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# Comparative analysis for combination of unwrapping and de-noising of phase data with high speckle decorrelation noise

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ABSTRACT

Unwrapping and de-noising are key processes for the restoration of phase data in the presence of high speckle decorrelation noise. Usually, there are two strategies to deal with noisy wrapped phase: de-noising before unwrapping, or unwrapping before de-noising. This paper aims at comparing the robustness and efficiency of the strategies. Six combinations which belong to different strategies are compared in this paper. Ten simulated phase maps with progressive noise standard deviations are generated based on the realistic speckle decorrelation noise to evaluate the performances of the approaches. The results of simulation show that de-noising with windowed Fourier transform filtering before unwrapping with the algorithm based on least-squares and iterations which belongs the first strategy has the best accuracy and acceptable computation speed for the restoration of high noisy phase data. Application of selected methods to experimental phase data from digital holography validated the analysis.



## 1. Introduction

Phase unwrapping [1] is a necessary procedure in many applications such as magnetic resonance imaging (MRI) [2–4], synthetic aperture radar imaging (SAR) [5–7], interferometry [8–10], profilometry [11–13], tomography [14–16] and holography [17–19]. The presence of noise makes phase unwrapping very difficult and decreases the accuracy of measurement. In the last decades, many phase unwrapping methods have been developed to deal with phase data with noise. Among them, quality guided approaches [1,20,21], Flynn's minimum discontinuity approaches [1,22], minimum $L^p$-norm ($L^0$) algorithms [1,23], PUMA [24,25] and CPULSI [26] exhibit good performance for unwrapping phase data with high noise. Usually, these approaches can directly unwrap phase data with noise and the acquired unwrapped phase data need to be de-noised to restore true phase data [27]. Another processing strategy is that de-noising is carried out for wrapped phase before unwrapping and then some fast phase unwrapping algorithms are utilized to unwrap the de-noised phase data [28,29]. For the last strategy, de-noising is carried out on the wrapped phase data. In order to preserve the $2\pi$ phase jump, some filtering methods are carried out on the sine and cosine images calculated from the raw phase, and some others are carried out on the complex representation. In these de-noising approaches, windowed Fourier transform filtering (WFTF), curvelet transform filtering (CTF) and median filtering (MF) exhibit excellent performance for wrapped phase de-noising [30]. In practice, these two strategies are both used in many applications. In this paper, we aim at comparing the performances of these two strategies.

This paper is organized as follows: in Section 2, the theoretical basics for the selected phase unwrapping and de-noising approaches are described; Section 3 proposes the processing strategies and corresponding approaches to be compared; Section 4 gives the simulated phase maps with speckle decorrelation noise. In Section 5, we discuss on the evaluation of the different approaches. Errors are quantitatively evaluated thanks to the data base constituted with the realistic simulation. Section 6 gives an application of these approaches in experimental data. Conclusions and perspectives to the study are drawn in Section 7.

## 2. Principles of phase unwrapping and de-noising algorithms

In our previous studies, we proposed two phase unwrapping algorithms, phase unwrapping based on least-squares and iterations (PULSI) [31] and calibrated phase unwrapping based on least-squares and itera-

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tions (CPULSI) [26]. We have demonstrated that CPULSI is more robust than other phase unwrapping algorithms in the presence of high speckle noise and PULSI is the fastest algorithm for unwrapping the noise-free or low-noise phase data [26]. In another study [30], we compared different algorithms for reduction of speckle decorrelation noise. We demonstrated that the windowed Fourier transform filtering (WFTF), curvelet transform filtering (CTF) and median filtering (MF) are the best approaches for phase de-noising. These algorithms are introduced briefly in the following.

### 2.1. Phase unwrapping based on least-squares and iteration (PULSI)

In this paper, $\varphi_{ij}$ represents the true phase and $\psi_{ij}$ ($\in[-\pi,+\pi]$, that is modulo $2\pi$) the wrapped phase at the grid point ($i,j$) of a phase map. The wrapped phase is extracted from a computation process based on the arctangent operator. This process is currently referred as the "wrapping operator". In this paper, the wrapping operator is symbolized as [1]:

$$W\left(\varphi_{ij}\right)=\psi_{ij}\quad (i=0,1...,M-1;j=0,1...N-1), \tag{1}$$

where $-\pi\le\psi_{ij}\le\pi$, $M$, $N$ are respectively the number of grid points with respect to the $i$ and $j$ index. The 1st order spatial wrapped phase derivatives are defined as:

$$\begin{aligned}&\Delta_{ij}^{x}=W\left(\psi_{(i+1)j}-\psi_{ij}\right)\quad (i=0,1...,M-2;j=0,1...N-1)\\&\Delta_{ij}^{x}=0\quad \text{otherwise}\\&\Delta_{ij}^{y}=W\left(\psi_{i(j+1)}-\psi_{ij}\right)\quad (i=0,1...,M-1;j=0,1...N-2)\\&\Delta_{ij}^{y}=0\quad \text{otherwise,}\end{aligned} \tag{2}$$

where $\Delta_{ij}{}^{x}$ and $\Delta_{ij}{}^{y}$ are respectively the difference with respect to the $i$ and $j$ indexes.

In the least-squares sense, the optimal solution $\varphi_{ij}$ can be obtained from the discrete Poisson equation with Neumann boundary conditions [31]:

$$\left(\varphi_{(i+1)j}-2\varphi_{ij}+\varphi_{(i-1)j}\right)+\left(\varphi_{i(j+1)}-2\varphi_{ij}+\varphi_{i(j-1)}\right)=\rho_{ij}, \tag{3}$$

where

$$\rho_{ij}=\left(\Delta_{ij}^{x}-\Delta_{(i-1)j}^{x}\right)+\left(\Delta_{ij}^{y}-\Delta_{i(j-1)}^{y}\right). \tag{4}$$

The discrete Poisson equation can be solved by many methods such as fast Fourier transform (FFT), discrete cosine transform (DCT) or the multi-grid method. In this algorithm, the DCT method is selected to solve the least-squares phase unwrapping problem.

