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Compressive measurement and feature reconstruction method for autonomous star trackers

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ABSTRACT

Compressive sensing (CS) theory provides a framework for signal reconstruction using a sub-Nyquist sampling rate. CS theory enables the reconstruction of a signal that is sparse or compressible from a small set of measurements. The current CS application in optical field mainly focuses on reconstructing the original image using optimization algorithms and conducts data processing in full-dimensional image, which cannot reduce the data processing rate. This study is based on the spatial sparsity of star image and proposes a new compressive measurement and reconstruction method that extracts the star feature from compressive data and directly reconstructs it to the original image for attitude determination. A pixel-based folding model that preserves the star feature and enables feature reconstruction is presented to encode the original pixel location into the superposed space. A feature reconstruction method is then proposed to extract the star centroid by compensating distortions and to decode the centroid without reconstructing the whole image, which reduces the sampling rate and data processing rate at the same time. The statistical results investigate the proportion of star distortion and false matching results, which verifies the correctness of the proposed method. The results also verify the robustness of the proposed method to a great extent and demonstrate that its performance can be improved by sufficient measurement in noise cases. Moreover, the result on real star images significantly ensures the correct star centroid estimation for attitude determination and confirms the feasibility of applying the proposed method in a star tracker.

1. Introduction

CS theory [1,2] is an emerging field that can sense sparse or compressible signals at a sub-Nyquist sampling rate and overcome the sampling rate limitation of traditional methods. It exploits the property of the sensed signal, that is, its usual sparsity in some transformed domain, to recover from a small number of linear random measurements with high probability. In the optical field, the images that contain a common scenario are usually sparse in the frequency domain, and the transformation of projecting it into a known basis is required to obtain good sparsity. Most of the CS optical applications depend on the image sparsity in the frequency domain and perform the reconstruction through the regularized optimization methods such as l_0, l_1 minimization and so on [3–5]. The current CS applications aim to obtain the best approximation of the original image and conduct data processing after the whole image has been successfully reconstructed, which cannot effectively reduce the data processing rate. In this regard, extracting and reconstructing the image feature from the compressive measurements represents an attractive perspective of CS application.

The autonomous star tracker [6,7] is an avionics instrument used to measure the absolute three-axis attitude of a spacecraft by star observation. The tracker is currently the most accurate on-board attitude sensor and is widely used in space missions. The star tracker consists of an electronic camera and a microprocessor, which locates and identifies the stars within the field of view (FOV) and performs attitude determination based on the sensed star images. The star image varies from common images and represents a scene of star points, where the number of star pixels is significantly less than that of background pixels, thereby showing admirable sparsity in a spatial domain. Moreover, the useful information for attitude determination is the star point rather than the background. The particular spatial sparsity of star image makes it possible to apply a distinct compressive measurement and reconstruction method in star trackers.

Many researchers have applied CS theory in astronomy. Bobin et al. [8,9] introduce compressive measurement in astronomical data analysis. Yao et al. [10] propose an algorithm for small moving space object detection and localization. Also, some researchers have studied the CS application in spatially sparse image. Gupta

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et al. [11,12] propose a folding algorithm as compressive sampling scheme for images that are spatially sparse and contain distinguishable objects. Hamilton et al. [13] improve the folding algorithm and present a focal plane array folding for the muzzle flash image. However, their schemes aim to reconstruct the whole image, which cannot conduct feature extraction or reconstruction from compressive measurements. Furthermore, Gardiner [14] explores the compressive image features such as corners, rotation, and translation, and presents algorithms for reconstructing them from folding representation. Unfortunately, their conclusions on these image features are not applicable for star image.

Our research focuses on feature reconstruction from low-dimensional measurements for star trackers based on spatial sparsity. We inherit and extend the advantage of folding algorithm by deriving a pixel-based folding model from CS perspective, which enables feature reconstruction of the star image. We also propose an innovative compressive feature reconstruction (CFR) method that directly recovers the star feature from low-dimensional data termed by superposed space. This method extracts the star centroids in the superposed space and directly decodes their original locations instead of reconstructing the whole image. The significant advantage of CFR method is that it not only reduces the sampling rate as common CS does but also lowers the rate of data processing. The results of the numerical simulation confirm the feasibility of CFR method for star images and guarantee the successful rate of subsequent star identification and attitude determination to a large extent.

The remainder of this paper is organized as follows: Section 2 reviews the CS framework and gives the motivation of this research by analyzing the folding algorithm. Section 3 derives a pixel-based folding model from CS perspective. Section 4 proposes the CFR method in detail and theoretically analyzes its advantage. Section 5 gives the numerical simulation and analysis. Section 6 concludes the study.

