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IFAC-PapersOnLine 48-9 (2015) 120–125

Singular Perturbation Control of the Lateral-Directional Flight Dynamics of an UAV

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Abstract: This paper presents a singular perturbation control strategy for regulating the lateral-directional flight dynamics of an Unmanned Air Vehicle (UAV). The proposed control strategy is based on a four-time-scale (4TS) decomposition that includes the side-slip velocity, bank angle, yaw rate and roll rate dynamics, with the control signals being the aileron and rudder deflection. The nonlinear control strategy drives the system to follow a reference in load factor which in return provides references in bank angle, side-slip velocity and yaw rate. In addition, the control strategy permits to select the desired dynamics for all the singularly perturbed subsystems. Numerical results are included for a realistic nonlinear UAV model, including saturation on the control signals, and unmodeled dynamics.

Keywords: control strategies, singular perturbations, time scales, lateral-directional, UAV

1. INTRODUCTION

Historically, classical linear control techniques have been sufficient to obtain reasonable control responses of aerospace systems, but the evolution of the aerospace industry, and the consequent improvement of technologies, has increased the performance requirements of all systems in general which, in addition, has called for better control designs capable of dealing with more complex systems. Specifically, in the area of aerospace systems, a wide range of different nonlinear control techniques have been studied to deal with the nonlinear dynamics of such systems. From singular perturbation Kokotović et al. (1999); Esteban et al. (2013), feedback linearization Brockett (1978), dynamic inversion Buffington et al. (1993), sliding mode control Sira-Ramírez et al. (1994), backstepping control methods Khalil (1996); Gavilan et al. (2011), or neural networks Balakrishnan and Biega (1996); Balakrishnan and Esteban (2001).

One of the most challenging tasks in control is the modeling of systems in which the presence of parasitic parameters, such as small time constants, is often the source of an increased order and stiffness Naidu and Calise (2001). The stiffness, attributed to the simultaneous occurrence of slow and fast phenomena, gives rise to time-scales, and the suppression of the small parasitic variables results in degenerated, reduced-order systems called singularly perturbed systems (SPS) that can be stabilized separately, thus simplifying the burden of control design of high-order systems.

The application of singular perturbation and time-scale techniques in the aerospace industry can be traced back to the 1960s when it was first applied to solve complex flight optimization problems Mehra et al. (1979). Since then, singular perturbation and time-scale techniques have been extensively used in the aerospace industry Naidu and Calise (2001). In recent years these techniques have been also extended to UAVs Esteban et al. (2013); Esteban and Rivas (2012); Bertrand et al. (2011).

The objective of this paper is to develop a singular perturbation control strategy for the lateral-directional dynamics of an aircraft, that will be able to follow load factor reference that provides additional references in bank angle, side-slip velocity and yaw rate, using as control actuators the aileron and rudder deflection. In addition, the proposed singular perturbation control strategy permits to select the desired closed-loop dynamics of each of the resulting reduced-order and boundary-layer subsystems using a time-scale analysis similar to those presented in Esteban (2011); Esteban et al. (2013); Esteban and Rivas (2012). Simulations are included for a realistic UAV model including nonlinear dynamics and actuator saturation on both the aileron and rudder deflection. The model used corresponds to the Cefiro aircraft Bernal et al. (2009), an UAV recently designed and constructed by the authors at the University of Seville, which will be used for future flight simulations.

This paper is structured as follows: Section 2 presents the flight dynamics used throughout this work; Section 3 presents the time scales selection; Section 4 describes the proposed 4-time-scale analysis; the singular perturbed

Peer review under responsibility of International Federation of Automatic Control.
10.1016/j.ifacol.2015.08.070

control strategies are presented in Section 5; numerical results are given in Section 6; and finally, some conclusions are drawn in Section 7.

