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Spreading and evolution behavior of coherent vortices of multi-Gaussian Schell-model vortex beams propagating through non-Kolmogorov turbulence

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A B S T R A C T

The explicit closed-form expressions for the beam width and angular spread of multi-Gaussian Schell-model vortex (MGSMV) beams propagating through atmospheric turbulence are derived in this paper. The spreading and evolution behavior of coherent vortices of MGSMV beams in non-Kolmogorov turbulence are investigated quantitatively by some typical numerical examples, where the evolution behavior of coherent vortices is stressed in particular. It is illustrated that MGSMV beams are more resistant to atmospheric turbulence than multi-Gaussian Schell-model (MGSM) non-vortex beams. By increasing the beam index of MGSMV beams, the deleterious turbulence effects can be reduced gradually. As MGSMV beams propagate in non-Kolmogorov turbulence, the position and number of coherent vortices are changeable. The impact of the beam index and fluctuations of atmospheric turbulence on the conservation distance of the topological charge is also explored in depth.

1. Introduction

Over the past few decades, lots of work has been done both theoretically and experimentally to investigate the propagation properties of partially coherent beams (PCBs) in free space and turbulent atmosphere, due to their resistance to the deleterious effects of atmospheric turbulence and potential applications in free-space optical (FSO) communications, remote sensing, laser radar systems, optical imaging, second-harmonic generation, particle trapping, etc [1–13]. In most cases, however, the research on propagation of PCBs is based on the conventional spatial correlation functions (i.e., Gaussian correlated Schell-model functions) owing to the lack of other available mathematical forms of spatial correlation functions for optical fields.

Since Gori and his collaborators established a sufficient condition for devising genuine spatial correlations based on the theory of reproducing kernel Hilbert spaces [14,15], many different kinds of PCBs with non-conventional spatial correlation functions are introduced, such as the non-uniformly correlated (NUC) beam [16], multi-Gaussian Schell-model (MGSM) beam [17], Laguerre-Gaussian Schell-model (LGSM) and Bessel-Gaussian Schell-model (BGSM) beam [18]. It was demonstrated that these PCBs with non-conventional spatial correlation functions have some remarkable characteristics on propagation through free space, such as the peculiar self-focusing effect and a laterally shifted intensity maxima [16], far fields with flat-topped intensity profiles [17], and far fields with ring-shaped intensity distributions [18].

In recent years, optical beams carrying phase singularities (i.e., optical vortex beams) have attracted much attention because of their various applications ranging from being used as optical tweezers and spanners [19,20] to being employed as information carriers in FSO communications [21,22], and the related research forms a significant subfield of optics, known as singular optics [23]. The properties of partially coherent vortex beams with conventional spatial correlation functions propagating through random media have been discussed in some previous literatures, and the partially coherent vortex beams are shown to be less sensitive to atmospheric turbulence than partially coherent non-vortex beams [24–34]. Lately, the multi-Gaussian Schell-model vortex (MGSMV) beam was introduced by Zhang *et al.* as a natural extension of the aforementioned MGSM beam, and its focusing properties were studied in [35]. Accordingly, Tang *et al.* investigated the propagation characteristics of MGSMV beams in isotropic random media, and derived the analytical formulae for the cross-spectral density function and mean-squared beam width of MGSMV beams [36]. As far as we know, however, the spreading and evolution behavior of coherent vortices of MGSMV beams propagating through turbulent atmosphere has not been discussed till now.

This paper is devoted to study the spreading and evolution

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behavior of coherent vortices of the MGSMV beam in non-Kolmogorov turbulence. Based on the extended Huygens-Fresnel principle and the paraxial approximation, the explicit analytical expressions for the spectral degree of coherence, normalized root mean square (rms) beam width and angular spread of the MGSMV beam with topological charge $l = \pm 1$ in atmospheric turbulence are obtained in Section 2. The spreading and the evolution behavior of coherent vortices of the MGSMV beam in non-Kolmogorov turbulence are illustrated and analyzed by a set of numerical examples in Section 3. Finally, the concluding remarks are given in Section 4.

