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Propagation of focused vector laser beams in turbulent atmosphere

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ABSTRACT

By adopting a combination of beam coherence-polarization matrix and extended Huygens–Fresnel integral formula, analytical formulae are derived for the average intensity, the degree of polarization and the RMS (Root-Mean-Square) beam width of focused vector laser beams propagating in a turbulent atmosphere, and the propagation of focused vector laser beams is studied in detail. The spreading properties of vector laser beams and fundamental Gaussian beams are studied comparatively. It reveals that the focused vector laser beam evolves into Gaussian profile in shorter distance compared with collimated vector laser beams, and that destroying of polarization structure of focused ones is more severe. The spreading properties of the focused vector laser beams become the same as that of fundamental Gaussian beams faster. The propagation properties of focused vector laser beams are closely related to the parameters of the beam and the structure constant of the atmospheric turbulence. The results can provide useful reference in designing fiber laser system.

1. Introduction

Polarization is one important property of light and this vector nature of light and its interactions with matter make many optical devices and optical system designs possible [1]. Due to the unique feature, vector beams have attracted significant interest in recent years [2–5]. Radially polarized and azimuthally polarized beams are two typical cases of vector beams. It is well known that a radially polarized beam can be focused to a spot significantly smaller than for linear polarization [6] and an azimuthally polarized beam can be focused into a hollow dark spot [7], which is useful for many applications. The focusing properties, paraxial and non-paraxial propagation properties through paraxial optical system or free space have been widely studied [5,8–11]. On the other hand, propagation of laser beams through a turbulent atmosphere has many important applications in free space optical communications (FSOC), Laser Radar (LADAR), Light Detection and Ranging (LIDAR), remote sensing and imaging [12–14]. It is important to investigate the propagation properties of such laser beams in atmosphere. The propagation of collimated vector beams has been studied analytically [13] and numerically [12]. Focused laser beams are superior in energy delivering performance within tactical range, which have drawn attention [15]. However, to the best of knowledge, the propagation properties of focused vector beams in atmosphere have not been examined.

It is known that optical fiber can generate vector laser beams as well as fundamental Gaussian laser beams [16]. It is necessary to study the propagation properties of such two kinds of beams. In this paper, the propagation properties of focused vector beam in turbulent atmosphere have been studied based on extended Huygens–Fresnel integral formula and beam coherence-polarization matrix. The spreading properties of vector laser beams and fundamental Gaussian beams are studied comparatively. Some interesting analytical formulae, which can describe the propagation properties of the focused vector laser beams, are derived and numerical examples are illustrated to investigate the propagation properties of the focused vector beams.

2. Formulation

Within the framework of the paraxial approximation, the vectorial electric field of a radially polarized laser beam can be expressed as the coherent superposition of a TEM01 with a polarization direction parallel to the x-axis and a TEM10 with a polarization direction parallel to the y-axis [6,17]

$$E_r(x,y)=E_1\vec{e_x}+E_2\vec{e_y}=E_0\left\{\frac{x}{w_0}\exp\left[-\frac{(\vec{r})^2}{w_0^2}\right]\vec{e_x}+\frac{y}{w_0}\exp\left[-\frac{(\vec{r})^2}{w_0^2}\right]\vec{e_y}\right\} \quad (1)$$

where $(\vec{r})^2=x^2+y^2$ and w_0 denotes the beam waist size of a Gaussian beam, E_0 is a constant. Then the focused radially polarized laser beam can be expressed as

$$E_r(x,y)=E_1\vec{e_x}+E_2\vec{e_y}=E_0\exp\left[-\frac{ik}{2F}(\vec{r})^2\right]\left\{\frac{x}{w_0}\exp\left[-\frac{(\vec{r})^2}{w_0^2}\right]\vec{e_x}+\frac{y}{w_0}\exp\left[-\frac{(\vec{r})^2}{w_0^2}\right]\vec{e_y}\right\} \quad (2a)$$

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with F being the focal length. $F>0$ represents a convergent beam and $F<0$ represents a divergent beam. In the paper, $F>0$ is adopted.

