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A novel dual-wavelength iterative method for generalized dual-wavelength phase-shifting interferometry with second-order harmonics

Xiaoqing Xu[a,c], Yawei Wang[b,*], Ying Ji[b], Yuanyuan Xu[b], Ming Xie[b], Hao Han[b]

[a] School of Mechanical Engineering, Jiangsu University, Zhenjiang, Jiangsu 212013, China
[b] Faculty of Science, Jiangsu University, Zhenjiang, Jiangsu 212013, China
[c] School of Mechanical Engineering, Yangzhou Polytechnic College, Yangzhou, Jiangsu 225009, China

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ABSTRACT

To address dual-wavelength interferograms with arbitrary phase shifts and second-order harmonics, a novel dual-wavelength iterative method (DWIM) based on the least-squares algorithm is proposed. In generalized dual-wavelength phase-shifting interferometry, to compensate for the phase-shift errors consisting of systematic and random phase-shift error, the wrapped phases of single-wavelength with high accuracy can be simultaneously obtained from generalized dual-wavelength interferograms without second-order harmonics. In addition, this method is also employed to deal with randomly phase-shifted dual-wavelength interferograms with the second-order harmonics, and then the effects of the fringe number in interferogram and the number of interferograms used on the accuracy of phase extraction are investigated by numerical simulations. Based on theoretical analysis and simulation results of DWIM, we present the basic relationship between the number of wavelengths, the second-order harmonics and the requirement of the minimum number of interferograms. Finally, the effectiveness of this method is proved by the simulation results of the spherical cap, and its applicability is verified with the results of the micro-sphere, the HeLa cell and the red blood cell, respectively.

1. Introduction

Optical phase-shifting interferometry is a well-established technique widely used in various applications, such as optical imaging [1,2] and surface topography measurement [3,4]. However, in single-wavelength interferometry (SWI), when the surface features of the object under test exceed half of the probe wavelength, the phase of the object cannot be determined unambiguously. So a complicated phase unwrapping algorithm must be presented to remove the phase discontinuities and then achieve true phase of the object in SWI. To overcome this problem, various dual-wavelength interferometry (DWI) [5–16,22] or three-wavelength interferometry (TWI) [19–21] techniques have been proposed since they can yield the phase of the synthetic beat wavelength by the subtraction operation between the wrapped phases of single-wavelength [5–7]. In [8], based on subtraction of two wrapped phases, DWI is first introduced to measure three-dimensional shape of millimeter-scale object with a scanning dye laser. In [9,10], in-line DWI (named as Abdelsalam's method) or TWI is employed to extract the quantitative phases at each wavelength from the interferograms in SWI by using traditionally temporal phase-shifting algorithms, so its phase retrieving process is very time-consuming and complicated. In [11–13], to realize real-time and dynamic measurement, the wrapped phases of single-wavelength can be retrieved from only a single-shot off-axis interferogram. But, off-axis DWI or TWI restricts the space-bandwidth product of the optical imaging system, and its phase retrieval accuracy is easily affected due to the use of spatial Fourier transform and a filter window. In [14], a simultaneous phase-shifting DWI (named as Zhang's method) is proposed to extract the wrapped phases of single-wavelength based on two-step approximate algorithm, which results in the measuring inaccuracy. In [15,16], a phase retrieval method of single-wavelength from a sequence of simultaneous multi-wavelength in-line phase-shifting interferograms is presented by using the least-squares iterative algorithm (LSIA). However, because of simply estimating initial phase shifts of single-wavelength, this method cannot exactly extract the quantitative phases at each wavelength from unknown phase-shifted interferograms in practice. Subsequently, in [17], the phase shifts at each wavelength are estimated by principle component analysis (PCA) [18], then the wrapped phases of single-wavelength are retrieved based on LSIA using these phase shifts. Though the accurate phase retrieval can be obtained, the temporal and spatial hybrid matching condition is not easily satisfied in reality. In [19,20], a number of red-green-blue three-wavelength interferograms are recorded by using a color CCD. Though the wrapped phases at each wavelength can be easily retrieved based

* Corresponding author.
E-mail address: jszjwyw@sina.cn (Y. Wang).

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on the color separation method, it cannot work well if the difference value between two wavelengths is too small. In [22], a method of phase extraction from five interferograms based on two intensities without the corresponding dc terms (named as Xu's method) is proposed. But, the strict requirements for the precision and environmental stability of the phase shifter are put forward due to the fact that this method requires 2π special phase shift.

