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IFAC-PapersOnLine 48-2 (2015) 155–160

Underwater Single-Beacon Localization: Optimal Trajectory Planning and Minimum-Energy Estimation

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Abstract: Single-beacon, range-based AUV localization systems work on the principle that a vehicle may find its position by maneuvering appropriately and acquiring measurements of its successive distances (ranges) to a stationary beacon deployed at a known location. This motivates the study of optimal trajectories to improve the accuracy of the vehicle's position estimate while respecting mission related criteria. In this work, the performance index used to compare different trajectories is the determinant of a properly defined Fisher Information Matrix (FIM). Assuming that heading measurements are available, the problem is studied in 2D, and a class of analytical and numerical solutions are derived. An approach to deal with the case where the initial position of the vehicle is known to lie in a region of uncertainty is also presented. Considering that depth measurements can be obtained, a 3D navigation algorithm consisting of an optimal trajectory planner and a minimum-energy estimator is proposed and its performance assessed via simulation of a practical scientific scenario.

Keywords: Underwater Range-Based Navigation; Single-Beacon Localization; Trajectory Optimization; Fisher Information Matrix

1. INTRODUCTION

A central problem in the field of marine robotics is that of estimating the position of a vehicle in a given inertial reference frame. As applications of marine vehicles become increasingly more diverse, underwater navigation systems are not only required to be reliable and accurate but also affordable and easy to install and operate. For these reasons, single-beacon navigation is steadily emerging as a reduced-cost alternative to conventional acoustic navigation methods such as those at the core of LBL (Long Baseline) and SBL (Short Baseline) systems. However, single-beacon navigation using range measurements introduces some difficulties as well, due to the fact that a single range measurement is clearly not enough to define the vehicle position with respect to a beacon installed at a fixed, known position. Thus, the vehicle must move around and acquire range measurements at different positions, while estimating the displacements in between measurements, in order to compute its own position - which implies that the navigation system is dependent on the type of motion imparted to the vehicle. Motivated by these considerations, this paper is focused on the optimization of trajectories to increase the expected accuracy of single-beacon navigation.

The literature on single-beacon navigation is extensive and defies a simple summary. For early work in the area and a detailed example of the implementation of single-beacon navigation algorithms the reader is referred to Larsen (2000). In what concerns the study of optimal trajectories for single-beacon navigation, the majority of the work reported addresses the study of the observability of the system undergoing specific trajectories, in a deterministic framework; see for instance Gadre and Stilwell (2005) and Crasta et al. (2013). However, these studies do not take into account neither the noise affecting the range measurements nor the uncertainty associated with the estimate of the initial position of the vehicle.

The work presented in this paper follows a different approach, similar to that exposed in Martínez and Bullo (2006), and Moreno-Salinas et al. (2011). We use the determinant of a properly defined FIM as a measure of the best accuracy with which the position of the vehicle can be computed along generic trajectories (using any non-biased estimator), assuming that the measurements are affected by additive Gaussian noise. Besides the quality of the position estimates, other mission related criteria are taken explicitly into account in the trajectory optimization procedure. Namely, energy consumption and how far the actual trajectory of the vehicle deviates from a nominal, desired trajectory. Additionally, we propose a method to deal explicitly with the uncertainty associated with the initial position of the vehicle. The procedures used to compute the trajectories are embodied into an optimal trajectory planner that, in combination with a minimum energy estimator, yields an algorithm for simultaneous trajectory generation and single-beacon navigation. The efficacy of the algorithm developed is assessed with results of simulations of a realistic scientific scenario involving homing in of an underwater vehicle on a deep sea laboratory.

The paper is organized as follows. In Section 2 optimal 2D trajectories are derived, taking into consideration the criteria described above. In Section 3 a 3D single-beacon navigation algorithm is described.

