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Simultaneous estimation of multiple phases in digital holographic interferometry using state space analysis

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ABSTRACT

A new approach is proposed for the multiple phase estimation from a multicomponent exponential phase signal recorded in multi-beam digital holographic interferometry. It is capable of providing multidimensional measurements in a simultaneous manner from a single recording of the exponential phase signal encoding multiple phases. Each phase within a small window around each pixel is appproximated with a first order polynomial function of spatial coordinates. The problem of accurate estimation of polynomial coefficients, and in turn the unwrapped phases, is formulated as a state space analysis wherein the coefficients and signal amplitudes are set as the elements of a state vector. The state estimation is performed using the extended Kalman filter. An amplitude discrimination criterion is utilized in order to unambiguously estimate the coefficients associated with the individual signal components. The performance of proposed method is stable over a wide range of the ratio of signal amplitudes. The pixelwise phase estimation approach of the proposed method allows it to handle the fringe patterns that may contain invalid regions.

1. Introduction

Optical interferometric techniques, such as, electronic speckle pattern interferometry, holographic interferometry, shearography, and moiré are at the heart of the methods dedicated to obtaining full-field measurements on rough object surfaces in a non-invasive and non-contact manner [1–3]. Encoding the information on the physical quantity under measurement in the form of a change in the phase of the optical wavefields lies at the core of these techniques. In general, a sinusoidal fringe pattern embedding the phase information is recorded. It is essential to demodulate the fringe pattern to retrieve the underlying phase distribution. Consequently, fringe analysis is of crucial importance in obtaining the accurate measurement of the physical quantities and an important amount of research has been reported over the years [4,5] in this regard.

Among different techniques, digital holographic interferometry (DHI) offers some unique advantages [6]. Since the numerical reconstruction of a single hologram provides the measurement of both the amplitude and phase of the optical wavefield associated with the object, digital holography is suitable for performing measurement of dynamic events. In deformation analysis applications, the use of DHI have been limited to the measurement of a single component of the displacement vector, i.e. out-of-plane displacement. However, until and unless restricted by the methods' capabilities, it is usually desirable to obtain the simultaneous measurement of multiple components of the displacement vector. This has indeed been a formidable problem and it is only recently that we have started to tackle it in a practical manner.

Multi-beam digital holographic interferometry setups have been proposed over the years to obtain multidimensional measurements. The principle underlying most of these experimental setups is to simultaneously record multiple holograms one each associated with a single illumination beam. For example, multiple pairs of object-reference beams are used in [7] which are made incoherent with respect to each other by the adjustment of their path lengths. Orthogonally polarized reference beams are used in [8,9] to record incoherently mixed holograms. The other popular way of simultaneous recording of multiple holograms is using multi-color multi-beam DHI setups [10–14]. Although multiple phase information associated with the object deformation can be derived in a simultaneous manner, the use of multiple object-reference beam pairs complicates these experimental setups. Moreover, the measurement accuracy is affected by the color channel cross-talk in case of multi-color laser setups. Phase-shifting DHI has been proposed for the measurement of multi-dimensional displacement and strain [15,16] involving the recording of multiple holograms. Apart from multi-beam DHI, other optical interferometry setups have also been proposed for multi-dimensional deformation measurement [17,18].

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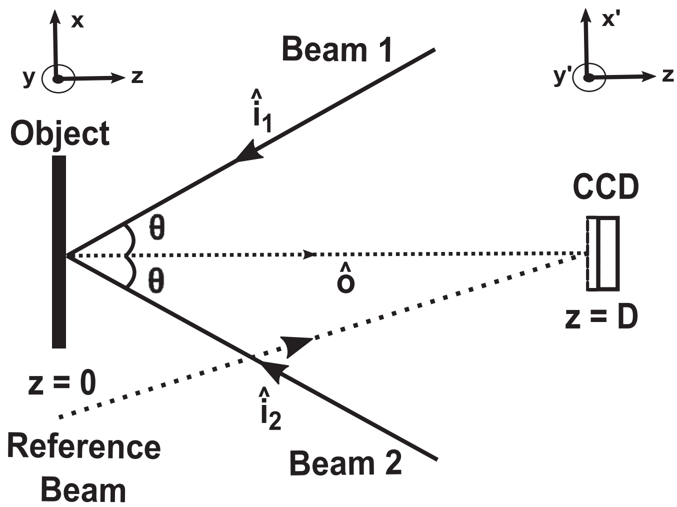

Fig. 1. Digital holographic interferometry setup consisting of two object illumination beams and single reference beam.

Recently, a multi-beam DHI configuration consisting of multiple object illumination beams and a *single* reference beam all derived from a single color laser source has been proposed [19,27]. The use of a single color laser source and a single reference beam simplifies the experimental setup and reduces the operational cost. In this paper, we consider a specific case of such multi-beam DHI configuration consisting of two object illumination beams.

In this paper, a new approach for the multiple phase estimation based on the approximation of phases as a first order two-dimensional polynomial functions of spatial coordinates within a small size window around each pixel is developed. Thus, the problem of phase estimation is converted into the problem of accurate estimation of the polynomial coefficients. An approach based on state space analysis is developed by assigning the polynomial coefficients and the signal amplitudes as the elements of the state vector. The state estimation performed using a nonlinear Kalman filter provides multiple phase estimates in the unwrapped forms. The pixel-wise approach of the phase estimation not only provides the unwrapped estimates of the two phases but also allows to demodulate the fringe patterns that are masked to distinguish the valid and invalid fringe regions.