In a similar way, the vectorial electric field of a focused azimuthally polarized laser beam can be expressed as follows:

$$E_\theta(x,y)=E_0\exp\left[-\frac{ik}{2F}(\vec{r})^2\right]\left\{-\frac{y}{w_0}\exp\left[-\frac{(\vec{r})^2}{w_0^2}\right]\vec{e_x}+\frac{x}{w_0}\exp\left[-\frac{(\vec{r})^2}{w_0^2}\right]\vec{e_y}\right\} \tag{2b}$$

It is known that the longitudinal electric field component is negligible [10,18], so we just consider the transverse component in Eqs. (1), (2a) and (2b). The beam coherence-polarization matrix (BCP) provides the information of polarization and spatial correlation, which is defined as [19]

$$\hat{\Gamma}(\vec{r_1},\vec{r_2},z)=\begin{bmatrix}\Gamma_{11}(\vec{r_1},\vec{r_2},z) & \Gamma_{12}(\vec{r_1},\vec{r_2},z)\\ \Gamma_{21}(\vec{r_1},\vec{r_2},z) & \Gamma_{22}(\vec{r_1},\vec{r_2},z)\end{bmatrix} \tag{3}$$

where

$$\Gamma_{\alpha\beta}(\vec{r_1},\vec{r_2},z)=\langle E_\alpha(\vec{r_1},\vec{r_2},z)E_\beta^*(\vec{r_1},\vec{r_2},z)\rangle,\quad(\alpha,\beta=1,2) \tag{4}$$

E_1 and E_2 are the component of the vectorial electric field in the x and y directions, respectively, and the angle brackets denote an ensemble average over the medium statistics. The equivalent irradiance distribution of a polarized beam is given by

$$I(\vec{r},z)=\Gamma_{11}(\vec{r},\vec{r},z)+\Gamma_{22}(\vec{r},\vec{r},z) \tag{5}$$

and the degree of polarization is expressed as

$$P(\vec{r},z)=\sqrt{1-\frac{4\det[\hat{\Gamma}(\vec{r},\vec{r},z)]}{\{Tr[\hat{\Gamma}(\vec{r},\vec{r},z)]\}^2}} \tag{6}$$

where det[•] and Tr[•] denote determinant and trace of the BCP matrix, respectively.

By applying Eqs. (2a) and (2b)–(4), the BCP matrices for a focused vector laser beam at source plane can be expressed as follows:

$$\hat{\Gamma}(\vec{r_1},\vec{r_2},z=0)=\frac{E_0^2}{w_0^2}\exp\left\{-\frac{1}{w_0^2}\left[(\vec{r_1})^2+(\vec{r_2})^2\right]\right\}\times\exp\left\{-\frac{ik}{2F}\left[(\vec{r_1})^2-(\vec{r_2})^2\right]\right\}\begin{bmatrix}x_1x_2 & x_1y_2\\ y_1x_2 & y_1y_2\end{bmatrix} \tag{7a}$$

$$\hat{\Gamma}(\vec{r_1},\vec{r_2},z=0)=\frac{E_0^2}{w_0^2}\exp\left\{-\frac{1}{w_0^2}\left[(\vec{r_1})^2+(\vec{r_2})^2\right]\right\}\times\exp\left\{-\frac{ik}{2F}\left[(\vec{r_1})^2-(\vec{r_2})^2\right]\right\}\begin{bmatrix}y_1y_2 & -x_2y_1\\ -x_1y_2 & x_1x_2\end{bmatrix} \tag{7b}$$

The paraxial propagation of a laser beam in a turbulent atmosphere can be treated with the well-known extended Huygens–Fresnel integral formula, and the elements of the BCP matrix $\Gamma_{\alpha\beta}(\vec{r_1},\vec{r_2},z)$ at the output plane are given as follows [15–20]:

$$\Gamma_{\alpha\beta}(\vec{r},\vec{r},z)=\frac{k^2}{4\pi^2z^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\Gamma_{\alpha\beta}(\vec{r_1},\vec{r_2},0)\exp\times\left[\frac{ik}{2z}(\vec{r_1}-\vec{r})^2-\frac{ik}{2z}(\vec{r_2}-\vec{r})^2\right]\times\exp\left[-\frac{1}{\rho_0^2}(\vec{r_1}-\vec{r_2})^2\right]d^2\vec{r_1}\,d^2\vec{r_2} \tag{8}$$

$\Gamma_{\alpha\beta}(\vec{r_1},\vec{r_2},0)$ is given by Eq. (4). $\rho_0=(0.545C_n^2k^2z)^{-3/5}$ is the coherence length of a spherical wave propagation with C_n^2 being the structure constant. We have employed Kolmogorov spectrum and a quadratic approximation for Rytov's phase structure function in the derivation of Eq. (8) [15–20]. The extended Huygens–Fresnel integral formula of Eq. (8) has been approved to be reliable in e.g. Refs. [21,27], and has been used widely (see e.g. Refs. [15–31]).

