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Accuracy-enhanced constitutive parameter identification using virtual fields method and special stereo-digital image correlation

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ABSTRACT

The virtual fields method (VFM) is generally used with two-dimensional digital image correlation (2D-DIC) or grid method (GM) for identifying constitutive parameters. However, when small out-of-plane translation/rotation occurs to the test specimen, 2D-DIC and GM are prone to yield inaccurate measurements, which further lessen the accuracy of the parameter identification using VFM. In this work, an easy-to-implement but effective “special” stereo-DIC (SS-DIC) method is proposed for accuracy-enhanced VFM identification. The SS-DIC can not only deliver accurate deformation measurement without being affected by unavoidable out-of-plane movement/rotation of a test specimen, but can also ensure evenly distributed calculation data in space, which leads to simple data processing. Based on the accurate kinematics fields with evenly distributed measured points determined by SS-DIC method, constitutive parameters can be identified by VFM with enhanced accuracy. Uniaxial tensile tests of a perforated aluminum plate and pure shear tests of a prismatic aluminum specimen verified the effectiveness and accuracy of the proposed method. Experimental results show that the constitutive parameters identified by VFM using SS-DIC are more accurate and stable than those identified by VFM using 2D-DIC. It is suggested that the proposed SS-DIC can be used as a standard measuring tool for mechanical identification using VFM.

1. Introduction

The Virtual Fields Method (VFM), proposed and advocated by Grédiac and Pierron [1–7], is an inverse method for identifying the parameters governing a constitutive equation. Based on the principle of virtual work, VFM can be used to estimate multiple constitutive parameters from a test specimen in a single test. Compared with the classical finite element model updating method (FEMU), the efficiency advantage of the VFM is evident, since it does not require performing heavy iterative finite element simulations of the test to find constitutive parameters that achieve the best match between computed and actual measurements. In addition, using suitable virtual fields causes the VFM to be insensitive to boundary conditions that usually affect the stress fields.

In earlier studies regarding VFM, the grid method (GM) was most commonly used to determine the full-field deformation of test specimens. However, the following inherent drawbacks of the GM limit the prevalence of VFM [8–11]. First, it may be tricky to fabricate and transfer good-quality grid pattern to the surface of a test specimen. Second, the CCD array and the grid should be carefully aligned to avoid the aliasing phenomenon in acquired grid images. In the last decades, the rapid development and popularity of digital image correlation (DIC) techniques [12] facilitates the easier implementation and wider acceptance of the VFM. With its outstanding advantages of simple optical arrangement and easy specimen preparation, low requirement on experimental environment, and wide applicability, DIC serves as an easy-to-implement and versatile full-field measurement technique for VFM. Nowadays, the DIC-based VFM has been widely used to identify constitutive parameters of various materials, including but not limited to metals [13–15], polymeric foams [16,17] and composites [18,19]. Recently, in combination with DIC techniques, VFM has also been used to identify elasticity parameters of graphite [20], 304-steel [21] under high temperature and biological materials [22]. Interestingly, an extensive comparison between VFM and FEMU was carried out in the latter. It was observed that VFM was 125 times faster and more robust than FEMU.

When using VFM, full-field kinematic measurements are used with the applied force amplitude as the only experimental data input for subsequent identification. The accuracy of the identified constitutive parameters therefore heavily depends on the accuracy of the full-field deformation measurements considered as input data. In the literature, 2D-DIC is used in most of the investigations and applications of DIC-

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based VFM to measure in-plane full-field deformations [18–20]. Strictly speaking, 2D-DIC using a single camera is limited to in-plane displacement/strain measurement of planar surfaces. In this case, the measured kinematic fields are however impacted by the out-of-plane motion/rotation of the specimen during loading. Furthermore, lens distortion, which has an adverse effect on the measured strains, would also lessen the accuracy of constitutive parameters identification using VFM [23–25]. Of course, a high-accuracy 2D-DIC measuring system with high-quality bilateral telecentric lenses can be used. Such a system is insensitive to out-of-plane motion of test object within its telecentric depth. It also demonstrates negligible lens distortion [26]. However, a high-quality bilateral telecentric lens is very expensive and with limited applicability, since its object distance shall range within the telecentric depth, and the field of view and magnification are fixed. Recently, Wang et al. [16,17] employed two cameras to measure the heterogeneous deformation fields on the two back-to-back specimen planes simultaneously. In this case, the effect of out-of-plane movements can be eliminated by averaging measured values from two cameras. Nevertheless, using two separate 2D-DIC systems bring additional difficulties, such as precise synchronization of the two systems and precise matching of the same measurement points.

