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Advanced Fourier transform analysis method for phase retrieval from a single-shot spatial carrier fringe pattern

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A R T I C L E I N F O

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A B S T R A C T

An advanced Fourier transform analysis (***AFTA***) method is proposed in this study for phase retrieval from a single-shot spatial carrier fringe pattern (***SCFP***). It firstly extracts four phase-shifted fringe patterns (***FP***) from the ***SCFP*** with one pixel malposition, and then performs Fourier transform to each of them for their Fourier spectrums. The innovation of ***AFTA*** is making a subtraction in frequency domain directly to calculate the spectrum difference, which can generally eliminate the direct current component, thus it could mitigate the spectrum leakage problem and the edge error of traditional Fourier transform analysis (***FTA***) method significantly. After that, by making an inverse Fourier transform to the spectrum difference and a simple average operation, the phase information could be reconstructed conveniently. The time cost is approximately twice of ***FTA***, which is still highly efficient and time-saving for dynamic or real-time measurement. ***AFTA*** is adequately validated as a promising method in fringe analysis by various simulations and comparative experiments, with emphasis of this study on its performance in terms of accuracy, noise-tolerance, carrier frequency, time cost, etc.

1. Introduction

The demodulation of wavefront phase from fringe patterns (***FP***) is one of the most important techniques for modern optical metrology [1]. Among various available methods, temporal phase shift (***TPS***) method, commonly known as phase-shifting interferometry [1–3], is a well-established and accurate method for measuring optical wavefronts. It usually captures three or more ***FP***s with relative phase shifts in a time-sequenced fashion, and then by applying appropriate phase shifting algorithm [2], the wavefront phase can be calculated with a high efficiency and accuracy. However, its accuracy is found to be sensitive to the environmental vibration and the error of phase shifter [2].

Fourier transform analysis (***FTA***), as a spatial-frequency analytical technique proposed by Takeda et al. in 1982 [4], is one famous method for phase demodulation, especially for the phase retrieval from a single-shot spatial carrier fringe pattern (***SCFP***), and has been developed vastly in the past decades [5–7]. ***FTA*** is an efficient complement to the ***TPS*** method [2,3] when just one ***FP*** can be captured rather than multiple ***FPs*** (e.g., in vibration analysis or real-time measurements), thus it becomes an important and promising technique in optical measurements, such as laser interferometry and fringe pattern projection based applications [7,8].

FTA requires the addition of large linear spatial carrier (i.e., tilt fringes) to the ***FP*** between the test and the reference beams to separate the signal from the background. A ***FP*** with a spatial linear carrier can be analyzed to obtain the wavefront shape by processing the information in the interferogram plane (i.e., space domain [9]) or in the Fourier plane (i.e., frequency domain). ***FTA*** method retrieves the phase in Fourier plane in the following steps [2]. Firstly, ***FFT*** is implemented to the fringe pattern to obtain the ±1 order spectrum of the signal from the background. Then either +1 or −1 order spectrum is filtered out by a suitable band-pass filter. Finally, the phase information is retrieved by implementing the inverse ***FFT*** to the selected order. Other mathematical transform methods, such as windowed Fourier transform (***WFT***) [10–12] developed vastly by Kemao Qian et al., and wavelet transform ***(WT)*** [13–17], can be employed to retrieve the phase in a similar way. However, compared with ***WFT*** and ***WT***, ***FTA*** has the advantage of high efficiency (i.e., time-saving) because of the ***FFT*** algorithm, thus making it still widely used in today's fringe analysis [18]. The detailed comparisons of them were made by Huang et al., regarding to their merits and limitations [18].

Unfortunately, ***FTA*** also suffers from some problems, e.g., spectral leakage problem due to the insufficient carrier frequency, the edge error caused by the Gibbs effect, etc.[19]. If the spatial carrier is not high enough and the pupil is finite, the side ripples of the Fourier transforms of each order interfere with each other, making the spectral leakage to be the most important source of phase errors in the Fourier method. This spectral leakage can be reduced by increasing the tilt, using win-

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dow functions such as the Hamming filter, or extrapolating fringes outside of the pupil limits [19]. To reduce the edge error of ***FTA***, Dong et al. proposed one hybrid algorithm combining the ***FTA*** and spatial carrier phase shifting method [20], in which the ***FTA*** was used to retrieve the phase of four sub-fringe-patterns, and then by a subtraction operation to get the phase shifts between them, and the phase information could be finally reconstructed by a least squared phase shifting method [21]. The hybrid algorithm exhibits high accuracy, but the time-cost is aggravated, not only because of the extensive computations of ***FFT***, inverse ***FFT*** and phase unwrapping, but also the final phase reconstruction. More recently, Dong et al. also proposed a noise-tolerant hybrid algorithm [22], where the ***FFT*** was replaced by ***WT*** to enhance the noise tolerance, and the least squared phase shifting method was replaced by a least squared gradient integration method [23]. Its accuracy and noise-tolerance were validated to be considerably high, however, it is also time-consuming compared with ***FTA*** method, thus not suitable for real-time measurements.

Based on the idea of making subtractions to the phase-shifted ***FP***s retrieved from a single-shot ***SCFP***, in this study, an advanced ***FTA*** method (***AFTA***) is proposed to mitigate the spectrum leakage problem due to the insufficient carrier, and reduce the edge error of ***FTA*** caused by the Gibbs effect. The details of ***AFTA*** are elaborated in Section 2. A detailed derivation of mathematics, and some possible variations for implementing ***AFTA*** are also introduced. Its feasibility and performances (in terms of accuracy, noise-tolerance, spatial carrier and time cost) are also investigated and validated by simulations and contrastive experiments to ***TPS***, ***FTA*** in Sections 3 and 4. The work is finally summarized in Section 5.