2. Basic principles

2.1. Fundamentals of CS

The CS framework enables a sparse or compressible signal to be sensed by obtaining fewer measurements than the number of samples that define the signal. CS theory states that a finite-dimensional signal that has a sparse representation in some transformed domain can be reconstructed from a small number of its non-adaptive linear projections with minimal loss of information. If the signal is an image, then it may be rearranged into a one-dimensional vector. In the CS model, an N-dimensional signal $x \in \mathbb{R}^N$ can be sparsely represented as

$$x = \Psi\alpha, \tag{1}$$

where Ψ is an $N \times N$ sparse representation matrix and α is the corresponding coefficient vector. The signal x is K-sparse if the vector α has only $K\,(K \ll N)$ nonzero components. Instead of acquiring all N samples, CS obtains a low-dimensional measurement result $y \in \mathbb{R}^M$ using an $M \times N\,(M \ll N)$ measurement matrix Φ such that

$$y = \Phi x = \Phi\Psi\alpha = \Theta\alpha, \tag{2}$$

where $\Theta = \Phi\Psi$ is a sensing matrix.

The purpose of CS is to reconstruct x or alternatively the coefficient vector α from the incomplete measurement y. The reconstruction of the signal x from the compressed samples y is an ill-posed problem because $M \ll N$. However, the sparsity assumption and the incoherence between the measurement matrix Φ and sparse representation matrix Ψ make reconstruction possible. In other words, if the sensing matrix Θ satisfies a sufficient condition-restricted isometry property (RIP), then the signal x can be successfully reconstructed from y with a high probability.

The sparse reconstruction of the CS model is performed through the optimization method to determine vector α, which satisfies the measurement result y while minimizing the l_0-norm formulated as

$$\hat{\alpha} = \arg\min_{\alpha} \|\alpha\|_0 \;\; \text{s. t.}\; y = \Phi\Psi\alpha. \tag{3}$$

Solving for Eq. (3) is a non-deterministic polynomial-time hard (NP hard) [15] problem. One solution is to use l_1-norm minimization such as

$$\hat{\alpha} = \arg\min_{\alpha} \|\alpha\|_1 \;\; \text{s. t.}\;\; y = \Phi x. \tag{4}$$

A range of reconstruction methods have been proposed to solve this problem, such as basis pursuit, greedy, stochastic algorithms and so on [16,17]. Theoretically, these algorithms aim to obtain the best approximation of the original image through optimization methods; this process can be regarded as an image-focused reconstruction. The current CS application shows that feature extraction and data processing must be conducted after the whole image has been successfully reconstructed. Thus, the amount of data processing is still a full-dimensional image, which cannot be reduced theoretically.

2.2. Folding algorithm

The folding algorithm partitions the original $n_1 \times n_2$ dimensional image $I[x_1, x_2]$ into sub-frames with the size of $p \times q$ by the greatest extent and then sums all the non-overlapping sub-frames to yield the superposed image $m[y_1, y_2]$, which consists of $p \times q$ elements, as shown in Fig. 1. The folding algorithm is expressed as

$$\text{FOLD}(I, p, q) = m[y_1, y_2] = \sum_{\substack{x_1 \equiv y_1 (\text{mod } p) \\ x_2 \equiv y_2 (\text{mod } q)}} I[x_1, x_2]. \tag{5}$$

In terms of folding, all the elements in the original image are mapped to its folded version in the superposed image. Then, the folding algorithm introduces the Chinese remainder theorem [18] (CRT) as a decoding rule to recover the area that contains the object from the superposed image to the original one. The CRT solves a series of simultaneous equations with respect to different moduli in considerable generality by the following definition: Let $m_1, m_2, \cdots, m_k$ be integers greater than 1 with $\gcd(m_i, m_j) = 1$, where $i \neq j$. For any integers $a_1, a_2, \cdots, a_k$, the set of congruent equations

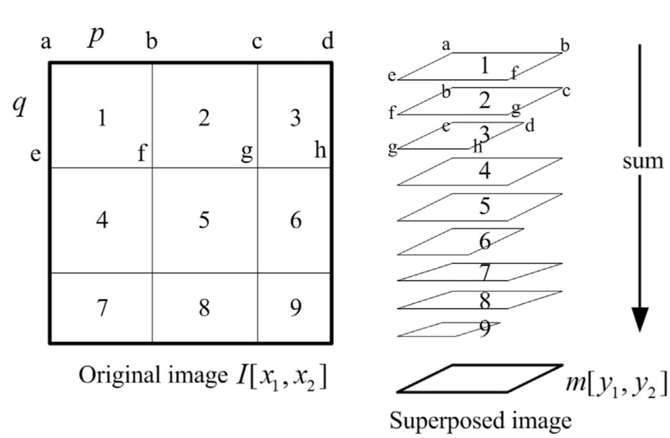

Fig. 1. Illustration of folding process [11,12].