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Frequency Response based Adaptive Vibration Control of Nonlinear Systems ⋆

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Abstract: The problem of active vibration control for a class of nonlinear mechanical system is addressed. A recently developed nonlinear frequency response analysis technique is utilized for the vibrational analysis and controller design. Unlike the linear case, the nonlinear system oscillations are complex, which can be observed via the nonlinear Frequency Response Function (FRF). In order to achieve a satisfactory vibration attenuation, an adaptive tuning method based on the FRF, for selecting the controller gains is proposed. The proposed algorithm performs sufficiently well for a given range of input excitation magnitudes and frequencies. A physically motivated example is given to demonstrate the application of these results. Finally, the feasibility of the proposed algorithm in a satellite vibration control application has been examined.

Keywords: Adaptive control, convergence analysis, cubic stiffness, frequency response function, satelitte, vibration control.

1. INTRODUCTION

Mechanical vibrations are present in countless real-life situations, where the mechanical system exhibit oscillations when subjected to certain disturbances. Most often these vibration phenomena are highly undesirable, which may even cause damage to the system itself. The vibration control is concerned with the prediction and controlling of these undesired oscillations. Vibration analysis and control are an active, vast, and growing research area, due to its practical importance and issues that arise in both linear and nonlinear system designs. The vibration can generally be controlled by adding controlling devices like dampers, isolators, and actuators to the system. These devices are added in such a way that the system's properties are modified to a desired one (Thenozhi and Yu (2013)). Most of the vibration control methods work based on time-domain techniques. These methods lack in describing how a closed-loop system respond to the input excitation at different frequencies and magnitudes. Since the vibration is characterized by its frequency (or frequencies), amplitude, and phase, it is important to study the frequency response of these systems.

Frequency domain techniques for linear systems have led to significant progress in analysis, modeling, and controller design (Kerschen et al. (2006); Tang (1993); Tang and Ortega (1993); Tang et al. (1995)). In reality, mechanical systems posses many physical properties such as material property, geometric nonlinearity, damping dissipation, and even due to boundary conditions, which lead to nonlinear vibration problems (Kerschen et al. (2006)). For that reason, these nonlinear systems do not posses simple oscillations as defined in the case of linear systems. These nonlinearities can result in some complex phenomenon, like jumping, chaos, secondary resonance, bifurcation, etc. (Nayfeh and Mook (2008)). In those instances, the classic linear frequency response analysis tools are insufficient to describe the behavior of the nonlinear system adequately.

The methods such as describing function (DF) lacks in accuracy due to its approximation scheme (Khalil and Grizzle (2002)). Nonlinear FRF such as the Generalized Frequency Response Function (GFRF) (George (1959)) is limited to second order due to its multi-dimensional characteristics. An extension of the GFRF, termed as Output Frequency Response Function (OFRF) was proposed in Lang and Billings (1996), which represents the relation between the system parameters and frequency response using finite Volterra series. However, this technique fails to detect some of the nonlinear phenomenons such as the subharmonics, jumping, etc. (Billings (2013)). In Pavlov et al. (2007), it has been shown that an FRF can be found for a class of nonlinear systems termed as convergent systems. The convergence property implies that the system trajectories tend towards the unique bounded solution. If the system under consideration is convergent, it signifies: 1) the system is stable (Lohmiller and Slotine (1998)); 2) an FRF can be obtained due to the existence of a unique steady-state solution (Pavlov et al. (2007)). Compared to the DF, GFRF, and OFRF, this FRF gives an exact frequency response via numerical or experimental approach.

The main objective of this paper is to introduce the potential of convergence analysis and nonlinear FRF in vibration control problems. If the system under consideration is: 1) convergent, it directly enables to derive a nonlinear FRF for a band of excitation inputs, 2) non-convergent, first a controller is used to obtain the convergence and then the corresponding FRF for a band of excitation inputs is derived. In terms of active vibration control, the controller

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gains are adapted based on the FRF of the system derived within the band of interest, which assures a satisfactory performance over that band. The control scheme developed here is applied to a mechanical system with cubic stiffness, similar to the work in Peng and Lang (2008) and to a satellite system, and the corresponding dynamic responses of both the controlled and uncontrolled cases are numerically evaluated.

2. FRF OF NONLINEAR CONVERGENT SYSTEMS

For a system to be satisfactory, it is necessary to analyze its stability. In general a system's stability is analyzed by examining, whether the equilibrium points so determined are stable. The convergence analysis also termed as contraction analysis, inspired by fluid mechanics, is the extension of the stability properties of asymptotically stable linear time-invariant systems. Unlike the Lyapunov stability theorem which defines the stability with respect to the equilibrium points, the convergence in convergent systems implies that the state trajectories with different initial conditions will converge to a unique bounded solution (Lohmiller and Slotine (1998)).

