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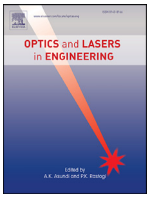

A novel approach for measuring nanometric displacements by correlating speckle interferograms

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ABSTRACT

Recently, two phase evaluation methods were proposed to measure nanometric displacements by means of digital speckle pattern interferometry when the phase changes introduced by the deformation are in the range $[0, \pi)$ rad. However, one of these techniques requires separate recording of the intensities of the object and the reference beams which correspond to both the initial and the deformed interferograms. The other technique only works to measure out-of-plane displacements. In this paper, we present a novel approach that overcomes these limitations. The performance of the proposed method is analyzed using computer-simulated speckle interferograms and it is also compared with the results obtained with a phase-shifting technique. Finally, an application of the proposed phase method used to process experimental data is illustrated.

1. Introduction

Whole-field optical techniques can be used as useful tools to test micro-system devices due to their advantages, which include robustness, high processing speed, and also non-contact and non-destructive nature [1–6]. One of these optical techniques is digital speckle pattern interferometry (DSPI), which has a high sensitivity and also has been widely used for the measurement of displacement and strain fields generated by rough object surfaces [7]. This technique is based on the evaluation of the optical phase changes that are coded in speckle interferograms, which are usually displayed in the form of fringe patterns.

In practical applications of DSPI, the phase-shifting and the Fourier-transform methods are the most common techniques used to retrieve the phase distribution introduced by the deformation [8]. As it is well known, phase-shifting methods have high accuracy, and the sign ambiguity is resolved automatically due to the recording of multiple interferograms. However, a mirror driven by a linear computer-controlled piezoelectric transducer must be introduced in the optical setup, thus generating an additional technical complexity. Moreover, these algorithms assume that phase-shifts between successive frames are all equal, which can be difficult to obtain experimentally. Phase-shifter miscalibrations and vibrations during the acquisition of multiple speckle interferograms also produce systematic errors which must be appropriately addressed [9]. On the contrary, the Fourier transform method has the advantage of requiring the acquisition of only two speckle interferograms to be analyzed. Even so, when the phase changes are non-monotonous, this method also needs the introduction of spatial carrier fringes to overcome the sign ambiguity [8]. Although there exist simple ways of introducing spatial carrier fringes in the optical setup, such as tilting the reference beam between the acquisition of both speckle interferograms to be correlated, this procedure also complicates the automation of the interferometer operation.

A novel phase evaluation method was recently proposed to measure nanometric displacements by means of DSPI when the phase change introduced by the deformation is in the range $[0, \pi)$ rad, i.e., when the generated correlation fringes show less than one fringe [10]. In this case, the wrapped phase map does not present the usual 2π phase discontinuities, and it is therefore unnecessary to apply a spatial phase unwrapping algorithm to obtain the continuous phase distribution. It must be noted that cases of correlation fringe patterns presenting less than one fringe can appear quite frequently when micro-systems are inspected [11]. This phase retrieval method is based on the calculation of the local Pearson's correlation coefficient between the two speckle interferograms generated by both deformation states of the object. Although this approach does not need the introduction of a phase-shifting facility or spatial carrier fringes in the optical setup, the intensities of the object and the reference beams corresponding to both the initial and the deformed interferograms must be recorded. It should be noted that this limitation complicates the automation of the interferometer operation. Moreover, this limitation does not allow the application of this method for the analysis of non-repeatable dynamic events by recording a sequence of interferograms throughout the entire deformation history of the testing object.

More recently, Tendela et al. [12] have presented a phase retrieval method based on the approach reported in Ref. [10] to be used in a

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DSPI. In this method there is no need to record the intensities of the object and the reference beams corresponding to both the initial and the deformed interferograms. However, when this technique is applied the rms phase errors for in-plane measurements are very large. Therefore, the approximations made in Ref. [12] are no longer valid for the in-plane case and this method is not suitable to measure in-plane displacements.

In this paper, we present a phase retrieval approach based on the methods reported in Ref. [10,12], which overcomes the aforementioned limitations. Therefore, there is no need to record the intensities of the object and the reference beams corresponding to both the initial and the deformed interferograms, and this technique can measure both in-plane and out-of-plane displacement fields. Furthermore, we show that the approximations made in Ref. [12] can be reached more naturally.

