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Per unit received power apertured averaged scintillation of partially coherent sinusoidal and hyperbolic Gaussian beams

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A B S T R A C T

We evaluate the per unit power received power aperture averaged scintillation performance of fully and partially coherent sinusoidal and hyperbolic Gaussian beams. Our analysis includes fundamental Gaussian, cosh Gaussian, cos Gaussian and annular Gaussian beams. The method is based on our earlier introduced semi-analytic approach. Scintillation performance is measured upon dividing the aperture averaged scintillation by the received power. Assessment is made both for aperture sizes that are adjusted separately for full and partially coherent beams to capture 10% and 20% of the equal source power and also for fixed aperture sizes. This way, the scintillation performance of the different beams in question is compared. From this comparison, we find that partially coherent beams have lower scintillation than the fully coherent ones, when adjustable aperture size is used. But upon switching to fixed aperture size, the reverse happens and coherent beams become more advantageous. In all cases of comparison, small source sized annular Gaussian beam and large source sized Gaussian beam seem to offer the lowest scintillation when aperture size is adjusted to capture 20% of the equal source power.

1. Introduction

To achieve a better performance, an optical receiver will attempt to enlarge its receiving aperture size. This way, it will gain two advantages. One is lowering the scintillation effects, by operating in aperture averaging mode. The other is collecting more power, which will help to increase the signal to noise ratio, thus lowering the error rate.

The positive consequences of increasing the collecting area of an optical receiver on reducing scintillation were recognized as early as 1950s and this idea was utilized in the astronomical telescopes of those days [1]. Aperture averaging occurs when the aperture size exceeds the correlation width of the intensity fluctuations and thus be considered as the simplest form of spatial diversity [2]. Aperture averaging was formulated during 1960s and 1970s by Fried, Lutomirski, Yura and Fante [3–5]. In these works and in the others [6–8], to express the advantage offered by the aperture averaging operation, a gain factor was defined as the ratio of aperture averaged (power) scintillation to that of point aperture. The evaluation of scintillation over a finite area requires the use of irradiance covariance function. For simple cases such as plane and spherical waves, this function takes relatively simpler forms [9]. But for specific beam profiles, irradiance covariance function becomes rather complicated to handle. In [10], for instance, the aperture averaged scintillation of fundamental Gaussian beam was formulated using ABCD matrix representation.

The experiments conducted demonstrate the clear benefit of aperture averaging on the reduction of scintillation [2,11–13]. Furthermore such practical work has also confirmed that the gains of aperture averaging are aperture shape insensitive [13].

In literature, there exist quite a number of papers emphasizing the advantages of partially coherent beams over the fully coherent ones. In this sense, it is reported in [14–16], that lowering coherence of the source beam will help reduce scintillation. In [17], it is found that partial coherence will bring about aperture size and relative speed detection dependent scintillation reductions. In [18], the spreading properties of partially coherent beams are examined and the turbulence resistive nature of partially coherent beams is exhibited in terms of source and propagation parameters. [19] is a brief review paper discussing how a partially coherent beam will provide a better free space optical link performance.

As noted above and also in [19], in designing a successful optical link, the aim should be to maximize the captured power and minimize scintillation. We can assign a formal metric to this aim by defining the quantity, "per unit received power aperture averaged scintillation", which is simply the aperture averaged scintillation divided by the power falling onto that aperture. In this paper, the

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scintillation assessment of beams is based on this metric, i.e., per unit received power aperture averaged scintillation.

As pointed out above, the evaluation of aperture averaged scintillation involves the formulation of two point irradiance covariance function [20], which is quite cumbersome to handle for the case of specific beams. To overcome this difficulty, here we benefit from the semi-analytic approach introduced earlier [21,22].

The purpose of this study is to investigate per unit received power aperture scintillation properties of a group of beams, namely coherent and partially coherent Gaussian, cosh Gaussian, cos Gaussian and annular Gaussian beams and draw conclusion on their suitability for the free space optical project currently under development in our university.

2. Formulation of per unit received power apertured averaged scintillation

In [22], a two point mutual coherence function of the source beams in question is given. From there, the four point mutual coherence function can be written as