In generalized dual-wavelength phase-shifting interferometry (GDWPSI), the accuracy of phase retrieval at each wavelength can be affected by the systematic errors. The two common sources of systematic errors are the phase-shift error, and non-sinusoidal waveform of the signal, which is due to nonlinearity of the detector or multiple-beam interference [23,24]; the phase-shift errors consist of systematic phase-shift error that is induced by linear miscalibration or nonlinear response of the phase shifter, and random phase-shift error that is caused by the imperfect phase-shifting mechanism or unstable environments. In the case that the effect of second-order harmonics on phase extraction accuracy can be ignored, we only need to deal with arbitrarily phase-shifted dual-wavelength interferograms. In the case that the coupled effects of arbitrary phase shifts and second-order harmonics on phase extraction accuracy can not be ignored, we need to address dual-wavelength interferograms containing arbitrary phase shifts and second-order harmonics. These two issues are very important for extracting the wrapped phases of single-wavelength with high accuracy in GDWPSI. Owing to the complex effects of these two errors on phase extraction of single-wavelength from dual-wavelength interferograms, the aforementioned algorithms in DWI or TWI do not efficiently deal with the two problems simultaneously to provide accurate phases.

In this paper, to deal with these two problems simultaneously, a novel dual-wavelength iterative method (DWIM) is presented based on the least-squares iterative algorithm [25,26]. As an example, DWIM is employed to analyze dual-wavelength interferograms with arbitrary phase shifts as well as deal with dual-wavelength interferograms containing arbitrary phase shifts and intensity nonlinearity. Subsequently, the phases and the phase shifts with high accuracy at each wavelength can be simultaneously achieved by using DWIM, and then the phase at synthetic beat wavelength can be easily obtained by the subtraction operation. The validity of this method is demonstrated by the simulation results of the spherical cap, and its applicability is investigated through the results of the micro-sphere, the HeLa cell and the red blood cell, respectively. Finally, discussions on DWIM and GDWPSI are also depicted. To the best of our knowledge, this is the first time that this method is proposed.

2. Principle

Owing to the existence of nonsinusoidal waveforms in GDWPSI, the intensity possessing harmonics up to the p th order, can be theoretically expressed as an incoherent addition of the interferograms at $\lambda_j (j = 1, 2, \dots, q)$

$$I'_{mn} = \sum_{k=0}^{p} \sum_{j=1}^{q} b_{\lambda_j,mnk} \cos\left[k(\varphi_{\lambda_j,n} + \delta_{\lambda_j,m})\right], \tag{1}$$

where $'$ represents the theoretical value; $\sum_{j=1}^{q} b_{\lambda_j,mn0}$ is the total dc term; $b_{\lambda_j,mnk}$ denotes the modulation amplitude of the kth order harmonics corresponding to $\lambda_j (k \geq 1)$; m represents the mth phase-shifted interferogram $(m = 1, 2, \dots, M)$; each interferogram is reshaped into one column with size of N, and n denotes the pixel points in each image $(n = 1, 2, \dots, N)$; q represents the number of wavelengths, namely, $q = 2$ in DWIM; $\varphi_{\lambda_j,n}$ and $\delta_{\lambda_j,m}$ are the phases and the phase shifts corresponding to λ_j, respectively.

Defining a new set of variables as $X_{\lambda_j,0} = \sum_{j=1}^{q} b_{\lambda_j,mn0}$, $X_{\lambda_j,2k-1} = b_{\lambda_j,mnk} \cos(k\varphi_{\lambda_j,n})$, $X_{\lambda_j,2k} = -b_{\lambda_j,mnk} \sin(k\varphi_{\lambda_j,n})$, $Y_{\lambda_j,0} = 1$, $Y_{\lambda_j,2k-1} = \cos(k\delta_{\lambda_j,m})$, and $Y_{\lambda_j,2k} = \sin(k\delta_{\lambda_j,m})$ $(k = 1, 2, \dots, p)$, thus Eq. (1) is rewritten as

$$I'_{mn} = \sum_{i=0}^{2p} \sum_{j=1}^{q} X_{\lambda_j,i} Y_{\lambda_j,i}. \tag{2}$$

In Eq. (2), it can be supposed that the total dc term and the modulation amplitude do not vary with frames. If $\delta_{\lambda_j,m}$ corresponding to λ_j is known, the phases at each wavelength can be retrieved from at least $2pq + 1$ interferograms in GDWPSI. In general, the more interferograms, the higher accuracy could be got for phase extraction in DWIM. The accumulated error E_n, which results from the sum of squares of difference between the theoretical intensity and the measured one of the n th pixel point of M interferograms, can be described as

$$E_n = \sum_{m=1}^{M} \left\{ \sum_{i=0}^{2p} \sum_{j=1}^{q} X_{\lambda_j,i} Y_{\lambda_j,i} - I_{mn} \right\}^2, \tag{3}$$

where I_{mn} is the practically captured intensity of the interferogram.