Let a dynamical nonlinear system be described by the differential equation

$$\dot{x} = f(x,t) \tag{1}$$

with $x \in \mathbb{R}^n$ is the state vector, $t \in \mathbb{R}_+$, and $f : \mathbb{R}^n \times \mathbb{R}_+ \rightarrow \mathbb{R}^n$ is a smooth nonlinear function. For the above system, a convergence region (or contraction region), $\mathcal{X}$ is defined where the system's Jacobian matrix, $J(x) = \partial f/\partial x$ is uniformly negative definite. The convergence properties of the system (1) can be verified by performing a coordinate transformation on $J(x)$. The resulting generalized Jacobian is defined as

$$\mathcal{J} = \left(\dot{\Upsilon} + \Upsilon J(x)\right)\Upsilon^{-1} \tag{2}$$

where $\Upsilon(x,t)$ is a uniformly invertible square matrix. If $\mathcal{J}$ is uniformly negative definite such that

$$\mathcal{J} \leq -\lambda_{\max} I, \qquad \forall x \in \mathcal{X} \subset \mathbb{R}^n,\ \forall t \in \mathbb{R} \tag{3}$$

where $-\lambda_{\max}$ is the largest eigenvalue of $\mathcal{J}$, then the transformed system (2) is convergent, which implies that all the solutions of the original system (1) converge exponentially to a single trajectory, independently of the initial conditions. If $\mathcal{J}$ is negative semi-definite, then the system is semi-convergent under some mild conditions similar in Barbalat's lemma, this implies that the solutions converge each other asymptotically. The global convergence or semi-convergence is obtained when $\mathcal{X} = \mathbb{R}^n$.

FRF is the characteristics of a system that describes its response to an input excitation as the function of frequency. Consider a nonlinear time-invariant system, which is forced by the excitation input $w \in \mathcal{W}$

$$\begin{aligned} \dot{x} &= f(x,w) \\ y &= g(x) \end{aligned} \tag{4}$$

where y is the system output. If the above system is convergent, then there exists an uniformly bounded steady-state (UBSS) solution, if for any $\rho > 0$ there exists $\sigma > 0$ such that for any input $w \in \mathcal{W}$ the following implication holds (Pavlov et al. (2007)):

$$|w| \leq \rho \qquad \forall t \in \mathbb{R} \implies |\bar{x}_w| \leq \sigma \qquad \forall t \in \mathbb{R} \tag{5}$$

where $\bar{x}_w$ is the steady-state solution, which depends on the input excitation w. For the convergent systems, this bounded solution $\bar{x}_w$ is unique and $\lim_{t\rightarrow\infty}\|\bar{x}_w(t) - x_w(t)\| = 0$ for any $x_0 \in \mathcal{X}$ and hence the system (4) is exponentially stable.

If the system (4) is convergent with UBSS property for a certain class of harmonic inputs $w(t) = a\sin(\omega t) \in \mathcal{W}$, then there exists a nonlinear function $\alpha : \mathbb{R}^3 \rightarrow \mathbb{R}^n$ such that

$$\bar{x}_w(t) := \alpha(v_1, v_2, \omega) \tag{6}$$

where $v_1 = a\sin(\omega t)$ and $v_2 = a\cos(\omega t)$. For the system (4), this nonlinear function $\alpha(v_1, v_2, \omega)$ is known as the state-FRF and the function $g(\alpha(v_1, v_2, \omega))$ is known as the output-FRF, which relates a sinusoidal input to the corresponding steady-state output. Now the output of the forced system at steady-state can be expressed as $\bar{y}_w(t) = g(\bar{x}_w(t))$, which will have the same period of the input signal w, but not necessarily sinusoidal.

The output response of the system for various amplitude and frequency inputs can be represented using an amplification gain $\gamma_a(\omega)$, which is the ratio between the maximal absolute value at steady-state and the input signal amplitude, so that

$$\gamma_{a,\omega} = \frac{1}{a}\left(\sup_{v_1^2+v_2^2=a^2} |g(\alpha(v_1, v_2, \omega))|\right) = \frac{|\bar{y}_w(t)|}{a} \tag{7}$$

Analyzing the above function, one can find the critical amplitudes and frequencies for the system and design a controller to bypass any undesirable effects such as the vibrations.

3. VIBRATION CONTROL OF CONVERGENT MECHANICAL SYSTEMS

In order to present the main idea, let us consider the dynamics of a mechanical system of form

$$M\ddot{q} + C\dot{q} + Kq + \Phi(q) = \Lambda w + \Gamma u \tag{8}$$

where M, C, and $K \in^{n_q \times n_q}$ are the mass, damping, and stiffness matrices, respectively, $\ddot{q}, \dot{q}$, and $q \in \mathbb{R}^{n_q}$ are the relative acceleration, velocity, and displacement vectors respectively, $\Lambda \in \mathbb{R}^{n_q \times n_q}$ denotes the influence of the excitation force w on the system, $\Gamma \in \mathbb{R}^{n_q \times n_u}$ is the actuator location matrix, $u \in \mathbb{R}^{n_u}$ [2] denotes the input by means of which the system can be controlled, and $\Phi(q) \in \mathbb{R}^{n_q}$ is the Odd Polynomial Nonlinearity (OPN), which has the following of form

$$\Phi(q) = \left[\phi_1(q_1), ..., \phi_{n_q}(q_{n_q})\right]^T \tag{9}$$

where

$$\phi_i(q_i) = \sum_{p=1}^{\overline{p}} b_{i,2p+1} q_i^{2p+1}, \qquad b_{i,2p+1} > 0,\ i = 1, ..., n_q$$

with $\overline{p}$ as the highest order of the nonlinearity.

3.1 Problem Formulation

Let us consider a feedback controller of form

$$u = -\Theta x \tag{10}$$

[2] In the case of full-state feedback control $n_q = n_u$ and for the second-order mechanical systems $n = 2n_q$.