Compared with 2D-DIC, stereo-digital image correlation (stereo-DIC) can provide more accurate deformation measurements without being affected by out-of-plane movements. Therefore, VFM, in combination with stereo-DIC, is expected to generate more accurate identification of constitutive parameters. However, in using regular stereo-DIC for full-field deformation measurements, the region of interest (ROI) and grid step are defined in the reference image captured by the left camera. Since the optical axis of the left camera is generally oblique to the test specimen surface (i.e., the sensor plane of the left camera is not parallel to the specimen surface), the calculation points are non-uniformly distributed on the specimen surface. Naturally, non-uniformly distributed points within the specified ROI increases the complexity in calculating the internal virtual work when applying the VFM. If the presence of non-uniformly distributed deformation data is neglected, the internal virtual work is estimated inaccurately. As a result, the accuracy of the constitutive parameters identified by VFM is deteriorated. Some researchers [14,15] employed the finite element method (with C° continuity) to eliminate the effect of non-uniformly distributed measured points on the specimen surface, but both the finite element calculation and data smoothing process dramatically increase the computational complexity.

In this paper, a special stereo-DIC (SS-DIC) method is proposed for accuracy-enhanced VFM identification. In the proposed SS-DIC, the sensor plane of the left camera is carefully aligned to be parallel to the specimen surface (just like any regular 2D-DIC system), which ensures spatially homogeneous measurement points on the surface of the test planar specimen. The right camera is inclined with respect to the object surface to switch from 2D to 3D deformation measurement in combination with the left camera. Using the SS-DIC, VFM is fed by high-accuracy full-field deformation measurement, which is not deteriorated by out-of-plane motion. Also, based on the spatially homogeneous deformation data on the specimen surface, VFM identification can be readily implemented. In addition, the method can be directly used to compare the accuracy of 2D-DIC and stereo-DIC measurements by using only the left camera instead of both cameras. In order to verify the effectiveness of the proposed SS-DIC-based VFM method, tensile tests on a perforated aluminum specimen and pure shear tests on a prismatic aluminum specimen were carried out. The identified constitutive parameters are also compared with those obtained with 2D-DIC-based VFM.

2. The virtual fields method and special stereo-DIC

2.1. The virtual fields method

The VFM is based on the principle of virtual work. For a continuous deformable solid with any shape, an integral form of the mechanical equilibrium equation can be written as follows:

$$-\int_V \boldsymbol{\sigma} : \boldsymbol{\varepsilon}^* \mathrm{d}V + \int_{\partial V} \bar{\mathbf{T}} \cdot \mathbf{u}^* \mathrm{d}A + \int_V \mathbf{b} \cdot \mathbf{u}^* \mathrm{d}V = \int_V \rho \boldsymbol{a} \cdot \mathbf{u}^* \mathrm{d}V \quad \forall \mathbf{u}^* \mathrm{KA} \tag{1}$$

where V is the volume of the studied part of the specimen, ∂V is the solid boundary where the external force is applied, $\boldsymbol{\sigma}$ is the stress tensor, $\mathbf{u}^*$ is the virtual displacement vector, $\boldsymbol{\varepsilon}^*$ is the corresponding virtual strain tensor, $\bar{\mathrm{T}}$ is the external surface force density applied on the solid boundary, $\mathbf{b}$ is the volume force, ρ is the mass density, $\boldsymbol{a}$ is the acceleration vector, KA represents the kinematically admissible condition.