In the following section, a description of the proposed phase retrieval method is presented. Afterwards, the performance of the proposed method is analyzed using computer-simulated speckle interferograms for in-plane and of out-of-plane displacements. This analysis allows us to evaluate the rms phase errors introduced by the novel approach and also to compare its performance with the one given by a phase-shifting algorithm. Finally, an application of the phase retrieval method used to process experimental data is also illustrated.

2. Theorical concepts

As it is well known, DSPI is based on the recording of the coherent superposition of two optical fields, at least one of them being a speckle field generated by the scattered light coming from the rough surface of the specimen. The result of the superposition is another speckle field called interferogram and its intensity I can be expressed as [7]

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos(\phi_1 - \phi_2) = I_0 + I_M \cos(\phi), \tag{1}$$

where I_1 and I_2 are the intensities of the object and the reference optical fields and ϕ_1 and ϕ_2 are their associated phases, respectively, $I_0 = I_1 + I_2$ is the intensity bias, $I_M = 2(I_1 I_2)^{1/2}$ is the modulation intensity, and $\phi = \phi_1 - \phi_2$ accounts for the optical path difference from the light source to the observation point considered.

If the scattering surface undergoes a deformation, the resulting intensity changes accordingly. The intensities I_a and I_b corresponding to the speckle interferograms recorded in the initial (a) and the deformed states (b), respectively, are determined by

$$\begin{aligned} I_a &= I_{a0} + I_{aM}\cos\phi_a = I_{a0} + I_{aM}\cos\phi_s \\ I_b &= I_{b0} + I_{bM}\cos\phi_b = I_{b0} + I_{bM}\cos(\phi_s + \Delta\phi), \end{aligned} \tag{2}$$

where $\phi_s = \phi_a$ accounts for the random change in the optical path due to the roughness of the scattering surface and $\Delta\phi = \phi_b - \phi_a$ corresponds to the deterministic change in the path introduced by the underwent deformation.

Below, it is presented a relationship to characterize the deterministic phase change $\Delta\phi$ as a function of the Pearson's correlation coefficient between the two interferograms described by Eq. (2). The Pearson's correlation coefficient $C(p, q)$ between two random variables p and q is defined as the covariance of the two variables divided by the product of their standard deviations and can be estimated as [13]

$$C(p,q) = \frac{\langle (p - \langle p\rangle)(q - \langle q\rangle)\rangle}{\left[(\langle p^2\rangle - \langle p\rangle^2)(\langle q^2\rangle - \langle q\rangle^2)\right]^{1/2}}, \tag{3}$$

where $\langle\,\rangle$ stands for the mean value of the sampled random variable.

Taking into account general hypotheses about the speckle distribution generated by the rough object, the correlation coefficient $C(I_a, I_b)$ for the two recorded interferograms I_a and I_b is given by

$$C = \frac{\langle (I_a - \langle I_a\rangle)(I_b - \langle I_b\rangle)\rangle}{[(\langle I_a^2\rangle - \langle I_a\rangle^2)(\langle I_b^2\rangle - \langle I_b\rangle^2)]^{1/2}}, \tag{4}$$

where the operator $\langle\,\rangle$ is evaluated by using a sliding window technique on each recorded image, and Eq. (2) can be used. The reader should note that the spatial coordinates of the pixel (m, n) at the CCD for $m, n = 1, \ldots, L$, where L is the number of pixels along the horizontal and vertical directions, were omitted intentionally for the sake of clarity.

Assuming that the intensity and phase of fully developed and polarized speckle fields are statistically independent, the following relationships are valid [14]

$$\begin{aligned} \langle I_M\cos\phi_s\rangle &= \langle I_M\rangle\langle\cos\phi_s\rangle, \\ \langle I_M\sin\phi_s\rangle &= \langle I_M\rangle\langle\sin\phi_s\rangle, \\ \langle I_M^2\sin^2\phi_s\rangle &= \langle I_M^2\cos^2\phi_s\rangle, \\ \langle\sin\phi_s\rangle &= \langle\cos\phi_s\rangle \approx 0, \\ \langle\sin\phi_s\cos\phi_s\rangle &\approx 0. \end{aligned} \tag{5}$$

