Available online at www.sciencedirect.com

ScienceDirect

IFAC-PapersOnLine 48-9 (2015) 061–066

Black-box and grey-box identification of the attitude dynamics for a variable-pitch quadrotor

Pietro Panizza * **Fabio Riccardi** * **Marco Lovera** *

* *Dipartimento di Scienze e Tecnologie Aerospaziali, Politecnico di Milano, Via la Masa 34, 20156 Milano, Italy, (e-mail: pietro.panizza@polimi.it, fabio.riccardi@polimi.it)*

Abstract: System identification is now a well established approach for the development of control-oriented models in the rotorcraft field (see, *e.g.*, the survey paper Hamel and Kaletka (1997), the recent books Tischler and Remple (2006), Jategaonkar (2006) and the references therein). Though the application to full scale rotorcraft is by now fairly mature, less experience has been gathered on small-scale vehicles, such as, *e.g.*, quadrotors. This paper deals with the problem of characterizing the attitude dynamics of a variable-pitch quadrotor from data and presents the results obtained in an experimental identification campaign. More precisely, on-line and off-line methods have been considered and the performance of black-box versus grey-box models has been compared.

1. INTRODUCTION AND MOTIVATION

The interest in quadrotors as platforms for both research and commercial unmanned aerial vehicle (UAV) applications is steadily increasing. In particular, some of the envisaged applications for quadrotors lead to tight performance requirements on the attitude control system, so wide bandwidth controllers must be designed. This, in turn, calls for increasingly accurate dynamic models of the vehicle's response to which advanced controller synthesis approaches can be applied. The problem of mathematical modelling of quadrotor dynamics has been studied extensively in the literature, see, *e.g.*, Mahony and Kumar (2012) and the references therein. In particular, it is apparent from the literature that mathematical models for quadrotor dynamics are easy to establish as far the kinematics and dynamics of linear and angular motion are concerned, so that a large portion of the literature dealing with quadrotor control is based on such models. Unfortunately, characterizing aerodynamic effects and additional dynamics such as, *e.g.*, due to actuators and sensors, is far from trivial, and has led to an increasing interest in the experimental characterization of the dynamic response of the quadrotor. More precisely, two classes of methods to deal with this problem can be envisaged. The first class of methods is based on black-box identification and aims at modeling the dynamics of the system directly (and solely) from measured input-output data (see for example La Civita et al. (2002)). The second class of methods is based on the calibration of the parameters of detailed physical models, see for example Kim and Tilbury (2004). In the present framework, key requirements for the identification method and the model class are the degree of automation of the identification procedure and the compatibility of the model class with existing control synthesis tools. Meeting such requirements would enable a fast and reliable deployment of the vehicle's control system.

In view of the above discussion, this paper aims at characterizing the attitude dynamics of a variable-pitch quadrotor directly from data and presents the results obtained in an experimental identification campaign based on the Aermatica Anteos quadrotor UAV, a platform having a MTOW of about 5 kg and an arm length of $d = 0.415\,\mathrm{m}$ with variable collective pitch - fixed rotor RPM architecture. More precisely, a number of different model identification methods have been considered in this study, with the aim of covering: on-line and off-line estimation, input-output and state space models, black-box and grey-box modeling approaches. With respect to preliminary results presented in Riccardi et al. (2014), more advanced subspace identification algorithms have been considered, with the ability of dealing with data generated in closed-loop. This paper is organized as follows: Section 2 presents the approach to model identification of the pitch dynamics as well as the corresponding experiments. In Section 3 the black-box model identification methods are illustrated. Subsequently, the grey-box methods are described in Section 4. Finally, Section 5 presents the results of the identification process; these results are then validated in the same section.

Fig. 1. Aermatica Anteos on laboratory test-bed.

2. IDENTIFICATION EXPERIMENTS

The pitch attitude identification experiments discussed in this paper have been carried out in laboratory conditions, with the quadrotor placed on a test-bed that constrains all translational and rotational degrees of freedom (DoFs) except for pitch rotation, as shown in Figure 1. Similar

10.1016/j.ifacol.2015.08.060

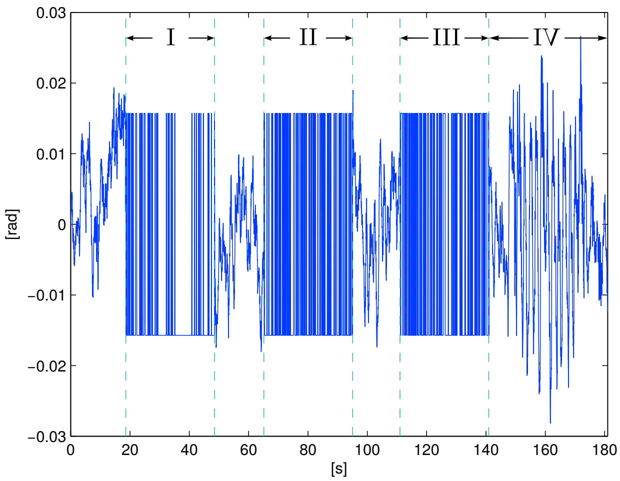

Fig. 2. Input signal of an identification test, namely the difference between collective blade pitch on opposite rotors (I, II and III are three different PRBS excitation sequences; IV represents a typical flight condition where a desired angular reference is imposed).

