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# Averaging of receiver aperture for flat-topped incidence

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ABSTRACT

Using a flat-topped profile for the incident beam, the power scintillation index for weak atmospheric turbulence is formulated and analytically evaluated. Through the use of the aperture averaging factor, the averaging effect of the finite receiver aperture on the intensity fluctuations for a flat-topped incident beam is examined. The influence of the order of flatness on the averaging is investigated. At large propagation lengths, increasing the flatness parameter decreases the power scintillations and it is possible to further reduce the scintillation by increasing the receiver aperture. Increasing the structure constant increases this effect.



## 1. Introduction

The fluctuation of the received intensity imposes signal dependent noise deteriorating the performance in the design of a wireless optical link operating in turbulent atmosphere. Various techniques are applied to reduce the effect of this noise which is quantified by the scintillation index. One of these techniques is known as the receiver aperture averaging in which a large area collecting aperture is employed. Receiver aperture averaging has been reported and discussed by many researchers. If the collecting diameter of a receiver aperture is larger than the spatial scale of the optical scintillations, the receiver can reduce the intensity fluctuations [1] and this is known as the receiver aperture averaging. The decrease in the scintillations with an increasing telescope collecting area had been recognized in early astronomical measurements made in the 1950s [2]. Aperture averaging effects have been studied in the context of laser beam propagation through atmospheric turbulence [3–8]. The receiver aperture averaging effects for the plane wave intensity fluctuation have been studied by Tatarskii [9]. Closed-form representation of the receiver aperture averaging effect is obtained for a beam wave in turbulent atmosphere where the Gaussian weighting function for the receiver aperture is utilized [10]. The propagation properties of a flattened Gaussian beam with a circular [11] and misaligned circular [12] transmitter aperture in turbulent atmosphere have been examined. The effects of turbulence on the scintillation index of flat-topped beams when a point detector is used are also studied in detail [13–15]. Recently, the aperture averaging effect for Gaussian beams has been analyzed in weak and moderate turbulence conditions by using the statistics of the exponentiated Weibull distribution family [16].

In this paper, we investigate the behavior of the intensity fluctuations when a realistic receiver possessing a finite sized aperture is employed in an atmospheric optics link. Our aim is to find out whether the use of flat-topped incident structures will help in further reducing the scintillation noise in an atmospheric optical link that uses a finite sized receiver aperture, and to examine the effect of the receiver aperture size in this reduction.

## 2. Formulation

The flat-topped field is expressed as a superposition of Gaussian beams as [17]

$$u(\mathbf{s}) = \sum_{n=1}^{N} \frac{(-1)^{n-1}}{N} \binom{N}{n} \exp\left[-\frac{n(s_x^2+s_y^2)}{2\alpha_s^2}\right], \tag{1}$$

where $s=(s_x, s_y)$ is the source transverse coordinate, $N$ is the number of Gaussian beams, also known as the flatness parameter and $\alpha_s$ is the Gaussian source size. From the extended Huygens–Fresnel integral, the average intensity at the receiver plane is found to be [10]

$$\langle I(\mathbf{p})\rangle = \frac{1}{(\lambda L)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathbf{ds}_1^2\mathbf{ds}_2^2 u(\mathbf{s}_1)u^*(\mathbf{s}_2) \times \exp\left\{\frac{jk}{2L}\left[(\mathbf{p}-\mathbf{s_1})^2-(\mathbf{p}-\mathbf{s_2})^2\right]-\rho_0^{-2}(\mathbf{s}_1-\mathbf{s}_2)^2\right\} \tag{2}$$

where $L$ is the propagation length, $\lambda$ is the wavelength, $j=\sqrt{-1}$, $*$ denotes complex conjugate, $p=(p_x,\ p_y)$ is the receiver