In addition, considering that $\Delta\phi$ is a deterministic magnitude, and after some mathematical manipulations, the numerator N_{ab} of Eq. (4) can be expressed as a function of $\cos\Delta\phi$ as follows

$$N_{ab} = \langle I_{a0}I_{b0}\rangle - \langle I_{a0}\rangle\langle I_{b0}\rangle + \frac{1}{2}\langle I_{aM}I_{bM}\rangle\cos\Delta\phi. \tag{6}$$

In a similar way, the denominator D_{ab} of Eq. (4) can be computed as

$$D_{ab} = \left[\left(\langle I_{a0}^2\rangle - \langle I_{a0}\rangle^2 + \frac{1}{2}\langle I_{aM}^2\rangle\right)\times\left(\langle I_{b0}^2\rangle - \langle I_{b0}\rangle^2 + \frac{1}{2}\langle I_{bM}^2\rangle\right)\right]^{1/2} \tag{7}$$

Replacing Eqs. (6) and (7) into Eq. (4), the correlation coefficient C can be estimated as

$$C(I_a, I_b) = \frac{\langle I_{a0}I_{b0}\rangle - \langle I_{a0}\rangle\langle I_{b0}\rangle + \frac{1}{2}\langle I_{aM}I_{bM}\rangle\cos\Delta\phi}{\left[(\langle I_{a0}^2\rangle - \langle I_{a0}\rangle^2 + \frac{1}{2}\langle I_{aM}^2\rangle)(\langle I_{b0}^2\rangle - \langle I_{b0}\rangle^2 + \frac{1}{2}\langle I_{bM}^2\rangle)\right]^{1/2}}. \tag{8}$$

Rearranging Eq. (8), the $\cos\Delta\phi$ can be written as

$$\begin{aligned} \cos\Delta\phi = {} & C(I_a, I_b) \\ & \times\frac{2\left[(\langle I_{a0}^2\rangle - \langle I_{a0}\rangle^2 + \frac{1}{2}\langle I_{aM}^2\rangle)(\langle I_{b0}^2\rangle - \langle I_{b0}\rangle^2 + \frac{1}{2}\langle I_{bM}^2\rangle)\right]^{1/2}}{\langle I_{aM}I_{bM}\rangle} \\ & -2\frac{\langle I_{a0}I_{b0}\rangle - \langle I_{a0}\rangle\langle I_{b0}\rangle}{\langle I_{aM}I_{bM}\rangle}. \end{aligned} \tag{9}$$

As before, considering that the intensity bias and the modulation intensity of fully developed and polarized speckle fields are statistically independent, the following relationships are also valid [14]

$$\begin{aligned} \langle I_{a0}\rangle &= \langle I_{b0}\rangle = \langle I_0\rangle, \\ \langle I_{a0}I_{b0}\rangle &= \langle I_0^2\rangle, \\ \langle I_{a0}^2\rangle &= \langle I_{b0}^2\rangle = \langle I_0^2\rangle, \\ \langle I_{aM}\rangle &= \langle I_{bM}\rangle = \langle I_M\rangle, \\ \langle I_{aM}I_{bM}\rangle &= \langle I_M^2\rangle, \\ \langle I_{aM}^2\rangle &= \langle I_{bM}^2\rangle = \langle I_M^2\rangle. \end{aligned} \tag{10}$$

After some mathematical manipulations, the phase change $\Delta\phi$ needed to determine the displacement components can be evaluated by inverting Eq. (9) as follows

$$\Delta\phi = \mathrm{acos}\left[C(I_a, I_b)(\alpha + 1) - \alpha\right], \tag{11}$$

where acos[] is the inverse of the cosine function, and the coefficient α is defined as

$$\alpha = 2\frac{\langle I_0^2\rangle - \langle I_0\rangle^2}{\langle I_M^2\rangle}. \tag{12}$$

For further analysis, it will be useful to express the coefficient α as a function of the intensities of the object and reference fields I_1 and I_2

$$\alpha = \frac{\langle I_1^2\rangle - \langle I_1\rangle^2 + \langle I_2^2\rangle - \langle I_2\rangle^2}{2\langle I_1\rangle\langle I_2\rangle}. \tag{13}$$