experiments have been carried out in flight to ensure that the indoor setup is representative of the actual attitude dynamics in flight for near hovering conditions. The manipulated variable of the real system is the difference between collective blade pitch on opposite rotors. Even in controlled laboratory conditions, the design of excitation sequences for the attitude dynamics of the quadrotor is a critical issue because of the inherent (fast) instability. In the present study a Pseudo Random Binary Sequence (PRBS, see Ljung (1999)) was selected and applied in quasi open-loop conditions: while the nominal attitude and position controllers were disabled, a supervision task enforcing attitude limits during the experiment was left active. The parameters of the PRBS sequence (signal amplitude and min/max switching interval) were tuned to obtain an excitation spectrum consistent with the expected dominant attitude dynamics, between 3 rad/s and 6 rad/s. As illustrated in Figure 2, the input signal of each identification experiment consists of three different PRBS excitation sequences (I, II, III in Figure 2) with the same switching interval and the same amplitude while in the last section of each identification test (IV in Figure 2), the nominal attitude controller was reactivated and a desired angular reference was manually imposed. This latter portion of each dataset is not tied to the parameters of the PRBS in the identification experiment and is collected for validation purposes since it representative of a typical closed-loop flight condition. For further details on the design of the identification experiments and the construction of the identification and validation datasets the interested reader is referred to Riccardi et al. (2014). Finally, during the tests the following variables were logged, with sampling time equal to 0.02 s: input manipulated variable u, pitch angular acceleration $\dot{q}$, angular velocity q and angle θ measured by the on-board Inertial Measurement Unit (IMU).

3. BLACK-BOX MODEL IDENTIFICATION

The problem of black-box model identification for the attitude dynamics of hovering quadrotors has been studied extensively in the literature (see, *e.g.*, Bergamasco and Lovera (2011, 2013, 2014) and the references therein for a detailed discussion). In particular, from the cited references, subspace model identification (SMI) methods emerge as a viable approach for the task. In view of this, the identification algorithm selected for this work is the PBISD subspace identification method(see, *e.g.*, Chiuso (2007)). This algorithm, which is briefly described in the following, considers the finite dimensional, linear time-invariant (LTI) state space model class

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) + w(k) \\ y(k) &= Cx(k) + Du(k) + v(k) \end{aligned} \tag{1}$$

where $x(k) \in \mathbb{R}^n$, $u(k) \in \mathbb{R}^m$, $y(k) \in \mathbb{R}^p$ and $\{v(k), w(k)\}$ are ergodic sequences of finite variance satisfying

$$E[\begin{bmatrix} w(t) \\ v(t) \end{bmatrix} \begin{bmatrix} w(s)^T & v(s)^T \end{bmatrix}] = \begin{bmatrix} Q & S \\ S^T & R \end{bmatrix} \delta_{s,t},$$

with $\delta_{s,t}$ denoting the Kronecker delta function, possibly correlated with the input u.

Let now

$$z(k) = \begin{bmatrix} u^T(k) & y^T(k) \end{bmatrix}^T$$

and

$$\bar{A} = A - KC, \quad \bar{B} = B - KD, \quad \widetilde{B} = \begin{bmatrix} \bar{B} & K \end{bmatrix},$$

where K is the Kalman gain associated with (1), and note that system (1) can be written as

$$\begin{aligned} x(k+1) &= \bar{A}x(k) + \widetilde{B}z(k) \\ y(k) &= Cx(k) + Du(k) + e(k), \end{aligned} \tag{2}$$

where e is the innovation vector. The data equations for the PBSID algorithm can be then derived by noting that propagating $p-1$ steps forward the first of equations (2), where p is the so-called past window length, one gets

$$\begin{aligned} x(k+2) &= \bar{A}^2 x(k) + \begin{bmatrix} \bar{A}\widetilde{B} & \widetilde{B} \end{bmatrix} \begin{bmatrix} z(k) \\ z(k+1) \end{bmatrix} \\ &\vdots \\ x(k+p) &= \bar{A}^p x(k) + \mathcal{K}^p Z^{0,p-1} \end{aligned} \tag{3}$$

where

$$\mathcal{K}^p = \begin{bmatrix} \bar{A}^{p-1}\widetilde{B}_0 & \dots & \widetilde{B} \end{bmatrix} \tag{4}$$

is the extended controllability matrix of the system and

$$Z^{0,p-1} = \begin{bmatrix} z(k) \\ \vdots \\ z(k+p-1) \end{bmatrix}.$$

Under the considered assumptions, $\bar{A}$ represents the dynamics of the optimal one-step ahead predictor for the system and therefore has all the eigenvalues inside the open unit circle, so the term $\bar{A}^p x(k)$ is negligible for sufficiently large values of p and we have that

$$x(k+p) \simeq \mathcal{K}^p Z^{0,p-1}.$$

As a consequence, the input-output behaviour of the system is approximately given by

$$\begin{aligned} y(k+p) &\simeq C\mathcal{K}^p Z^{0,p-1} + Du(k+p) + e(k+p) \\ &\vdots \\ y(k+p+f) &\simeq C\mathcal{K}^p Z^{f,p+f-1} + Du(k+p+f) + \\ &+ e(k+p+f), \end{aligned} \tag{5}$$

so that, introducing the matrix notation defined in the previous subsection, the data equations are given by

$$\begin{aligned} X^{p,f} &\simeq \mathcal{K}^p \bar{Z}^{p,f} \\ Y^{p,f} &\simeq C\mathcal{K}^p \bar{Z}^{p,f} + DU^{p,f} + E^{p,f}. \end{aligned} \tag{6}$$

Considering $p = f$, estimates for the matrices $C\mathcal{K}^p$ and D are first computed by solving the least-squares problem

$$\min_{C\mathcal{K}^p, D} \| Y^{p,p} - C\mathcal{K}^p \bar{Z}^{p,p} - DU^{p,p} \|_F. \tag{7}$$