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10.1016/j.ifacol.2015.06.025

2. 2D SINGLE-BEACON NAVIGATION

2.1 System model

Consider a vehicle moving in 2D while measuring its distance $d(t)$ with respect to a stationary beacon. The motion of the vehicle is controlled through its forward speed $v(t) > 0$ and yaw rate $r(t)$. The location of the beacon is known with respect to some inertial reference frame $\{\mathcal{I}\}$, with North-East-Down orientation (NED). The vehicle has access to the heading angle $\psi(t)$, which provides the orientation of the body-frame with respect to the inertial frame $\{\mathcal{I}\}$. Let the vehicle and the beacon positions in $\{\mathcal{I}\}$ be denoted by $\mathbf{p}(t) = [\, p^n(t)\ p^e(t)\,]^{\mathrm{T}}$ and $\mathbf{b_0} = [\, b_0^n\ b_0^e\,]^{\mathrm{T}}$, respectively. Without loss of generality, we will assume the beacon to be located at the origin of the reference frame $\{\mathcal{I}\}$, that is, $\mathbf{b}_0 = [\,0\ 0\,]^{\mathrm{T}}$. Thus, the distance between the vehicle and the beacon is given by $d(t) = \|\mathbf{p}(t)\|$. The kinematic model of the system with state $\mathbf{x}(t) = [\, p^n(t)\ p^e(t)\ \psi(t)\,]^{\mathrm{T}}$, input $\mathbf{u}(t) = [\, v(t)\ r(t)\,]^{\mathrm{T}}$, and output $\mathbf{y}(t) = [\, d(t)\ \psi(t)\,]^{\mathrm{T}}$ over the time-interval $[0, t_f]$ is given by

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} v(t)\cos(\psi(t)) \\ v(t)\sin(\psi(t)) \\ r(t) \end{bmatrix}, \tag{1}$$

$$\mathbf{y}(t) = [\|\mathbf{p}(t)\| \quad \psi(t)]^{\mathrm{T}}. \tag{2}$$

2.2 Determinant of the FIM

Let z_i with $i = 0, ..., m-1$ denote a set of m measurements of $d(t)$ corrupted by additive noise, obtained at different time instants t_i. Moreover, let d_i with $i = 0, ..., m-1$ denote the actual distances at the time instants of the measurements. The measurement model is given by

$$\begin{aligned} z_i &= d_i + w_i\,, \qquad i = 0, ..., m-1, \\ w_i &\sim \mathcal{N}(0, \sigma^2)\,, \qquad i = 0, ..., m-1. \end{aligned}$$

Assume that the model introduced in Section 2.1 and the measurement model above describe the system perfectly. Additionally, assume that the initial position of the vehicle $\mathbf{p}_0$ is unknown, but that from the set of measurements $\mathbf{z} = (z_0, \ldots, z_{m-1})$ we can obtain an unbiased estimate $\hat{\mathbf{p}}_0$. Under these assumptions, the determinant of the FIM associated with the above problem varies inversely with the volume of the uncertainty ellipsoid of the estimation error. See for example Jauffret (2007) where it is shown that the underlying system is locally observable if the FIM is nonsingular, meaning that trajectories that result from the maximization of the FIM determinant (with $|FIM| > 0$) render the system observable. Classical results in estimation theory (see Van Trees (2001)) dictate that the FIM with respect to the unknown initial position of the vehicle $\mathbf{p}_0$ is given by

$$FIM_{\mathbf{p}_0} \triangleq \mathbb{E}\left\{ [J_{\mathbf{p}_0}(\ln \mathcal{L}_{\mathbf{p}_0}(\mathbf{z}))]\,[J_{\mathbf{p}_0}(\ln \mathcal{L}_{\mathbf{p}_0}(\mathbf{z}))]^{\mathrm{T}} \right\}$$

where $J_{\mathbf{p}_0}(\ln \mathcal{L}_{\mathbf{p}_0}(\mathbf{z}))$ denotes the Jacobian of the log-likelihood function of the range measurements with respect to $\mathbf{p}_0$ and $\mathbb{E}$ is the expectation operator. As a consequence, and considering the model introduced in Section 2.1, the determinant of the FIM is given by

$$|FIM_{\mathbf{p}_0}| = \frac{1}{\sigma^4}\left[\sum_{i=0}^{m-1} \left(\frac{p_i^n}{d_i}\right)^2 \sum_{i=0}^{m-1} \left(\frac{p_i^e}{d_i}\right)^2 - \left(\sum_{i=0}^{m-1} \frac{p_i^n}{d_i}\frac{p_i^e}{d_i} \right)^2 \right] \tag{3}$$

where $\mathbf{p}_i = [\, p_i^n\ p_i^e\,]^{\mathrm{T}}$ denotes the vehicle's position at t_i.