2. MODEL DEFINITION

The problem discussed in this article considers a constant-mass UAV with an electrical propulsion plant, for which the point-mass lateral flight dynamics equations are given by

$$\dot{\beta} = \frac{\dot{v}V + \dot{V}v}{V^2 \cos\beta} \quad (1)$$

$$\dot{v} = -ru + pw + Y/m + g\cos\theta\sin\phi \quad (2)$$

$$\dot{\phi} = p - (q\sin\phi + r\cos\phi)\tan\theta \quad (3)$$

$$\dot{\psi} = (q\sin\phi + r\cos\phi)\sec\theta \quad (4)$$

$$N = I_z\dot{r} - I_{xz}\dot{p} + (I_y - I_x)pq - I_{xz}qr, \quad (5)$$

$$\bar{L} = I_x\dot{p} - I_{xz}\dot{r} + (I_z - I_y)qr - I_{zz}pq, \quad (6)$$

where u, v and w are the components of the aerodynamic velocity along the x,y and z components respectively, ϕ, θ and ψ represent bank, pitch and yaw angles respectively, and p, q and r the roll, pitch and yaw velocities; β is the side-slip velocity and V is de aerodynamic speed; Y is de aerodynamic side force, L and N are the yawing and roll moment components respectively; m the mass, and I_y, I_x, I_z, I_{xz} the moments of inertia of the UAV. The aerodynamic forces and moments are defined as $Y = q_\infty SC_y$, $\bar{L} = q_\infty SbC_l$, and $N = q_\infty SbC_n$, with $q_\infty = 1/2\rho V^2$ being the dynamic pressure, S the reference wing area, b the wing span, and C_y, C_l, and C_n being the aerodynamic force and moments coefficients expressed in wind-axes, which are given by the following standard models Roskam (2001)

$$C_y = C_{y_s}, \quad (7)$$

$$C_l = C_{l_s}\cos\alpha - C_{n_s}\sin\alpha, \quad (8)$$

$$C_n = C_{l_s}\sin\alpha + C_{n_s}\cos\alpha, \quad (9)$$

where α is the angle of attack, given by $\alpha = \theta - \gamma$, γ is the flight path angle, and C_{y_s}, C_{l_s}, and C_{n_s} are the aerodynamic force and moments coefficients expressed in stability axis given by the following approximations

$$C_{y_s} = C_{y_\beta}\beta + C_{y_p}c_d p + C_{y_r}c_d r + C_{y_{\delta_r}}\delta_r, \quad (10)$$

$$C_{l_s} = C_{l_\beta}\beta + C_{l_p}c_d p + C_{l_r}c_d r + C_{l_{\delta_r}}\delta_r + C_{l_{\delta_a}}\delta_r, \quad (11)$$

$$C_{n_s} = C_{n_\beta}\beta + C_{n_p}c_d p + C_{n_r}c_d r + C_{n_{\delta_r}}\delta_r + C_{n_{\delta_a}}\delta_a, \quad (12)$$

with $c_d = b/2V$, δ_r and δ_a are the rudder and aileron deflection, with the following saturation limits $-30° \leq \delta_r \leq 30°$ and $-30° \leq \delta_a \leq 30°$; the stability coefficients C_{y_β}, C_{y_p}, C_{y_r}, $C_{y_{\delta_r}}$, C_{l_β}, C_{l_p}, C_{l_r}, $C_{l_{\delta_r}}$, $C_{l_{\delta_a}}$, C_{n_β}, C_{n_p}, C_{n_r}, $C_{n_{\delta_r}}$, and $C_{n_{\delta_a}}$ are known Bernal et al. (2009). The simplifying assumption of constant air density is considered throughout this paper, and it is also assumed that the longitudinal dynamics are stabilized at a desired aerodynamic speed (V_1) and altitude (h_1), therefore considering that the longitudinal variables (u_1, w_1, θ_1, α_1, and q_1), are maintained constant by means of the singular perturbation control strategy presented in Esteban and Rivas (2012). It is also assumed that no perturbations are introduced in the model, such wind or unmodelled dynamics, which will be dealt in a future adaptive version of this control strategy. With this in mind, Equations (1–6) are expanded using Eqns. (10-12), resulting in

$$\dot{\beta} = \frac{1}{\cos\beta}\left[(-\beta_1 r + \beta_2 p)\cos\beta + \beta_3\beta + \beta_4 p + \beta_5 r + \beta_6\delta_r + \beta_7\sin\phi\right] \quad (13)$$