2. Theoretical model

In the Cartesian coordinate system, the field of a vortex beam at the source plane $z=0$ can be expressed as [37]

$$U(\mathbf{r}, 0) = u(\mathbf{r})\left[r_x + i\,\mathrm{sgn}(l) r_y\right]^{|l|} \tag{1}$$

where $\mathbf{r} \equiv (r_x, r_y)$ denotes the arbitrary transverse position vector at the source plane $z=0$, $u(\mathbf{r})$ represents the profile of the background beam envelope, $\mathrm{sgn}(\cdot)$ is the sign function, and l specifies the topological charge, also named as spiral number.

The MGSMV beam can be acquired by setting a MGSM beam as the background beam. In what follows, we consider a MGSMV beam at the source plane $z=0$ propagating into the half-space $z > 0$ where the turbulent atmosphere exits, and the possible dependence of all field statistics on the angular frequency ω is implied but omitted for brevity.

The second-order statistical properties of a partially coherent beam can be characterized by the cross-spectral density (CSD) function. The CSD of a MGSMV beam at the source plane $z=0$ can be written as [35]

$$\begin{aligned} W^{(0)}(\mathbf{r}_1, \mathbf{r}_2, 0) = &\frac{1}{C_0}\sum_{m=1}^{M}\frac{(-1)^{m-1}}{m}\binom{M}{m} \\ &\times \exp\left[-\frac{\mathbf{r}_1^2 + \mathbf{r}_2^2}{4\sigma^2}\right]\exp\left[-\frac{(\mathbf{r}_1 - \mathbf{r}_2)^2}{2m\delta^2}\right] \\ &\times \left[r_{1x}r_{2x} + r_{1y}r_{2y} + i\,\mathrm{sgn}(l)(r_{1x}r_{2y} - r_{2x}r_{1y})\right]^{|l|} \end{aligned} \tag{2}$$

where $\mathbf{r}_1 \equiv (r_{1x}, r_{1y})$ and $\mathbf{r}_2 \equiv (r_{2x}, r_{2y})$ denote two arbitrary transverse position vectors at the source plane $z=0$, $C_0 = \sum_{m=1}^{M}\frac{(-1)^{m-1}}{m}\binom{M}{m}$ is the normalization factor with M being the beam index, $\binom{M}{m}$ stands for binomial coefficients, σ and δ is the waist width and spatial correlation width of the MGSMV beam, respectively. It can be clearly seen from Eq. (2) that the MGSMV beam can degenerate into a Gaussian Schell-model vortex (GSMV) beam under the condition of $M=1$ and into a classic Gaussian Schell-model (GSM) non-vortex beam when $M=1$ and $l=0$. For simplicity, the topological charge l is assumed to be ± 1 in the following.

Within the validity of the paraxial approximation and based on the extended Huygens-Fresnel principle, the CSD of the MGSMV beam propagating through turbulent atmosphere at the z plane can be obtained by [10,26,32]

$$\begin{aligned} W(\rho_1, \rho_2, z) = &\left(\frac{k}{2\pi z}\right)^2\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} W^{(0)}(\mathbf{r}_1, \mathbf{r}_2, 0) \\ &\times \exp\left\{-\frac{ik}{2z}\left[(\rho_1 - \mathbf{r}_1)^2 - (\rho_2 - \mathbf{r}_2)^2\right]\right\} \\ &\times \left\langle \exp\left[\psi^*(\rho_1, \mathbf{r}_1, z) + \psi(\rho_2, \mathbf{r}_2, z)\right]\right\rangle_R d^2\mathbf{r}_1 d^2\mathbf{r}_2 \end{aligned} \tag{3}$$