2.3 Problem setup

The setup for the trajectory optimization considers the model of the system introduced in Section 2.1 under the assumptions below.

Assumption 1. The vehicle speed is constant and the yaw rate is piecewise constant, that is,

$$\begin{aligned} &v(t) \equiv \bar{v}, \\ &r(t) \equiv r_k,\ \ t \in [t_k, t_{k+1}),\ \ k = 0, ..., m-1 \end{aligned}$$

with $|r_k| \le r_{\mathrm{b}}$, where r_{b} is a bound on the vehicle yaw rate.

Assumption 2. The time instants at which the inputs may change are given by $t_k = k\,T$, for $k = 0, ..., m-1$, with T the cycle interval. In view of the above, the vehicle position at any t can be written as a function of $\mathbf{p}_0$, ψ_0, $\bar{v}$, $\mathbf{r} = (r_0, \ldots, r_{m-1})$ and T, and is given by

$$\mathbf{p}(t) = \mathbf{p}_0 + \bar{v}\sum_{j=0}^{k-1} \frac{1}{r_j} \begin{bmatrix} \sin(\psi((j+1)\,T)) - \sin(\psi(j\,T)) \\ -\cos(\psi((j+1)\,T)) + \cos(\psi(j\,T)) \end{bmatrix} + \frac{\bar{v}}{r_k} \begin{bmatrix} \sin(\psi(t)) - \sin(\psi(k\,T)) \\ -\cos(\psi(t)) + \cos(\psi(k\,T)) \end{bmatrix}, \tag{4a}$$

$$\psi(t) = \psi_0 + T\sum_{j=0}^{k-1} r_j + (t - k\,T)\,r_k, \tag{4b}$$

$$k = \mathrm{floor}(t/T), \tag{4c}$$

where the function $\mathrm{floor}(x)$ returns the largest integer not greater than x. If any r_k is zero the expression for $\mathbf{p}(t)$ has a singularity, which is removable.

Assumption 3. The time instants at which the range measurements are obtained are given by $t_i = i\,\Delta t$, for $i = 0, ..., m-1$, with Δt the sampling interval.

Assumption 4. The cycle interval of the yaw rate function, T, is a multiple of Δt. We introduce a new tuning parameter c, such that $T = c\,\Delta t$. The vehicle positions and the actual ranges, at the time instants t_i, are given by (4) for $t = i\Delta t$ and $T = c\Delta t$. Hence, the FIM determinant is completely defined as a function of the problem parameters and variables, which are summarized in Table 1. Note that

Table 1. Setup parameters and optimization variables.

setup parameters	$\mathbf{p}_0$, ψ_0, Δt, m, σ, c, $\bar{v}$, r_{b}
problem variables	$r_0, \ldots, r_{N-1}$ with $N = c^{-1}(m-1)$

the speed $\bar{v}$ is a parameter because small AUVs usually keep the speed approximately constant during missions; a typical value is 1.5 $[m/s]$. The bound on the yaw rate is set to $\pi/9\,[rad/s]$, a typical value for the MEDUSA [1] class of AUVs.

2.4 Maximizing the FIM determinant

The problem of maximizing the FIM determinant including the vehicle dynamics explicitly is too complex for an analytical solution to be obtained for a general scenario. Therefore, we start by looking for a solution in two steps: (i) compute, analytically, the optimal locations for the measurement points neglecting the vehicle dynamics and (ii) find trajectories that cover all the optimal measurement points and check if these are compatible with the vehicle dynamics. As an alternative non-analytic approach,

[1] MEDUSA is an AUV for scientific research developed and operated by the Institute for Systems and Robotics of IST.