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transverse coordinate, $\rho_0=(0.546C_n^2k^2L)^{-3/5}$ is the coherence length of a spherical wave propagating in the turbulent medium, $k=2\pi/\lambda$, $C_n^2$ is the structure constant. Solving Eq. (2) by the repeated use of Eq. (3.323.2) in [18]

$$\langle I(\mathbf{p})\rangle=\frac{\pi^2}{(\lambda L)^2}\sum_{n=1}^{N}\sum_{m=1}^{N}\frac{(-1)^{n+m}N^{-2}}{t_{1x}t_{2x}t_{1y}t_{2y}}\binom{N}{n}\binom{N}{m}\exp\left(-\frac{k^2p_x^2}{4t_{1x}^2L^2}\right)$$
$$\times\exp\left[0.25t_{2x}^{-2}\left(-\frac{k^2p_x^2}{L^2}-\frac{k^2p_x^2}{t_{1x}^4L^2\rho_0^4}+\frac{2k^2p_x^2}{L^2t_{1x}^2\rho_0^2}\right)\right]$$
$$\times\exp\left(-\frac{k^2p_y^2}{4t_{1y}^2L^2}\right)\exp\left[0.25t_{2y}^{-2}\left(-\frac{k^2p_y^2}{L^2}-\frac{k^2p_y^2}{t_{1y}^4L^2\rho_0^4}+\frac{2k^2p_y^2}{L^2t_{1y}^2\rho_0^2}\right)\right],\tag{3}$$

where $t_{1x}=(0.5n\alpha_s^{-2}-0.5jkL^{-1}+\rho_0^{-2})^{0.5}$, and $t_{2x}=(0.5m\alpha_s^{-2}+0.5jkL^{-1}+\rho_0^{-2}-t_{1x}^{-2}\rho_0^{-4})^{1/2}$. $t_{1y}$ and $t_{2y}$ are attained by changing all $x$ subscripts to $y$ in $t_{1x}$ and $t_{2x}$, respectively.

The average power detected by a finite sized receiver having Gaussian aperture function is

$$\langle P\rangle=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle I(\mathbf{p})\rangle\exp\left(-\frac{|\mathbf{p}|^2}{R^2}\right)d^2\mathbf{p},\tag{4}$$

where $R$ is the radius of the receiver size. Substituting Eq. (3) in Eq. (4) and performing the integrations the average power can be found as

$$\langle P\rangle=\sum_{n=1}^{N}\sum_{m=1}^{N}\frac{\pi}{t_{1x}t_{2x}t_{1y}t_{2y}}\frac{(-1)^{n+m}}{N^2}\binom{N}{n}\binom{N}{m}$$
$$\times\left(\frac{4L^2}{k^2R^2}+\frac{1}{t_{1x}^2}+\frac{1}{t_{2x}^2}+\frac{1}{t_{2x}^2t_{1x}^4\rho_0^4}-\frac{2}{t_{2x}^2t_{1x}^2\rho_0^2}\right)^{-0.5}$$
$$\times\left(\frac{4L^2}{k^2R^2}+\frac{1}{t_{1y}^2}+\frac{1}{t_{2y}^2}+\frac{1}{t_{2y}^2t_{1y}^4\rho_0^4}-\frac{2}{t_{2y}^2t_{1y}^2\rho_0^2}\right)^{-0.5}\tag{5}$$

The average of the square of the power as detected by a finite sized receiver having a Gaussian aperture function is found as [10]

$$\langle P^2\rangle=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle I(\mathbf{p}_1)I(\mathbf{p}_2)\rangle\exp\left(-\frac{|\mathbf{p}_1|^2+|\mathbf{p}_2|^2}{R^2}\right)d^2\mathbf{p}_1\,d^2\mathbf{p}_2\tag{6}$$

where the derivation and the resulting formula for the lengthy $\langle P^2\rangle$ is provided in Appendix A.

