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Model Predictive Control for Spacecraft Rendezvous in Elliptical Orbits with On/Off Thrusters ⋆

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Abstract: In previous works, the authors have developed a trajectory planning algorithm for spacecraft rendezvous which computed optimal Pulse-Width Modulated (PWM) control signals, for circular and eccentric Keplerian orbits. The algorithm is initialized by solving the impulsive problem first and then, using explicit linearization and linear programming, the solution is refined until a (possibly local) optimal value is reached. However, trajectory planning cannot take into account orbital perturbations, disturbances or model errors. To overcome these issues, in this paper we develop a Model Predictive Control (MPC) algorithm based on the open-loop PWM planner and test it for elliptical target orbits with arbitrary eccentricity (using the linear time-varying Tschauner-Hempel model). The MPC is initialized by first solving the open-loop problem with the PWM trajectory planning algorithm. After that, at each time step, our MPC saves time recomputing the trajectory by applying the iterative linearization scheme of the trajectory planning algorithm to the solution obtained in the previous time step. The efficacy of the method is shown in a simulation study where it is compared to MPC computed used an impulsive-only approach.

Keywords: Spacecraft autonomy, Space robotics, Pulse-width modulation, Trajectory planning, Optimal trajectory, Linearization.

1. INTRODUCTION

Technology enabling simple autonomous spacecraft rendezvous and docking is becoming a growing field as access to space continues increasing. The field has become very active in recent years, with an increasingly growing literature. Among others, approaches based on trajectory planning and optimization (Breger and How (2008); Arzelier et al. (2013, 2011)) and predictive control (Richards and How (2003); Rossi and Lovera (2002); Asawa et al. (2006); Gavilan et al. (2009, 2012); Larsson et al. (2006); Hartley et al. (2012); Leomanni et al. (2014)) are emerging.

Classically, in these approaches the problem of rendezvous is modeled by using impulsive maneuvers; one computes a sequence of (possibly optimal) impulses (usually referred to as ΔV's) to achieve rendezvous. Other methods allow the control signal (thrust) to take any value inside an allowed range. This type of control signal is usually referred to as Pulse-Amplitude Modulated (PAM).

However, neither impulsive actuation nor PAM actuation capture with precision the behavior of real spacecraft thrusters. A more realistic model has to take into account that, typically, thrusters are ON-OFF actuators, i.e., the thrusters are not able to produce arbitrary forces, but instead can only be switched on (producing the maximum amount of force) or off (producing no force). These switching times are the only signals that can be controlled. This type of control signal is usually referred to as Pulse-Width Modulated (PWM). Trajectory planning in the rendezvous problem with PWM actuation poses a challenge because the system becomes nonlinear in the switching times.

Recently, Vazquez et al. (2011, 2014) introduced a trajectory planning algorithm algorithm for spacecraft rendezvous that was able to incorporate PWM control signals. The former considered the linear time-invariant Clohessy-Wiltshire model (target orbiting in a *circular* Keplerian orbit, see Clohessy and Wiltshire (1960)). The latter extended the approach to *elliptical* target orbits by using the linear time-varying Tschauner-Hempel model (see Tschauner and Hempel (1965)). Both methods start from an initial guess computed by solving an optimal linear program with PAM or impulsive actuation, approximate the solution with ON-OFF thrusters, and then iteratively linearize around the obtained solutions to improve the PWM solution. For both circular and elliptical target orbits the algorithms are simple and reasonably fast, and we showed simulations of its application favorably comparing it with an impulsive-only approach.

However, these methods are based on trajectory planning which cannot take into account orbital perturbations, disturbances or model errors. To overcome these issues, in this paper we develop a Model Predictive Control (MPC) algorithm. The term Model Predictive Control does not designate a specific control strategy but rather

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an ample range of control methods which make explicit use of a model of the process to obtain the control signal by minimizing an objective function over a finite receding horizon. In MPC the process model is used to predict the future plant outputs, based on past and current values and on the proposed optimal future control actions. These actions are calculated by the optimizer taking into account the cost function (where the fuel cost and the future tracking error are considered) as well as the constraints. The MPC algorithm developed in this paper is based on the previous open-loop PWM planner for elliptical target orbits with arbitrary eccentricity (Vazquez et al. (2014)). The MPC is initialized by first solving the open-loop problem with the PWM trajectory planning algorithm. After that, at each time step, our MPC saves time recomputing the trajectory by applying the iterative linearization scheme of the trajectory planning algorithm to the solution obtained in the previous time step.

The structure of the paper is as follows. In Section 2 we introduce the Tschauner-Hempel model, both in the impulsive and PWM case. We follow with Section 3 where we formulate the underlying optimization problems. Section 4 describes a method that solves the planning problem using PWM signals. Section 5 develops the model predictive controller. In Section 6 we show simulations of the method compared to MPC computed used an impulsive-only approach. We finish with some remarks in Section 7.

