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Review

Full field modal measurement with a single standard camera

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ABSTRACT

The full field measurement of 8 vibration modes of a cantilever plate, up to the kHz frequency is performed using a single standard camera. The image acquisition is carried out at low rate but using fast exposure time (1/4000 s) that “freezes” a part of the motion. A deflectometry setup coupled with digital image correlation analysis allows angle fields, then deflection fields, to be measured. An out–of–plane displacement database is generated from all stored fields. This snapshot library is finally post-processed using principal component analysis techniques and adapted weights that enable space vibration fields to be extracted. A test case is performed on a reflecting cantilever steel plate. 370 deflection fields are measured allowing 8 space vibration fields to be extracted. The natural frequencies of those modes are estimated using the Rayleigh ratio. A coarse and visual comparison with a numerical simulation shows similar results for 6 of the 8 space modes.

1. Introduction

The analysis of the displacement fields of a sample during a mechanical test is a key element for model validation and identification. One of the developed full field measurement procedure is Digital Image Correlation (DIC) [1] that provides 2D displacement fields from a series of images. This technique is now widely used in laboratories because of its robustness, its accuracy and the generally accessible required experimental setup. Based on reduced predefined kinematic solutions (*e.g.,* finite element mesh kinematic with global DIC [2]), this method has been applied for complex measurements and identifications (*e.g.,* measurement on images with poor texture, with gray level and blur changes [3], localized motions as crack initiation and propagation [4] or shear bands etc.). It provides accurate displacement fields, up to centi-pixel, with well characterized uncertainties [5]. Nevertheless, DIC is generally limited to the analysis of in–plane displacement fields. To circumvent this limitation, an extension of DIC for the measurements of 3D surface motions called stereo-DIC has been developed.

In stereo-DIC methods, the different views of the same surface enable, in addition to the in-plane motion, the out–of–plane displacement to be measured. Stereo-DIC has been applied in many fields including kinematic identification, material characterization [6], surface topology measurement [7,8] etc. Stereo-DIC also appears to be an efficient tool for the analysis of non-flat structures with complex 3D shapes. Different experimental setups have been studied in the literature, generally combining multiple cameras, but also with a stereo-setup based on a single camera with multiple mirrors (mirror based single-camera stereo-DIC methods [9,10]) or a prism [11] in order to have multiple views of the same characterized surface. Combined with fast cameras, stereo-DIC has been developed for the measurement of vibration displacement fields [12–14] (often applied for the study of the deflection of beams or plates). Nevertheless, those stereo-approaches may require a setup composed of multiple fast cameras [15] and a complex procedure for the calibration and DIC measurement. The analysis is also limited to the first vibration modes until the deflection becomes too small (particularly with fast cameras that often have low spatial resolutions).

Deflectometry is an experimental method that allows small out–of–plane displacements to be assessed [16–18]. A grid or fringe pattern is imaged through a reflective sample by a camera. The deflection of the sample deforms the acquired grid patterns that are analyzed from phase measurements. This method is a gradient measurement technique that provides a full field slope identification of the surface. The height can then be obtained after integration. This method has been developed for different applications such as topography measurements [19–21], dynamic measurements for vibration, damage detection [22], mechanical identification [23,24].

A similar method to deflectometry has been developed in the field of fluid mechanics in the past few years [25,26]. An angle field is measured from the refraction of water instead of mirror samples. The bottom face of a water tank is covered by speckle patterns. A camera is located outside, at the top of the reservoir and images this bottom face throughout the water. A change of the fluid height (generated by wave propagation) distorts the acquired images and hence can be read as surface angle using digital image correlation and geometrical relations. The surface heights can thus be obtained from the integration of the measured angle fields. The method has also been used for transparent film motion analysis [27].

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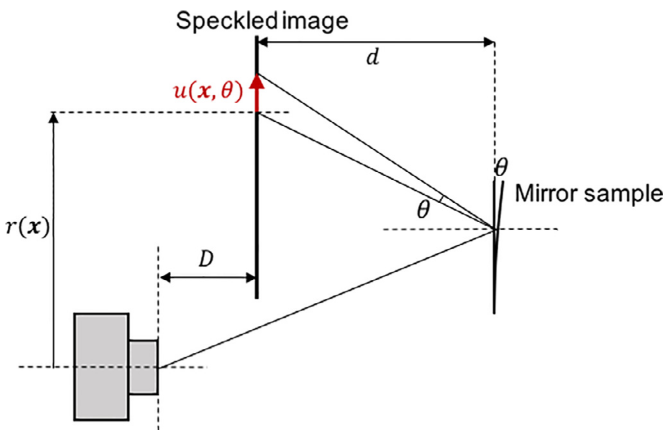

Fig. 1. Principle of deflectometry based on a mirror sample that reflects a speckled image. The bending of the measured plate deforms the acquired images.

Many models and data reduction techniques [28,29] (*e.g.,* Principle Component Analysis (PCA), Singular Value Decomposition (svd), Karhunen–Loeve decomposition) provide modal decomposition of a problem with separated variables. The analysis of space-time fields allows modes, product of separated space and time functions, to be generated. This may constitute, when truncated up to a certain number of modes, a reduced basis. The relationship between vibration modes and PCA decomposition [29,30] has been studied. It has been shown that, considering a light damping system composed of an identity mass matrix (that can be easily obtained from a change of variable), the space eigenmodes of the PCA decomposition could be confused with vibration modes [31,32]. For an accurate modal measurement, the number of snapshots used for the decomposition has to be large [33].