$$\begin{aligned}\Gamma(\mathbf{s}_1,\mathbf{s}_2,\mathbf{s}_3,\mathbf{s}_4) &= \exp\left[-\frac{(\mathbf{s}_1-\mathbf{s}_2)^2+(\mathbf{s}_3-\mathbf{s}_4)^2}{2\sigma_s^2}+\frac{2\pi(\mathbf{s}_1^2-\mathbf{s}_2^2+\mathbf{s}_3^2-\mathbf{s}_4^2)}{j\lambda F_s}\right]\\ &\times\sum_{i1=1}^{2}\sum_{i2=1}^{2}\sum_{i3=1}^{2}\sum_{i4=1}^{2}A_{i1}A_{i2}A_{i3}A_{i4}\\ &\times\exp\left[-\left(\frac{\mathbf{s}_1^2}{w_{i1}^2}+\frac{\mathbf{s}_2^2}{w_{i2}^2}+\frac{\mathbf{s}_3^2}{w_{i3}^2}+\frac{\mathbf{s}_4^2}{w_{i4}^2}\right)+D_{i1}\mathbf{s}_1+D_{i2}^*\mathbf{s}_2\right.\\ &\left.+D_{i3}\mathbf{s}_3+D_{i4}^*\mathbf{s}_4\right]\end{aligned}\tag{1}$$

where $\mathbf{s}_n=(s_{xn},\ s_{yn})$, $n=1,2,3,4$ are the source transverse plane coordinates, σ_s and F_s are the partial coherence and focusing parameter, λ is the wavelength, A_i is the amplitude coefficient, w_i is the source size, D_i is the displacement parameter. By using the appropriate settings for A_i, w_i, D_i, Eq. (1) turns into the mutual coherence functions of Gaussian, cosh Gaussian, cos Gaussian and annular Gaussian beams.

As stated in [10] and [23], on a receiver plane, situated at an L distance from the source plane, the aperture averaged scintillation is given by

$$b^2(L)=\frac{\langle P^2(L)\rangle}{\langle P(L)\rangle^2}-1\tag{2}$$

where $P(L)$ stands for the power captured by the aperture opening of the receiver and can simply be obtained by integrating the well known extended Huygens–Fresnel integral over the aperture area. $\left\langle P^2(L)\right\rangle$ on the other hand is related to irradiance covariance function. It includes fourfold integration for the irradiance and double integration to implement the aperture overlap [21,23]. Using [21,22], $\left\langle P^2(L)\right\rangle$ will become

$$\begin{aligned}\langle P^2(L)\rangle &= \frac{1}{(\lambda z)^4}\int_{-0.5S_a}^{0.5S_a}d^2\mathbf{r}_1\int_{-1.5S_a-\mathbf{r}_1}^{1.5S_a+\mathbf{r}_1}d^2\mathbf{r}_2\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\\ &\times d^2\mathbf{s}_1d^2\mathbf{s}_2d^2\mathbf{s}_3d^2\mathbf{s}_4\Gamma(\mathbf{s}_1,\mathbf{s}_2,\mathbf{s}_3,\mathbf{s}_4)\\ &\times G(\mathbf{r}_1,\mathbf{r}_2,\mathbf{s}_1,\mathbf{s}_2,\mathbf{s}_3,\mathbf{s}_4)H(\mathbf{r}_1,\mathbf{r}_2,\mathbf{s}_1,\mathbf{s}_2,\mathbf{s}_3,\mathbf{s}_4)\end{aligned}\tag{3}$$

where S_a is the side length of the square aperture opening. $\mathbf{r}_n=(r_{xn},r_{yn})$, $n=1,2$ are the receiver transverse plane coordinates. $G(\mathbf{r}_1,\mathbf{r}_2,\mathbf{s}_1,\mathbf{s}_2,\mathbf{s}_3,\mathbf{s}_4)$ and $H(\mathbf{r}_1,\mathbf{r}_2,\mathbf{s}_1,\mathbf{s}_2,\mathbf{s}_3,\mathbf{s}_4)$ are paraxially approximated Green's function and the fourth order moment for two receiver coordinates, respectively representing the diffractive and fluctuating properties of the atmosphere. From [21,22], it is possible to express $G(\mathbf{r}_1,\mathbf{r}_2,\mathbf{s}_1,\mathbf{s}_2,\mathbf{s}_3,\mathbf{s}_4)$ and $H(\mathbf{r}_1,\mathbf{r}_2,\mathbf{s}_1,\mathbf{s}_2,\mathbf{s}_3,\mathbf{s}_4)$ as

