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Two-step fringe pattern analysis with a Gabor filter bank

Mariano Rivera [a,*], Oscar Dalmau [a], Adonai Gonzalez [b], Francisco Hernandez-Lopez [a]

[a] Centro de Investigacion en Matematicas AC, 36240 Guanajuato Gto., Mexico
[b] Departamento de Ciencias e Ingenieria, Universidad Iberoamericana Leon, 37238 Leon Gto., Mexico

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ABSTRACT

We propose a two-shot fringe analysis method for Fringe Patterns (FPs) with random phase-shift and changes in illumination components. These conditions reduce the acquisition time and simplify the experimental setup. Our method builds upon a Gabor Filter (GF) bank that eliminates noise and estimates the phase from the FPs. The GF bank allows us to obtain two phase maps with a sign ambiguity between them. Due to the fact that the random sign map is common to both computed phases, we can correct the sign ambiguity. We estimate a local phase-shift from the absolute wrapped residual between the estimated phases. Next, we robustly compute the global phase-shift. In order to unwrap the phase, we propose a robust procedure that interpolates unreliable phase regions obtained after applying the GF bank. We present numerical experiments that demonstrate the performance of our method.

1. Introduction

Despite the advances in single-shot algorithms for Fringe Pattern (FP) with closed fringes, in recent years there has been an interest in developing two-step algorithms with random phase-shift since such an approach simplifies and strengthens the phase recovering procedure. See, for example, the methods reported in Refs. [1–10] and references therein. Those techniques have significantly reduced the acquisition time and simplified the experimental setups. In this work, we propose a two-step analysis method that assumes the following FP model:

$$I_j(x) = a_j(x) + b_j(x)\cos(\phi(x) + \delta_j) + \eta_j(x); \quad j = 1, 2 \tag{1}$$

where $x = [x_1, x_2]^T$ denotes the pixel position in a regular lattice $\mathcal{L}$. The unknowns in (1) are the background illumination, a_1 and a_2; the local fringe contrast, b_1 and b_2; the phase-shift, δ_1 and δ_2; the phase map we are interested in computing, ϕ; and, of course, the independent noise, η_1 and η_2. Without loss of generality, we define the random phase-shifts $\delta_1 = 0$ and $\delta_2 = \delta$, with $\delta \in [-\pi, \pi)$. In this work, we assume a_j, b_j (for $j=1,2$) and ϕ are smooth. Fig. 1 shows a couple of interferograms with the afore-mentioned characteristics.

In this paper, we propose a robust algorithm that overcomes the limitation of random two-step algorithm in the literature. Our method estimates the phase from two noisy FPs with spatial–temporal variations in the illumination conditions and a random phase-shift, δ. Our method consists of three stages:

1. *Normalisation*: We propose to normalise the FP (eliminating the illumination components a and b) with the application of a Gabor Filter (GF) bank. By means of this filter bank, we compute the local phase of each FP, except for a common random sign map. We also compute a quality map that indicates the pixels where the FP is correctly normalised. GFs [11] are widely used in image processing, computer vision and other areas [12–14].
2. *Phase computing*: In order to compute the final phase, we propose a method for estimating the phase shift between the FPs. We also show that procedures for phase shift computing based on the cross-correlation factor or Gram–Schmidt orthonormalisation fail to estimate the phase shift.
3. *Phase unwrapping*: We present a variant for robust phase unwrapping. Our variant incorporates the quality map and interpolates unreliable regions.

The FP normalisation consists in removing the background illumination and the contrast variations. Recently, Trusiak and Patosky proposed a normalisation procedure based on the Hilbert–Huang Transform (HHT) [9]. However, HHT is prone to fail when the FPs have regions with high frequencies that are corrupted by noise: the FP's extrema corresponding to high frequencies can be confused with noise. An alternative is to reduce the noise previous to the normalisation. For this task, Villa et al. propose a method for FPs filtering that can be understood in two stages: estimation of the local fringe's orientation, and low-pass filtering along the fringes [15]. Abramovich et al. investigate other alternatives for FP filtering that can be considered as directional low-pass filters [16]. Enguita et al. proposed the use of GF bank (set of narrow-band filters) for denoising FPs produced by conoscopic holography [17].

* Corresponding author.
E-mail address: mrivera@cimat.mx (M. Rivera).

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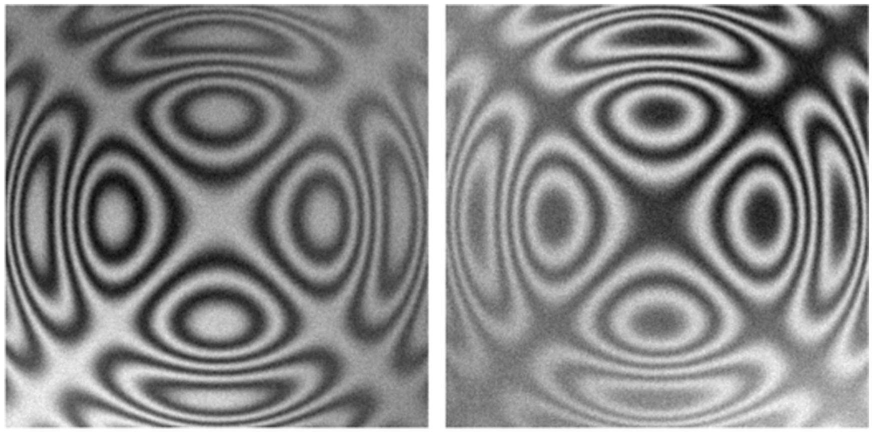

Fig. 1. Noisy interferograms with random phase-shift and spatial–temporal variations in the illumination components.