where $\rho_1 \equiv (\rho_{1x}, \rho_{1y})$ and $\rho_2 \equiv (\rho_{2x}, \rho_{2y})$ denote two arbitrary transverse position vectors at the z plane, perpendicular to the direction of propagation of the beam, $k = 2\pi/\lambda$ is the wave number with λ being the wavelength of the source, $\langle\cdot\rangle$ represents the average over the ensemble, the asterisk specifies the complex conjugate, and the expression in the angular brackets with subscript R is the complex phase correlation of a spherical wave propagating in the turbulent medium. The phase correlation term describing the influence of atmospheric turbulence can be well approximated by [28,38,39]

$$\begin{aligned} &\left\langle \exp\left[\psi^*(\rho_1, \mathbf{r}_1, z) + \psi(\rho_2, \mathbf{r}_2, z)\right]\right\rangle_R \\ &= \exp\left\{-\frac{\pi^2k^2Tz}{3}\left[(\mathbf{r}_1 - \mathbf{r}_2)^2 + (\mathbf{r}_1 - \mathbf{r}_2)(\rho_1 - \rho_2) \right.\right. \\ &\quad \left.\left. + (\rho_1 - \rho_2)^2\right]\right\} \end{aligned} \tag{4}$$

where

$$T = \int_0^{\infty}\kappa^3\Phi_n(\kappa)d\kappa \tag{5}$$

with $\Phi_n(\kappa)$ is the one-dimensional spatial power spectrum of the refractive-index fluctuations of atmospheric turbulence, and κ is the scalar spatial wave number.

In order to cover a wide scope of atmospheric conditions, our discussion on the propagation of the MGSMV beam is based on a generalized power spectrum model introduced by Toselli *et al.* in [40], which is valid in non-Kolmogorov turbulence

$$\Phi_n(\kappa, \alpha) = A(\alpha)\tilde{C}_n^2\frac{\exp\left[-(\kappa^2/\kappa_m^2)\right]}{(\kappa^2 + \kappa_0^2)^{\alpha/2}}, \quad 0 \le \kappa < \infty,\ 3 < \alpha < 4 \tag{6}$$

where the term $\tilde{C}_n^2$ is the generalized refractive-index structure constant with units $m^{3-\alpha}$, $\kappa_0 = 2\pi/L_0$, $\kappa_m = c(\alpha)/l_0$, with L_0 and l_0 being the outer and inner scales of atmospheric turbulence, respectively, and

$$A(\alpha) = \frac{1}{4\pi^2}\Gamma(\alpha - 1)\cos\left(\frac{\alpha\pi}{2}\right) \tag{7}$$

$$c(\alpha) = \left[\frac{2\pi}{3}\Gamma\left(\frac{5-\alpha}{2}\right)A(\alpha)\right]^{\frac{1}{\alpha-5}} \tag{8}$$

with $\Gamma(\cdot)$ being the Gamma function. On setting $\alpha = 11/3$, $A(\alpha) = 0.033$, $\tilde{C}_n^2 = C_n^2$, $L_0 \to \infty$ and $l_0 \to 0$, the generalized power spectrum reduces to the conventional Kolmogorov spectrum.

By use of the power spectrum in Eq. (6), the integral in Eq. (5) becomes [11,28,36,39]

$$T = \frac{A(\alpha)}{2(\alpha - 2)}\tilde{C}_n^2\left[\kappa_m^{2-\alpha}\mu\exp\left(\frac{\kappa_0^2}{\kappa_m^2}\right)\Gamma\left(2 - \frac{\alpha}{2}, \frac{\kappa_0^2}{\kappa_m^2}\right) - 2\kappa_0^{4-\alpha}\right] \tag{9}$$

where $\mu = 2\kappa_0^2 + (\alpha - 2)\kappa_m^2$ and $\Gamma(\cdot,\cdot)$ denotes the incomplete Gamma function.

Introducing two variables of integration, namely the center of gravity and difference vectors

$$\mathbf{r}_s = \frac{\mathbf{r}_1 + \mathbf{r}_2}{2}, \quad \mathbf{r}_d = \mathbf{r}_1 - \mathbf{r}_2 \tag{10}$$

and substituting Eqs. (2) and (4) into Eq. (3), one obtains