It is proposed in this article a modal measurement of a vibrating plate using a deflectometry setup and a single standard camera. A series of speckled images are acquired at a slow rate and fast exposure time. The global DIC post-process gives a series of slope measurement snapshots with a very high sensitivity to the deflection. The slope fields are then integrated in height fields and stored in a database. Then a modal basis composed of the space modes associated to natural frequencies can be obtained using PCA decomposition techniques with a particular attention on the experimental uncertainties. An application on a vibrating cantilever plate is carried out. The 420 acquired images with a standard digital camera allows 8 vibration modes (space and time) to be identified up to 1 kHz frequency. A comparison with the 8 first modes from a finite element simulation gives 6 similar space shapes.

2. Method

2.1. Deflectometry

The out–of–plane measurement method used in this article is based on deflectometry [16–18]. Instead of measuring the displacement field on the surface of a sample, as it would be done in DIC, it is proposed to work on the reflection of a fixed speckled image by a mirror specimen. With the deflection of the sample, the reflected image captured by the camera is distorted. Contrary to standard full field measurement methods as DIC, the deflectometry fields are slope fields that can be integrated in one single height field. For the experimental measurement on non-reflecting materials, a thin mirror film could be added on the surface. A drawing of the deflectometry setup is shown Fig. 1.

A very high out–of–plane sensitivity is expected, related to the geometrical parameters (*i.e.,* length from the mirror to the speckled image). It can be noted that this length is easily adaptable depending on the required angle amplification and allowing to avoid caustic effects (the size of the speckles can also be adjusted).

Multiple sources of image distortion may appear in this procedure and can be corrected:

- distortion of the image due to the deformation of the mirror sample. This is the identified distortion, related to the bending of the imaged surface. Then, the slope fields have to be post-processed in order to obtain the deflection.
- distortion introduced by the imaging device and setup (tilted position of the camera, conic angle of view etc.). This can be measured and corrected for example with geometric models or with the comparison of a known speckled target (calibration printed sample) with its distorted acquisition.
- distortion introduced by an initial curvature on the reflecting sample. This can be identified using a calibration sample.

The two last distortions can be identified experimentally or analytically (with assumptions on the geometry). Because the following application is the measurement of an assumed plane sample (a steel cantilever plate), the last distortion is not considered.

2.2. Digital image correlation

Digital image correlation [1] relies on the registration of an image $f(\boldsymbol{x})$, defined for every pixels of the selected region of interest $\boldsymbol{x} = [x, y]$ in the reference configuration and a series of pictures $g(\boldsymbol{x})$ in the deformed configurations. The registration operation consists of minimizing the sum of squared differences between the deformed image corrected for its displacement and the reference image. Hence

$$\boldsymbol{u}(\boldsymbol{x}) = \mathrm{Argmin}_{u\in E}\ \frac{1}{2\sigma^2 N_x}\sum_{\boldsymbol{x}}(f(\boldsymbol{x}) - g(\boldsymbol{x}+\boldsymbol{u}(\boldsymbol{x})))^2 \tag{1}$$

with E, a subspace composed of a reduced kinematic basis, σ the standard deviation of the supposed white Gaussian noise and N_x the number of pixels per image. With global DIC [2], the motion is described on a finite element mesh kinematic basis using standard shape functions $\boldsymbol{\phi}_i(\boldsymbol{x})$ and a reduced number of unknowns u_i such that

$$\boldsymbol{u}(\boldsymbol{x}) = \sum_{i=1}^{N_{\mathrm{dof}}} u_i \boldsymbol{\phi}_i(\boldsymbol{x}) \tag{2}$$

with N_{dof} the total number of degrees of freedom (2 displacements per node). The minimization of the functional is carried out with a Newton descent algorithm. The DIC is carried out using the Correli-3.0 framework [34] with an update at each iteration l of the displacement field $\{\boldsymbol{u}\}^l = \{\boldsymbol{u}\}^{l-1} + \{\delta\boldsymbol{u}\}$. The correction of the displacement $\{\delta\boldsymbol{u}\}$ for the linearized problem can be written

$$\{\delta\boldsymbol{u}\} = [\boldsymbol{H}]^{-1}\{\boldsymbol{b}\} \tag{3}$$

with $[\boldsymbol{H}]$ the Hessian function with respect to $\{\boldsymbol{u}\}$ and $\{\boldsymbol{b}\}$ the second member vector

$$H_{ij} = \frac{1}{2\sigma^2}\sum_{\boldsymbol{x}}\left(\nabla f(\boldsymbol{x})\boldsymbol{\phi}_i(\boldsymbol{x})\right)\left(\boldsymbol{\phi}_j(\boldsymbol{x})\nabla f(\boldsymbol{x})\right) \tag{4}$$

$$b_j = \frac{1}{2\sigma^2}\sum_{\boldsymbol{x}}\eta^l(\boldsymbol{x})\boldsymbol{\phi}_j(\boldsymbol{x})\nabla f(\boldsymbol{x}) \tag{5}$$

with ∇ the gradient operator and $\eta^l(\boldsymbol{x})$ the residual field at iteration l that represents what has not been captured by the kinematic correction. This residual allows the procedure to be (in)validated

$$\eta^l(\boldsymbol{x}) = f(\boldsymbol{x}) - g(\boldsymbol{x} + \boldsymbol{u}^l(\boldsymbol{x})) \tag{6}$$

From geometrical or experimental relations, the measured displacement fields on the deformed surface can be read as angle fields. With $\{\boldsymbol{\theta}\} = [\theta_x, \theta_y]^t$ the angles in x and y directions

$$\{\boldsymbol{\theta}\} = \Gamma(\{\boldsymbol{u}\}) \tag{7}$$

This function will be expressed for the test case in the application part considering a simple cone field of view correction.