2. Procedures of *AFTA*

2.1. Basis and characters of Fourier transform

Fourier theory is an important mathematical tool for the digital processing of interferograms. The core of Fourier theory is, any nonperiodical function can be regarded as a periodical function with an infinite period, and thus a nonperiodical continuous function can be represented by an infinite number of sinusoidal functions.

For Fourier transform operation to a signal $h(x, y)$, it can be expressed as

$$H(u, v) = F(h(x, y)) = \iint_{-\infty}^{\infty} h(x, y) e^{-i2\pi(ux+vy)} dx dy \tag{1}$$

where, $H(u, v)$ is the amplitude spectrum of the signal $h(x, y)$ at its frequency domain, (u, v) is the frequency coordinate, (x, y) is the spatial coordinate, and F represents the Fourier transform operation.

In a similar rule, the inverse Fourier transform to $H(x, y)$ can be expressed as

$$h(x, y) = F^{-1}(H(u, v)) = \iint_{-\infty}^{\infty} H(u, v) e^{i2\pi(ux+vy)} du dv \tag{2}$$

where, F^{-1} represents the inverse Fourier transform operation.

Some important characters of Fourier transform will be used in following contexts are

(1) Linear rule of Fourier transform

$$F(ah_1(x, y) + bh_2(x, y)) = aH_1(u, v) + bH_2(u, v) \tag{3}$$

(2) Displacement rule of Fourier transform (i.e., translation property):

$$F\left\{h(x, y) e^{i2\pi(f_a x + f_b y)}\right\} = H(u - f_a, v - f_b) \tag{4}$$

where, f_a, f_b are the frequency shifts.

The demonstration of them are not the aim of this study. The readers could find them in many related books dealing with Fourier transform.

2.2. FTA *algorithm review*

The intensity of a single-shot fringe pattern with spatial carrier is expressed as

$$g(x, y) = a(x, y) + b(x, y) cos[\phi(x, y) + 2\pi f_x x + 2\pi f_y y] \tag{5}$$

where, $a(x, y)$ denotes the background intensity, $b(x, y)$ denotes the modulating amplitude, and $\phi(x, y)$ is the phase map to be determined, f_x f_y are carrier frequencies in X and Y directions (for separating the signal from the background), respectively.

Base on the fact of that

$$\cos\phi = \frac{1}{2}[\exp(i\phi) + \exp(-i\phi)] \tag{6}$$

Eq. (5) can be written as

$$g(x, y) = a(x, y) + c(x, y)\exp[2i\pi(f_x x + f_y y)] + c^*(x, y)\exp[-2i\pi(f_x x + f_y y)] \tag{7}$$

where,

$$c(x, y) = \frac{1}{2} b(x, y) \exp[i\phi(x, y)] \tag{8}$$

Performing Fourier transform to both sides of Eq. (7), and based on the displacement rule of Fourier transform (see Eq. (4)), we get the spectrum of the fringe pattern

$$G(u, v) = A(u, v) + C(u - f_x, v - f_y) + C^*(u + f_x, v + f_y) \tag{9}$$

where, the Fourier transform uses a Fast Fourier transform(***FFT***) algorithm as its high efficiency. The spectrum in Eq. (9) is comprised of three components, the zero order of spectrum $A(u, v)$, and ± 1 order of spectrum, C and C^*, respectively.

The remaining operation of ***FTA*** separates the spectrum of C or C^* by proper filters, and then shifts it to the origin, and makes inverse Fourier transform to C or C^* obtaining $c(x, y)$ or $c^*(x, y)$, finally retrieves the wrapped phase (ϕ_w) by

$$\phi_w(x, y) = \operatorname{atan}\left(\frac{imag(c(x, y))}{real(c(x, y))}\right) \tag{10}$$

where, the subscript of "*w*" means a wrapped phase. The phase information could be obtained by various phase unwrapping algorithms [25–27].

However, in the proposed ***AFTA*** method, these remaining operations are replaced by the contents of Section 2.3.

2.3. AFTA *method*

From here, we take care of the four sub-fringe-patterns extracted from a carrier fringe pattern with one pixel malposition, with certain phase shift to each other as expressed by

$$\begin{cases} g_1(x, y) = g(x, y) = a_1(x, y) + b_1(x, y) cos(\Phi(x, y) + \delta_1(x, y)) \\ g_2(x, y) = g(x+1, y) = a_2(x, y) + b_2(x, y) cos(\Phi(x, y) + \delta_2(x, y)) \\ g_3(x, y) = g(x, y+1) = a_3(x, y) + b_3(x, y) cos(\Phi(x, y) + \delta_3(x, y)) \\ g_4(x, y) = g(x+1, y+1) = a_4(x, y) + b_4(x, y) cos(\Phi(x, y) + \delta_4(x, y)) \end{cases} \tag{11}$$

where, we define the phase map with spatial carrier as

$$\Phi(x, y) = \phi(x, y) + f_x x + f_y y \tag{12}$$

and $\delta_1(x, y)$, $\delta_2(x, y)$, $\delta_3(x, y)$, $\delta_4(x, y)$ are the relative phase shifts of the four sub-fringe-patterns (also are known as the phase gradients of $\phi(x, y)$), with $\delta_1(x, y) = 0$ as the reference. In Eq. (11), there is a one pixel boundary problem if the ***FP*** has even number, while not for odd number. For the case of even number, we can omit one row and one column to make it odd, or we can make an interpolation operation to make it odd.

We assume

$$\begin{cases} a = a_1 \approx a_2 \approx a_3 \approx a_4 \\ b = b_1 \approx b_2 \approx b_3 \approx b_4 \end{cases} \tag{13}$$