2. TSCHAUNER-HEMPEL MODEL OF SPACECRAFT RENDEZVOUS

The Tschauner-Hempel model (see Tschauner and Hempel (1965) or Carter (1998)) assumes that the target vehicle is passive and moving along an elliptical orbit with semi-major axis a and eccentricity e. Following Vazquez et al. (2014), we write the Tschauner-Hempel using eccentric anomaly instead of time. Let us first establish some notation. Note that t and E are related in a one-to-one fashion by using Kepler's equation:

$$n(t - t_p) = E - e \sin E, \tag{1}$$

where t_p is the time at periapsis, a parameter of the target's which we use as a starting point to measure the eccentric anomaly E. This equation is numerically invertible (see any Orbital Mechanics reference, such as Wie (1998)), and we will represent its inverse by the function K, i.e. $E = K(t)$. Denote by E_0 the eccentric anomaly corresponding to t_0, this is, $E_0 = K(t_0)$, and $E_k = K(t_k) = K(t_0 + kT)$, where T is an adequately chosen sampling time. Call as x_k, y_k, and z_k the position of the chaser in a local–vertical/local–horizontal (LVLH) frame of reference fixed on the center of gravity of the target vehicle at time t_k. In the (elliptical) LVLH frame, x refers to the radial position, z to the out-of-plane position (in the direction of the orbital angular momentum), and y is perpendicular to these coordinates (no longer aligned with the target velocity given that its orbit is not circular). The velocity and inputs of the chaser in the LVLH frame at time t_k are denoted, respectively, by $v_{x,k}$, $v_{y,k}$, and $v_{z,k}$, and by $u_{x,k}$, $u_{y,k}$, and $u_{z,k}$.

If there is no actuation (i.e. $u_{x,k} = u_{y,k} = u_{z,k} = 0$), the resulting transition equation was obtained exactly by Yamanaka and Ankersen (2002) as follows:

$$\mathbf{x}_{k+1} = A(t_{k+1}, t_k)\mathbf{x}_k \tag{2}$$

where

$$\mathbf{x}_k = [x_k \ y_k \ z_k \ v_{x,k} \ v_{y,k} \ v_{z,k}]^T, \tag{3}$$

and where $A(t_{k+1}, t_k) = Y_{K(t_{k+1})} Y^{-1}_{K(t_k)}$, with $Y_{K(t_k)}$ being the fundamental matrix solution of the Tschauner-Hempel model. Working expressions of this matrix and its inverse can be found in Vazquez et al. (2014). They are as in Yamanaka and Ankersen (2002) but using eccentric anomaly and a different definition of the reference axes.

Next, we formulate two versions of the discretized equations. In the first version, the control inputs are considered impulses which are applied at the middle of the sampling interval. This is referred to as the impulsive discrete model. In a second, more realistic version, thrusters can only be switched on (producing the maximum force) or off (producing no force), and only once during each sampling time. This is referred to as the PWM discrete model.

2.1 Impulsive discrete model

For the impulsive model, we assume that we can apply an impulse u in any axis and at any given sample time. For simplicity's purpose, we assume that only one impulse per axis is allowed at each time interval and model the impulse at the beginning of the time interval. We also assume that impulses are limited above and below:

$$u_{min} \leq u \leq u_{max}.$$

Exploiting the linearity of the system, it can be easily shown that

$$\mathbf{x}_{k+1} = A(t_{k+1}, t_k)\mathbf{x}_k + B(t_{k+1}, t_k)\mathbf{u}_k, \tag{4}$$

where $\mathbf{u}_k = [u_{x,k} \ u_{y,k} \ u_{z,k}]^T$ and, calling m the mass of the spacecraft (assumed constant)

$$B(t_{k+1}, t_k) = A(t_{k+1}, t_k) \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1/m & 0 & 0 \\ 0 & 1/m & 0 \\ 0 & 0 & 1/m \end{bmatrix}. \tag{5}$$

Compact formulation

Next we develop a compact formulation that simplifies the notation of the problem. The state at time t_{j+k+1}, given the initial state at time t_j (which is denoted as $\mathbf{x}_j$) and the input signals from t_j to time $t_j + k$, is computed by applying recursively Equation (4) and using the fact that $A(t_{i+1}, t_i)A(t_i, t_{i-1}) = A(t_{i+1}, t_{i-1})$:

$$\mathbf{x}_{j+k+1} = A(t_{j+k+1}, t_j)\mathbf{x}_j + \sum_{i=j}^{j+k} A(t_{j+k+1}, t_{i+1})B(t_{i+1}, t_i)\mathbf{u}_i, \tag{6}$$

where it must be noted that $A(t_i, t_i) = \mathrm{Id}$, where Id is the identity matrix. Define now $\mathbf{x_S}(j)$ and $\mathbf{u_S}(j)$ as a stack of $N_p - j$ states and input signals, respectively, spanning from time t_j to time t_{N_p} for the state and from time t_{j-1} to time t_{N_p-1} for the controls, where N_p is the initial MPC horizon (and desired time of rendezvous):

$$\mathbf{x_S}(j) = \begin{bmatrix} \mathbf{x}_{j+1} \\ \vdots \\ \mathbf{x}_{N_p} \end{bmatrix}, \ \mathbf{u_S}(j) = \begin{bmatrix} \mathbf{u}_j \\ \vdots \\ \mathbf{u}_{N_p-1} \end{bmatrix}.$$

Then,