Based on the principle of the least-squares algorithm, Eq. (3) can achieve the global minimum when the derivative of E_n with respect to $X_{\lambda_j,i}$ is equal to zero, which can be described as

$$\frac{\partial E_n}{\partial X_{\lambda_j,i}} = 0 (i = 0, 1, 2, \dots, 2p, j = 1, 2, \dots, q). \tag{4}$$

Consequently, according to Eq. (4), we can obtain a new equation as follows:

$$A^{(n)}_{\lambda_j,il} X^{(n)}_{\lambda_j,i} = B^{(n)}_{\lambda_j,l}, \tag{5}$$

where $A^{(n)}_{\lambda_j,il} = \sum_{m=1}^{M} Y_{\lambda_j,i} Y_{\lambda_j,l}$, $B^{(n)}_{\lambda_j,l} = \sum_{m=1}^{M} I_{mn} Y_{\lambda_j,l}$ $(i, l = 0, 1, 2, \dots, 2p)$.

From Eq. (5), the unknown $X_{\lambda_j,i}$ can be solved and the quantitative phases $\varphi_{\lambda_j,n}$ at each wavelength λ_j can be determined by

$$\varphi_{\lambda_j,n} = \frac{1}{k} \tan^{-1}\left(-\frac{X_{\lambda_j,2k}}{X_{\lambda_j,2k-1}}\right) (k = 1, 2, \dots, p, j = 1, 2, \dots, q). \tag{6}$$

In Eq. (6), pq phase images can be yielded, but most phases are unavailable. In the case of $k > 1$, for dual-wavelength interferograms with second-order harmonics, $b_{\lambda_j,mn1}$ is much larger than $b_{\lambda_j,mnk}$; thus, the calculated phase $\varphi_{\lambda_j,n}$ can have a large error by using the ratio of $-X_{\lambda_j,2k}$ to $X_{\lambda_j,2k-1}$ for estimating phases in DWIM. Consequently, Eq. (6) should be rewritten as

$$\varphi_{\lambda_j,n} = \tan^{-1}\left(-\frac{X_{\lambda_j,2k}}{X_{\lambda_j,2k-1}}\right) (k = 1, j = 1, 2, \dots, q). \tag{7}$$

Once the phases $\varphi_{\lambda_j,n}$ at each wavelength are estimated with correct global sign by using Eq. (7), the phase shifts can be determined with correct direction from at least $2pq + 1$ interferograms in DWIM.

Defining another set of variables as $X'_{\lambda_j,0} = 1$, $X'_{\lambda_j,2k-1} = \cos(k\varphi_{\lambda_j,n})$, $X'_{\lambda_j,2k} = \sin(k\varphi_{\lambda_j,n})$, $Y'_{\lambda_j,0} = \sum_{j=1}^{q} b_{\lambda_j,mn0}$, $Y'_{\lambda_j,2k-1} = b_{\lambda_j,mnk} \cos(k\delta_{\lambda_j,m})$, and $Y'_{\lambda_j,2k} = -b_{\lambda_j,mnk} \sin(k\delta_{\lambda_j,m}) (k = 1, 2, \dots, p)$, Eq. (1) is rewritten as

$$I'_{mn} = \sum_{i=0}^{2p} \sum_{j=1}^{q} X'_{\lambda_j,i} Y'_{\lambda_j,i}. \tag{8}$$

In Eq. (8), it can be assumed that the total dc term and the modulation amplitude do not have pixel-to-pixel variation. The accumulated error E_m, which results from the sum of squares of difference between the theoretical intensity and the measured one of the m th interferogram of N pixel points, can be expressed as

$$E_m = \sum_{n=1}^{N} \left\{ \sum_{i=0}^{2p} \sum_{j=1}^{q} X'_{\lambda_j,i} Y'_{\lambda_j,i} - I_{mn} \right\}^2. \tag{9}$$

To obtain the global minimum of Eq. (9), we have