$$\begin{aligned}G(\mathbf{r}_1,\mathbf{r}_2,\mathbf{s}_1,\mathbf{s}_2,\mathbf{s}_3,\mathbf{s}_4) &= \exp\left\{\frac{j\pi}{\lambda L}\left[\mathbf{s}_1^2-\mathbf{s}_2^2+\mathbf{s}_3^2-\mathbf{s}_4^2-2\mathbf{r}_1(\mathbf{s}_1-\mathbf{s}_2)-2\mathbf{r}_2(\mathbf{s}_3-\mathbf{s}_4)\right]\right\}\\ &\times H(\mathbf{r}_1,\mathbf{r}_2,\mathbf{s}_1,\mathbf{s}_2,\mathbf{s}_3,\mathbf{s}_4)\\ &=\exp\left\{-j\rho_p\left[\mathbf{s}_1^2-\mathbf{s}_2^2+\mathbf{s}_3^2-\mathbf{s}_4^2-2\mathbf{s}_1\mathbf{s}_3+2\mathbf{s}_2\mathbf{s}_4\right.\right.\\ &\left.\left.+(\mathbf{s}_1-\mathbf{s}_2-\mathbf{s}_3+\mathbf{s}_4)(\mathbf{r}_1-\mathbf{r}_2)\right]\right\}\times\left[\exp\left[-\frac{1}{\rho_0^2}\left(\mathbf{s}_1^2\right.\right.\right.\\ &+\mathbf{s}_2^2+\mathbf{s}_3^2+\mathbf{s}_4^2-2\mathbf{s}_1\mathbf{s}_2+2\mathbf{s}_1\mathbf{s}_3-2\mathbf{s}_1\mathbf{s}_4-2\mathbf{s}_2\mathbf{s}_3\\ &\left.\left.+2\mathbf{s}_2\mathbf{s}_4-2\mathbf{s}_3\mathbf{s}_4\right)\right]+\rho_t\exp\left[-\frac{1}{\rho_0^2}(\mathbf{r}_1-\mathbf{r}_2)^2\right]\\ &\times\left(\exp\left\{-\frac{1}{\rho_0^2}\left[2\mathbf{s}_1^2+\mathbf{s}_2^2+2\mathbf{s}_3^2+\mathbf{s}_4^2-2\mathbf{s}_1\mathbf{s}_2\right.\right.\right.\\ &\left.\left.-2\mathbf{s}_1\mathbf{s}_4-2\mathbf{s}_2\mathbf{s}_3+2\mathbf{s}_2\mathbf{s}_4-2\mathbf{s}_3\mathbf{s}_4+(\mathbf{s}_1-\mathbf{s}_3)(\mathbf{r}_1-\mathbf{r}_2)\right]\right\}\\ &+\exp\left\{-\frac{1}{\rho_0^2}\left[\mathbf{s}_1^2+2\mathbf{s}_2^2+\mathbf{s}_3^2+2\mathbf{s}_4^2-2\mathbf{s}_1\mathbf{s}_2+2\mathbf{s}_1\mathbf{s}_3\right.\right.\\ &\left.\left.\left.\left.-2\mathbf{s}_1\mathbf{s}_4-2\mathbf{s}_2\mathbf{s}_3-2\mathbf{s}_3\mathbf{s}_4+(\mathbf{s}_2-\mathbf{s}_4)(\mathbf{r}_1-\mathbf{r}_2)\right]\right\}\right)\right]\ (4)\end{aligned}\tag{4}$$

In Eq. (4), the vectorial transverse plane coordinates obey the dot product rules, such that

$$\begin{aligned}&\mathbf{s}_n^2=s_{xn}^2+s_{yn}^2,\ \ \mathbf{s}_n\mathbf{s}_p=s_{xn}s_{xp}+s_{yn}s_{yp},\ n\ ,\ p=1,2,3,4\\ &\mathbf{r}_m^2=r_{xm}^2+r_{ym}^2,\ \ \mathbf{s}_n\mathbf{r}_m=s_{xn}r_{xm}+s_{yn}r_{ym},\\ &n=1,2,3,4\quad m=1,2\end{aligned}\tag{5}$$

The other terms in Eq. (4) are defined as

$$\rho_p=6.0814\frac{C_n^2L^{5/6}}{\lambda^{13/6}},\quad \rho_t=2.1167\frac{C_n^2L^{11/6}}{\lambda^{7/6}},\quad \rho_0=0.1586\frac{\lambda^{6/5}}{\left(C_n^2L\right)^{3/5}}\tag{6}$$

C_n^2 being the structure constant.

After inserting Eq. (4) into Eq. (3) and collecting the coefficients of $\mathbf{s}_n^2$ and $\mathbf{s}_n\mathbf{s}_p$ in the exponential arguments in separate x and y, 4×4 and 1×4 matrices, it is possible to solve the most inner quadruple integration in Eq. (3) in a semi-analytic fashion as described in [21,22]. From there, the aperture averaged scintillation results can eventually be attained by converting the outer double integration in Eq. (3) into grid summations over the aperture overlap area.

As explained in Section **1**, optical receiver will be vulnerable both to scintillation effects and amount of received power. In this sense, Eq. (2) is a unitless ratio of powers and do not carry any information about absolute power quantities. As a better measure of performance of the optical receiver, we rewrite Eq. (2) so that it expresses the per unit received power aperture averaged scintillation, hence

$$b_n^2(L)=\frac{b^2(L)}{\langle P(L)\rangle}\tag{7}$$

In order to assess the usefulness of Eq. (7), we provide the following examples based on the numeric values used in the next section.