The tedious but straightforward integral calculations in Eq. (8) deliver

$$\Gamma_{r11}(\vec{r},\vec{r},z)=\frac{E_0^2}{w_0^2\tau^4}\left[\frac{w_0^2\tau_2^2}{4}+\frac{\tau_1^2+\tau_3^2}{\tau^2}x^2\right]\exp\left[-\frac{2}{w_0^2\tau^2}(\vec{r})^2\right] \tag{9a}$$

$$\Gamma_{r12}(\vec{r},\vec{r},z)=\Gamma_{r21}(\vec{r},\vec{r},z)=\frac{E_0^2}{w_0^2}\frac{\tau_1^2+\tau_3^2}{\tau^6}xy\exp\left[-\frac{2}{w_0^2\tau^2}(\vec{r})^2\right] \tag{9b}$$

$$\Gamma_{r22}(\vec{r},\vec{r},z)=\frac{E_0^2}{w_0^2\tau^4}\left[\frac{w_0^2\tau_2^2}{4}+\frac{\tau_1^2+\tau_3^2}{\tau^2}y^2\right]\exp\left[-\frac{2}{w_0^2\tau^2}(\vec{r})^2\right] \tag{9c}$$

with

$$\tau^2=\tau_1^2+\tau_2^2+\tau_3^2 \tag{10}$$

Here $\tau_1=2z/kw_0^2$, $\tau_2=2\sqrt{2}z/kw_0\rho_0$ and $\tau_3=1-(z/F)$. The parameters τ_1, τ_2, and τ_3 can be interpreted as the factors that describe the beam spreading due to diffraction, turbulence, and geometrical magnification, respectively. When $F\to\infty$, Eqs. (9a), (9b), (9c) and (10) can be reduced into the expressions in Ref.[13]. In a similar way, we obtain the following expressions for the elements of BCP matrix of an azimuthally polarized laser beam

$$\Gamma_{\theta11}(\vec{r},z)=\Gamma_{r22}(\vec{r},z),\quad\Gamma_{\theta22}(\vec{r},z)=\Gamma_{r11}(\vec{r},z),$$
$$\Gamma_{\theta12}(\vec{r},z)=\Gamma_{\theta21}(\vec{r},z)=-\Gamma_{r12}(\vec{r},z) \tag{11}$$

At the focal plane, where $\tau_3=0$, Eqs. (9a), (9b), (9c) and (10) can be simplified as

$$\Gamma_{r11}(\vec{r},\vec{r},z=F)=\frac{E_0^2}{w_0^2\tau^4}\left[\frac{w_0^2\tau_2^2}{4}+\frac{\tau_1^2}{\tau^2}x^2\right]\exp\left[-\frac{2}{w_0^2\tau^2}(\vec{r})^2\right] \tag{12a}$$

$$\Gamma_{r12}(\vec{r},\vec{r},z=F)=\Gamma_{r21}(\vec{r},\vec{r},z=F)=\frac{E_0^2}{w_0^2}\frac{\tau_1^2}{\tau^6}xy\exp\left[-\frac{2}{w_0^2\tau^2}(\vec{r})^2\right] \tag{12b}$$

$$\Gamma_{r22}(\vec{r},\vec{r},z=F)=\frac{E_0^2}{w_0^2\tau^4}\left[\frac{w_0^2\tau_2^2}{4}+\frac{\tau_1^2}{\tau^2}y^2\right]\exp\left[-\frac{2}{w_0^2\tau^2}(\vec{r})^2\right] \tag{12c}$$

with

$$\tau^2=\tau_1^2+\tau_2^2 \tag{13}$$

From Eqs. (5), (11), (12a), (12b) and (12c), the intensity distribution in a turbulent atmosphere at focal plane is

$$I(\vec{r},z)=\frac{E_0^2}{w_0^2\tau^4}\left[\frac{w_0^2\tau_2^2}{2}+\frac{\tau_1^2}{\tau^2}(\vec{r})^2\right]\exp\left[-\frac{2}{w_0^2\tau^2}(\vec{r})^2\right] \tag{14}$$

The spreading property of the laser beams propagating in turbulence can be characterized by the RMS (Root-Mean-Square) beam width at the z plane, which is defined as [31]

$$w_x^2(z)=\frac{4\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}x^2I(\vec{r},z)d^2\vec{r}}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}I(\vec{r},z)d^2\vec{r}} \tag{15a}$$

$$w_y^2(z)=\frac{4\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}y^2I(\vec{r},z)d^2\vec{r}}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}I(\vec{r},z)d^2\vec{r}} \tag{15b}$$

The RMS beam width at focal plane for focused vector laser beam can be expressed as

$$w_x^2(z)=\frac{4\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}x^2I(\vec{r},z)d^2\vec{r}}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}I(\vec{r},z)d^2\vec{r}}=\left(1+\frac{\tau_1^2}{\tau_1^2+\tau_2^2}\right)w_0^2\tau^2 \tag{16a}$$