The FPs, with almost constant frequency, are normalised. In order to simultaneously filter noise and compute the phase, Jun and Asundi use a GF bank tuned to the dominant phase peak (carrier frequency) [18]. As we are not assuming a carrier frequency, we propose the use of a GF bank that covers half of the Discrete Fourier Domain and rejects the low-pass region spectra (related to the background illumination) and very-high frequencies (assumed as noise). In this paper, we assume that the phase map and the illumination components of the FPs are smooth. Thus, it is natural to compute the normalised FP as the cosine of the phase corresponding to the GF's response with maximum magnitude.

After normalising the FPs, one can proceed with the phase computation. This requires the implicit or explicit computation of the phase-shift. A popular strategy for estimating the random phase-shift build upon the definition of the cross-correlation factor (*c.c.f.*): the cosine of the angle between two multidimensional vectors [7–10]. However, as we demonstrate, the angle between multidimensional vectors is not the phase-shift between FPs. Another strategy computes the ratio of the extreme values (maximum or minimum) of the FPs [6]. The disadvantage of these strategies are the computational cost of detecting the extrema and its limitation in dealing with noisy FP. Wang and Han propose a two-stage method to iteratively estimate the phase and the phase-shift [2]. Rivera et al. present a generalisation to deal with more than two FPs [19]. Despite the advantages of the methods of Wang and Han, and Rivera et al.; they are computationally expensive and require normalised FPs. On the other hand, our method computes the phase from two FPs with random phase-shift and spatio-temporal variations in their illumination components.

Since the computed phase can be unreliable within regions with very-low frequency, saddle sites and fringes centre, we modify the phase unwrapping method ARM [20] to interpolate the unreliable regions.

This paper is organised as follows. Section 2 presents our method. It begins with the normalisation stage, followed by the phase recovery stage and the unwrapping procedure. Section 3 discusses the limitations of other approaches in contrast to our proposal. Section 4 presents experiments in order to demonstrate our method's capabilities. Finally, conclusions are given in Section 5.

2. Methods

The proposed method consists of three stages. First, we use a GF bank to recover the phase from an FP, up to a random sign map s. We denote this phase extraction step by the operator $\mathcal{H}$. Next, we estimate the phase-shift between the pair of FPs and compute the sign-corrected phase. Finally, we unwrap the phase using a robust procedure (denoted by the operator $\mathcal{W}^{-1}$) that interpolates unreliable phase pixels.

2.1. Normalisation $\mathcal{H}$

The normalisation operator $\mathcal{H}$ is implemented as a procedure. It is defined in this subsection and is based on the application of a Gabor Filter (GF) bank $\{h_k\}_{k=1,2,\ldots,K}$. The GF bank allows us to estimate the local magnitude and phase of the FPs, I_1 and I_2. GFs are band-pass filters that are the result of modulating a complex sinusoid with a Gaussian [11]. The complex form of the convolution kernel is of the form

$$h_k(x) \stackrel{def}{=} g_k(x) c_k(x) \tag{2}$$

where

$$g_k(x) = \exp\left[-x^\top x/(2\sigma_k^2)\right] \quad \text{and} \quad c_k(x) = \exp\left[-i\omega_k^\top x\right], \tag{3}$$

where $i \stackrel{def}{=} \sqrt{-1}$, σ_k is the width of the Gaussian filter (bandwidth of the bandpass filter) and $\omega_k = [u_k, v_k]^\top$ is the central complex frequency (centre of the bandpass filter). The behaviour of a GF (2) can be easily understood in the Fourier space. The transformation of the c_k term corresponds to Kronecker's delta at ω_k. The transformation of the Gaussian g_k is another Gaussian G_k. By the convolution theorem of the Fourier transform, a Gabor filter in the frequency domain is given by a Gaussian centred at ω_k, *i.e.*, $H_k(\omega) = G_k(\omega - \omega_k)$; see Fig. 2. Let

$$\tilde{I}_j^k = h_k * I_j \quad \text{for } j = 1, 2; \quad k = 1, 2, \ldots, K, \tag{4}$$

the result of applying the kth GF to the jth FP; where $*$ denotes convolution. Since h_k is complex, then

$$\tilde{I}_j^k(x) \stackrel{def}{=} m_j^k(x) \exp\left(-i\,\psi_j^k(x)\right) \tag{5}$$

is complex too with magnitude $m_j^k(x)$ and phase $\psi_j^k(x)$. The filter with maximum response for the jth FP at pixel x is

$$k_j^*(x) = \arg\max_k m_j^k(x). \tag{6}$$

Thus, the local magnitude and phase are given by

$$m_j(x) \stackrel{def}{=} m_j^{k_j^*(x)}(x) \tag{7}$$

and

$$\psi_j(x) \stackrel{def}{=} \psi_j^{k_j^*(x)}(x) \tag{8}$$