The virtual displacement $\mathbf{u}^*$ shall be continuous and must satisfy the kinematically admissible condition. In the static or quasi-static cases, the inertial force can be ignored. In addition, the effect of the volume force is negligible compared to that of the external surface force, so the equation for the principle of virtual work reduces to

$$-\int_V \boldsymbol{\sigma} : \boldsymbol{\varepsilon}^* \mathrm{d}V + \int_{\partial V} \bar{\mathbf{T}} \cdot \mathbf{u}^* \mathrm{d}A = 0 \quad \forall \mathbf{u}^* \mathrm{KA} \tag{2}$$

In the above equation, the stress tensor $\boldsymbol{\sigma}$ cannot be measured directly by experiments. However, the stress can be expressed as a function of the strain tensor and constitutive parameters. This equation can be used with different and independent kinematically admissible fields $\mathbf{u}^*$ to identify the parameters governing a various type of constitutive models such as linear, non-linear elasticity and elasto-plasticity. More details can be found in Ref [2].

In this paper, the linear-elasticity model was considered to easily estimate the improvement brought about by using SS-DIC to measure the deformation fields. Indeed, the parameters identified with the VFM can be compared in this case to the Young's modulus and the Poisson's ratio obtained by standard tests such as strain gauges. For an in-plane problem and isotropic materials, the linear relation between stress and strain can be expressed as follows:

$$\begin{pmatrix} \sigma_x \\ \sigma_y \\ \tau_{\mathrm{xy}} \end{pmatrix} = \begin{bmatrix} Q_{xx} & Q_{xy} & 0 \\ Q_{xy} & Q_{xx} & 0 \\ 0 & 0 & (Q_{xx} - Q_{yy})/2 \end{bmatrix} \begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{pmatrix} \tag{3}$$

The components of the stress tensor in Eq. (2) can be rewritten as a function of the components of the strain tensor using Eq. (3). Thus

$$Q_{xx} \int_S \left(\varepsilon_x \varepsilon_x^* + \varepsilon_y \varepsilon_x^* + \frac{1}{2} \gamma_{xy} \gamma_{xy}^* \right) \mathrm{d}S + Q_{xy} \int_S \left(\varepsilon_x \varepsilon_y^* + \varepsilon_y \varepsilon_x^* - \frac{1}{2} \gamma_{xy} \gamma_{xy}^* \right) \mathrm{d}S = \frac{1}{t} \int_{\partial V} \bar{\mathbf{T}} \cdot \mathbf{u}^* \mathrm{d}A \tag{4}$$

where the left part of Eq. (3) is the internal virtual work, and the right part the external virtual work. Q_{xx} and Q_{xy} are the unknown parameters, S is the area of the selected region and t is the thickness of the specimen. In practice, $\bar{\mathbf{T}}$ is deduced from a concentrated force measured by a load cell. The strain components ε_x, ε_y and γ_{xy} are measured on the specimen surface using a full-field optical technique, here SS-DIC.

In practice, the measured strain fields are composed of series of discrete data points. Each data point represents the local deformation of a small element. In other words, the measured strain fields can be represented as numerical 2D matrices of $\mathrm{m} \times \mathrm{n}$ points, and the virtual ones have the same dimension. Consequently, Eq. (4) can be expressed as follows:

$$Q_{xx} \sum_{\mathrm{i}=1}^{\mathrm{m}} \sum_{\mathrm{j}=1}^{\mathrm{n}} \left(\varepsilon_x \varepsilon_x^* + \varepsilon_y \varepsilon_x^* + \frac{1}{2} \gamma_{xy} \gamma_{xy}^* \right) \mathrm{S_{ij}} + Q_{\mathrm{xy}} \sum_{\mathrm{i}=1}^{\mathrm{m}} \sum_{\mathrm{j}=1}^{\mathrm{n}} \left(\varepsilon_x \varepsilon_x^* + \varepsilon_y \varepsilon_x^* - \frac{1}{2} \gamma_{xy} \gamma_{xy}^* \right) \mathrm{S_{ij}} = \frac{1}{t} \int_{\partial V} \bar{\mathbf{T}} \cdot \mathbf{u}^* \mathrm{d}A \tag{5}$$

where the subscript ij (i, j represent the location in the matrix) after ε_x, ε_y, γ_{xy} and the corresponding virtual strain components are removed for notation brevity; $\mathrm{S_{ij}}$ is the area of each element.