The power scintillation is given by [19]

$$m_p^2=\frac{\langle(P-\langle P\rangle)^2\rangle}{\langle P\rangle^2}=\frac{\langle P^2\rangle}{\langle P\rangle^2}-1\tag{7}$$

The receiver aperture averaging factor $G_R$ is defined as [19]

$$G_R=\frac{m_p^2}{m_p^2|_{R=0}}\tag{8}$$

where $m_p^2|_{R=0}$ is the scintillation index for a point aperture.

## 3. Results

In this section by using Eqs. (7) and (8), the effects of the receiver aperture averaging against the variations in the radius of the receiver aperture, the Gaussian source size, the propagation length and the flatness parameter are analyzed. In all the figures $\lambda=1.55\ \mu m$ and except in Fig. 4, $L=3$ km are taken. Our results are checked with the existing receiver aperture averaging results for the Gaussian beams in turbulent medium [10] and with the scintillation index for the coherent flat-topped beam in turbulence [16]. In Fig. 1, the receiver aperture averaging factor against the radius of receiver aperture is plotted in the limiting case of a Gaussian beam for different $C_n^2$ values at $\alpha_s=5$ cm. Being a check case [10], it is seen from Fig. 1 that as $C_n^2$ increases, after a certain value of $R$, the power scintillation starts to decrease and thus the receiver aperture averaging effect increases. In Fig. 2, the receiver aperture averaging factor is plotted for the flat-topped beam with $N=4$ against the radius of receiver aperture. When Figs. 1 and 2 are compared, it is seen that the receiver aperture averaging is more effective at the larger flatness parameter with increasing receiver aperture sizes. In Fig. 3, the receiver aperture averaging factor against the Gaussian source size is shown for flat-topped and Gaussian beams at different $R$ values. We note that due to the normalization as defined in Eq. (8), the curves ($N$=1, $R$=0) and ($N$=8, $R$=0) are the same and attain unity values for all the source sizes. The conclusion drawn from Fig. 3 is that the receiver aperture averaging effect is strong for the flat-topped beam of larger source sizes whereas for the Gaussian beam, receiver aperture averaging occurs at moderately smaller source sizes. Fig. 4 shows the power scintillation against the propagation length at different Gaussian source sizes for the flat-topped and Gaussian beams where $C_n^2=1.5\times10^{-15}\ \mathrm{m}^{-2/3}$ and $R$=9 cm are taken. For relatively smaller propagation lengths, power scintillations for both flat-topped and Gaussian beams first increase as the

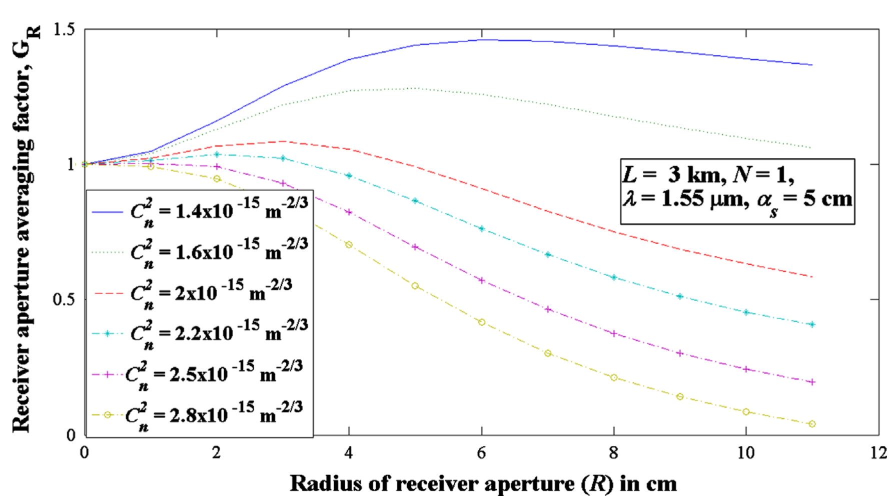


**Fig. 1.** The receiver aperture averaging factor $G_R$ versus the radius of the receiver aperture $R$ at $N$=1, $L$=3 km and $\alpha_s=5$ cm for different $C_n^2$ values.