \times \exp(jm\phi_r) \tag{9}$$

The integral in Eq. (9) evaluates to zero except at $l = p - m$, then

$$\begin{aligned}
e_p(r,L) &= e_i(r,L)g_{p-m}(r,L) = e_i(r,L)g_d(r,L)\\
&= \frac{e_i(r,\phi_r,L)}{\exp(jm\phi_r)}g_d(r,L)
\end{aligned} \tag{10}$$

This way, $e_p(r,L)$ represents the mode coupling from a transmitted intrinsic mode of m into a extrinsic mode of p which is different from m by an amount of d.From Eq. (6), we understand that difference is governed by the function, $g_d(r,L)$ which in turn is determined by the random quantity, i.e. the complex phase $\psi(r,\phi_r,L)$. To estimate the normalized average power coupled into a mode with a difference of d, we can perform the following integration