2. Recording of exponential phase signal containing two components

A two-beam DHI setup proposed in [19] is shown in Fig. 1. The object is symmetrically illuminated with two light beams with respect to the object surface normal. The hologram is recorded in the off-axis digital holographic configuration. Due the use of two object beams and a single reference beam, the two phase distributions associated with a particular object state are encoded in a single hologram. The intensity of such a hologram placed in the (x', y') plane can be represented as [19]

$$\begin{aligned} I(x,y) &= |R(x',y') + O_1(x',y') + O_2(x',y')|^2 \\ &= (R + O_1 + O_2)(R + O_1 + O_2)^* \\ &= I_0 + R(O_1 + O_2)^* + R^*(O_1 + O_2), \end{aligned} \tag{1}$$

where R represents the complex amplitude of reference beam, and O_1 and O_2 represent the complex amplitudes scattered by the object surface upon illumination by beam 1 and beam 2, respectively, reaching at the hologram plane; * represents the complex conjugation; $I_0 = |R|^2 + |O_1|^2 + |O_2|^2 + O_1O_2^* + O_1^*O_2$. The spatial dependence of the variables is dropped in the last two lines of Eq. (1) for the sake of brevity. The hologram is numerically reconstructed upon which the spatially separated dc, virtual and real images of the object are obtained corresponding to RI_0, $R^2(O_1 + O_2)^*$ and $|R|^2(O_1 + O_2)$. Typically, the reference beam is a plane wave and is taken to be a constant. Considering the contribution of $|R|^2(O_1 + O_2)$ only, the reconstructed wavefront scattered by the object can be represented as

$$\Gamma_b(x,y) = A_{O1}(x,y)\exp[j\psi_1(x,y)] + A_{O2}(x,y)\exp[j\psi_2(x,y)],$$

where, $j = \sqrt{-1}$; $x \in [1, N]$ and $y \in [1, M]$ indicate the pixels representing columns and rows of the object image, respectively; A_{O1}, A_{O2} and ψ_1, ψ_2 represent the amplitude and phases of the scattered wavefronts at the object plane associated the beam 1 and 2, respectively. Due to the rough object surface, the phases ψ_1 and ψ_2 are random in nature. Another hologram is recorded after the object is loaded. The associated wavefront can be represented as

$$\begin{aligned} \Gamma_a(x,y) = \; & A_{O1}(x,y)\exp[j(\psi_1(x,y) + \phi_1(x,y))] \\ & + A_{O2}(x,y)\exp[j(\psi_2(x,y) + \phi_2(x,y))], \end{aligned}$$

where, ϕ_1 and ϕ_2 represent the deterministic phases introduced by the object deformation along O_1 and O_2, respectively. Consequently, these phases carry the information on object deformation. A small deformation causes the changes mainly in the phases and amplitudes are assumed to remain constant. The exponential phase signal (EPS) carrying the deterministic phase terms can be obtained as

$$\begin{aligned} \Gamma(x,y) &= \Gamma_a(x,y)\Gamma_b^*(x,y) \\ &= A_{O1}^2\exp(j\phi_1) + A_{O2}^2\exp(j\phi_2) \\ &\quad + A_{O1}A_{O2}\exp(j(\psi_1 - \psi_2 + \phi_1)) + A_{O1}A_{O2}\exp(j(\psi_2 - \psi_1 + \phi_2)) \end{aligned} \tag{2}$$

In Eq. (2), the deterministic phases are associated with the first two signal components of the EPS. Since, random phases are associated with the last two signal components, these terms can be treated together as additive noise, ϵ. Thus we have,

$$\Gamma(x,y) = A_1(x,y)\exp[j\phi_1(x,y)] + A_2(x,y)\exp[j\phi_2(x,y)] + \epsilon(x,y), \tag{3}$$

where, A_1 and A_2 represent the amplitudes of the signal components with phases ϕ_1 and ϕ_2, respectively. In general, the EPS is corrupted by the speckle noise. For the purpose of analysis, the noise is modeled as a complex additive white Gaussian random variable with zero mean and variance σ^2. It has been shown in [19] that the DHI setup in Fig. 1 is sensitive to the object displacement along x and z direction. Furthermore, the sum and the difference of phases ϕ_1 and ϕ_2 are directly proportional to the displacement along z and x directions, respectively. The aim of the proposed algorithm is to obtain the estimate of these phases from the EPS. In presence of multiple unknowns associated with the EPS at each pixel in Eq. (3), it is essential to make certain assumptions on the spatial distribution of amplitudes and phases of the two signal components in order to obtain an unambiguous phase estimation.

3. Theory

According to the Weierstrass approximation theorem, a continuous function can be represented as a polynomial of an appropriate order [20]. Under the assumption of spatial continuity of the phase, it can also be represented as a polynomial function of the spatial coordinates x and y. Since the order of the polynomial function cannot be determined *a priori*, we represent the phase within a small area around each pixel as a pre-defined low order polynomial function. Not surprisingly, polynomial phase approximation has been applied extensively to the problem of phase unwrapping [21–23] in many areas of application. Under the assumption that the sought phase distribution is spatially continuous, the polynomial phase approximation allows us to obtain the unwrapped phase distribution from a noise corrupted EPS. Recently, the authors have proposed algorithms based on the polynomial phase approximation for the estimation of multiple phases from a single EPS [24–27]. While the methods described in [24–26] perform a segment-wise analysis of the EPS along its rows or columns to provide the estimation of two