Theoretically, the solution obtained from Eq. (3) is the exact one. However, there exist errors between the unwrapped phase and the true phase due to noise and to the smoothing performance of the least-squares method. In the proposed approach, iterations of unwrapped phase errors are utilized to seek more accurate results [31].

### 2.2. Calibrated phase unwrapping based on least-squares and iteration (CPULSI)

In practice, the presence of noise will generate errors between noisy and noise-free phase derivatives and makes unwrapping difficult, even failed. Here, we proposed a calibration approach in [26] to calibrate the phase derivatives exhibiting large errors:

$$\begin{aligned}&\Delta_{ij}^{x}=\operatorname{sgn}\left(\Delta_{ij}^{x}\right)\left|G_{x}\right|\quad \text{if}\quad \left|\Delta_{ij}^{x}\right|\ge T_{x}\\&\Delta_{ij}^{x}=\Delta_{ij}^{x}\quad \text{otherwise}\\&\Delta_{ij}^{y}=\operatorname{sgn}\left(\Delta_{ij}^{y}\right)\left|G_{y}\right|\quad \text{if}\quad \left|\Delta_{ij}^{y}\right|\ge T_{y}\\&\Delta_{ij}^{y}=\Delta_{ij}^{y}\quad \text{otherwise,}\end{aligned} \tag{5}$$

where sgn(…) is the signum function, $T_x$ and $T_y$ are the thresholds, $G_x$ and $G_y$ are the calibrated phase derivatives. These parameters are defined as ($E$[…] means statistical average):

$$\begin{cases}T_{x}=\sqrt{E\left[\left(\Delta_{ij}^{x}\right)^{2}\right]-\left(E\left[\Delta_{ij}^{x}\right]\right)^{2}}\\T_{y}=\sqrt{E\left[\left(\Delta_{ij}^{y}\right)^{2}\right]-\left(E\left[\Delta_{ij}^{y}\right]\right)^{2}}\end{cases}, \tag{6}$$

and,

$$\begin{cases}G_{x}=\frac{1}{MN}\sum_{i=0}^{i=M-1}\sum_{j=0}^{j=N-1}\Delta_{ij}^{x}\\G_{y}=\frac{1}{MN}\sum_{i=0}^{i=M-1}\sum_{j=0}^{j=N-1}\Delta_{ij}^{y}\end{cases}. \tag{7}$$

This calibration approach means that the phase derivatives whose values are larger than the standard deviation of phase derivatives are replaced by the average value of phase derivatives.

### 2.3. Windowed Fourier transform filtering (WFTF)

Windowed Fourier transform filtering (WFTF) [29,32] is a method which process phase maps locally, or block by block.

For the wrapped phase $\psi(x, y)$, we can convert it to exponential complex field as

$$f(x,y)=\exp[j\psi(x,y)] \tag{8}$$

where $j=\sqrt{-1}$. Subsequently, the signal $f(x, y)$ is transformed into its spectrum by a windowed Fourier transform (WFT). The WFT spectrum is thresholded in order to remove noise: the spectrum is set to zero if its amplitude is less than a preset threshold. The altered spectrum then undergoes an inverse windowed Fourier transform (IWFT) to reconstruct a filtered exponential phase field $\bar{f}(x,y)$. The whole process can be written as

$$\bar{f}(x,y)=\mathrm{IWFT(WFT(f(x,y)))} \tag{9}$$

Finally, the filtered phase can be given by the phase angle of $\bar{f}(x,y)$ as

$$\bar{\psi}(x,y)=\arg(\bar{f}(x,y)) \tag{10}$$

where arg(z) returns the principle argument ($\in[-\pi,+\pi]$) for each element of complex array z.

From the above algorithm, we can see that WFTF can only give the principle values of phase wrapped in the range [$-\pi$, $\pi$]. So WFTF is adapted to de-noise wrapped phase data. If we use this method to de-noise unwrapped phase, we can only acquire the wrapped results. In order to overcome the limitations of de-noising unwrapped phase with WFTF, we apply wrapping operator on the noisy unwrapped phase and subtract its WFTF values to obtain the filtered noise. Then, the de-noised unwrapped phase can be obtained by subtracting noise from noisy unwrapped phase.

### 2.4. Curvelet transform filtering (CTF)

Curvelet transform is based on wavelet transform and ridgelet transform to overcome the limitations of wavelet transform in de-noising the data with anisotropic features [33]. It is a multiscale geometric transform. The main idea of discrete curvelet transform is to decompose the signal into a set of wavelet bands and to analyze each band by a local ridgelet transform with a different block size for each scale level. The filtering in the curvelet domain is realized by performing curvelet transform to noisy signal, and using hard-thresholding rule to obtain filtered curvelet coefficients. Then the inverse curvelet transform is performed to restore the de-noised signal [33].

Curvelet transform filtering can be directly applied on the continuous unwrapped phase data. However, for the wrapped phase curvelet transform filtering will blur the phase jump. So we perform curvelet transform filtering on the sine and cosine of the wrapped phase and then calculate the de-noised wrapped phase from them.

### 2.5. Median filtering (MF)

Median filtering is one of the most commonly spatial filtering. It uses median value in the kernel to substitute the value of center of the kernel