$$\dot{\phi} = p - (q_1\sin\phi + r\cos\phi)\tan\theta_1 \quad (14)$$

$$\dot{\psi} = (q\sin\phi + r\cos\phi)\sec\theta \quad (15)$$

$$\dot{r} = \hat{r}_1\beta + \hat{r}_2 p + \hat{r}_3 r + \hat{r}_4\delta_a + \hat{r}_4\delta_r, \quad (16)$$

$$\dot{p} = \hat{p}_1\beta + \hat{p}_2 p + \hat{p}_3 r + \hat{p}_4\delta_a + \hat{p}_4\delta_r, \quad (17)$$

where $\beta_1 = \cos\alpha_1$, $\beta_2 = \sin\alpha_1$, $\beta_2 = \cos\alpha_1$, $\beta_3 = K_1 C_{y_\beta}$, $\beta_4 = K_2 C_{y_p}$, $\beta_5 = K_2 C_{y_r}$, $\beta_6 = K_1 C_{y_{\delta_r}}$, $K_1 = q_\infty S/Vm$, $K_2 = 2q_\infty Sb/V^2 m$, $r_1 = K_1 C_{n_\beta}$, $r_2 = K_2 C_{n_p}$, $r_3 = K_2 C_{n_r}$, $r_4 = K_1 C_{n_{\delta_a}}$, $r_5 = K_1 C_{n_{\delta_r}}$, $p_1 = K_1 C_{l_\beta}$, $p_2 = K_2 C_{l_p}$, $p_3 = K_2 C_{l_r}$, $p_4 = K_1 C_{l_{\delta_a}}$, $p_5 = K_1 C_{l_{\delta_r}}$.

The underactuated structure of the system, requires to define several variables as references. The maneuver being considered in this article is a steady state level turn, where $h_1 = cte.$ and $V_1 = cte.$, therefore $\theta_1 \approx \alpha_1$, and $\dot{V} = 0$, not to confuse with a coordinated level turn, in which the side-slip velocity is null. The particular steady state level turn considered in this article is conducted with $p_1 = 0$, which for a given radii of turn R_T, it can be proven that Roskam (2001) provides also a reference bank angle obtained with the expression $\phi_{ref} = R_T g/V^2$, which in addition results in a reference load factor $n_{ref} = 1/\cos\phi_{ref}$, $r_{ref} = g\sin\phi_{ref}/V$, and $q_{ref} = g\left(n_{ref} - 1/n_{ref}\right)/V$. It can be proven that analyzing the equilibrium conditions of Eqs. (13-16)by substituting the reference values, a β_{ref} can be obtained which will be also used in the control strategy. As it will be seen in the simulations, for a steady state level turn, the values of $\beta_{ref} < 0.5°$, as it can be seen in Table 2, so although the maneuver is not strictly a coordinated level turn, the lateral acceleration values can be considered small.

3. TIME SCALES SELECTION

The appropriate selection of time scales is an important aspect of singular perturbation and time-scales theory Naidu and Calise (2001); Ardema and Rajan (1985); Heiges et al. (1992), and can be categorized into three approaches: 1) direct identification of small parameters (such as small time constants); 2) transformation of state equations; and 3) linearization of the state equations. Ardema Ardema and Rajan (1985) proposes a rational method of identifying time scales separations that does not rely on an *ad hoc* selection of time scales based largely on physical insight and past experiences with similar problems. The proposed method only requires a knowledge of the state equations. Considering a dynamical systems of the form

$$\dot{x} = f(x, u),\ u \in U, \quad (18)$$

subject to suitable boundary conditions, where x is an n-dimensional state vector, u a r-dimensional control vector, and U the set of admissible controls. It is assumed that bounds have been established on the components of the state vector, either by physical limitations or by a desire to restrict the state to a certain region of state space, $x_{i,m} \leq x_i \leq x_{i,M}$, with $x_{i,m}$ and $x_{i,M}$ representing the minimum and maximum values of the state variables. As noted in Ardema and Rajan (1985), most *ad hoc* assessments of time-scale separation are based on the concept of state variable speed Mehra et al. (1979). The speed of a state