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# Effective angular and wavelength modeling of parallel aligned liquid crystal devices

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ABSTRACT

Parallel aligned liquid crystal (PA-LC) devices are widely used in many optics and photonics applications to control the amplitude, phase and/or state of polarization (SOP) of light beams. Simplified models yet with a good predictive capability are extremely useful in the optimal application of these devices. In this paper we propose and demonstrate the validity of a novel model enabling to calculate the voltage dependent retardance provided by parallel-aligned liquid crystal (PA-LC) devices for a very wide range of incidence angles and any wavelength in the visible. We derive the theoretical expressions, and both experimental and theoretical retardance results are obtained showing a very good agreement. The proposed model is robust and well adapted to a reverse-engineering approach for the calibration of its parameters, whose values are obtained without ambiguities. The model is based on only three physically related magnitudes: two off-state parameters per wavelength and one global voltage dependent parameter, the tilt angle of the LC molecules. To our knowledge it represents the most simplified model available for PA-LC devices yet showing predictive capability. Not only eases the design of experiments dealing with unconventional polarization states or complex amplitude modulation, but it also serves to analyze the physics and dynamics of PA-LC cells since we have estimation for their voltage dependent tilt angle within the device.



## 1. Introduction

Liquid crystal (LC) devices are commonly used in optics and photonics both in display [1] and in non-display applications [2]. In the latter, parallel-aligned cell geometry is usually the LC technology of choice since it enables the more energetic efficient phase-only operation without amplitude coupling [3,4]. Parallel-aligned liquid crystal (PA-LC) devices can be assimilated to linear variable retarders, and as such are characterized by their linear retardance. Then a number of methods typically used in the characterization of waveplates become available [5]. Recently, we demonstrated a novel method based on time-average Stokes polarimetry [6], able to provide robust and precise measurement of the linear retardance value even in the presence of flicker, exhibited by electrooptic devices such as liquid crystal on silicon (LCoS) displays [7–9].

Accurate calculation of the performance of liquid-crystal cells is possible when the different parameters characterizing the LC material and the LC cell are known [2,10]: such parameters are ordinary and extraordinary refractive indices, cell gap, pre-tilt angle, index of refraction of the glass window, viscosity and elastic coefficients, electrode structure, among others. In a first step, accurate calculation of the actual orientation of the LC director across the LC layer as a function of applied voltage applied is performed by minimizing the total free-energy of the LC cell [2,10]. In a second step, precise numerical methods are used to calculate the electromagnetic propagation of radiation across the cell. Among these propagation methods are the exact Berreman's $4\times4$ matrix approach [11] or the very precise extended Jones matrix calculation methods, proposed by Yeh, Gu or Lien [12–14], appropriate to be applied to oblique incidence in the general case of inhomogenous director axis orientation, such as in twisted-nematic LC cells [15,16]. Full advantage of this rigorous approach is possible for LC designers and manufacturers with access to all details to optimize the electro-optical properties of LC devices.

Most of the time users of LC devices have not access to this detailed information or the precision required in their application does not justify the use of the more complex rigorous approach. More simplified models and/or reverse-engineering approaches, enabling analytical expressions, are then highly desirable, as it was

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the case with transmissive twisted-nematic liquid crystal displays (TN-LCD) [17,18] which were the devices mostly used in spatial light modulation applications [2–4] until the appearance of modern parallel-aligned LCoS (PA-LCoS) panels. These models enabled the calculation of the complex amplitude transmittance at normal incidence, which is the typical working geometry used with transmissive LCDs in non-display applications. In LCoS panels, Lizana et al. analyzed experimentally the wavelength dependence of the phase-shift [19] and also its magnitude under oblique incidence [20], which is a fairly common working geometry in reflective devices such as LCoS devices. Recently, in the case of PA-LCoS panels [21] we showed angular dependence of retardance and flicker amplitude. A theoretical and analytical modeling of this dependence with a simplified approach compatible with a reverse-engineering strategy has not yet been attempted. If successful this provides the benefit of a deeper insight on the physics of the device together with a predictive capability useful to optimize the working conditions of PA-LC devices with angle and wavelength according to the specific application in mind.

In the present work we propose and demonstrate a novel physical model which, through a reverse-engineering approach, is able to provide with a good accuracy the linear retardance value versus applied voltage as a function of both the incidence angle and the illuminating wavelength for PA-LC devices. The model is based on only three physical parameters whose values are obtained without ambiguities by fitting a limited amount of calibration measurements. Experimental and simulated results will be provided, using a PA-LCoS microdisplay as the device under test. An excellent agreement is obtained under a wide range of situations, which is especially remarkable since the performance of such a complex device can be predicted with a highly reduced physical model. For the sake of comparison a rigorous model describing the retardance in a homogenous PA-LC cell with arbitrary director axis orientation [16] is presented and used as a reference. We show that it offers no better agreement with experiment, and furthermore is not able to provide the values for its parameters without ambiguities, which highlights the usefulness of our proposal. To our knowledge our proposal represents the most simplified model available for PA-LC devices yet showing a high predictive capability.

## 2. Theoretical model

A rigorous physical model for the phase retardance introduced by a homogeneous uniaxial anisotropic plate can be obtained by direct application of the Maxwell equations as it is for example developed by Gu and Yeh [10, pp. 326–328] or also by Lien [16]. Next we show the basic expressions resulting from this rigorous approach, whose diagram showing the meaning of the various magnitudes is presented in Fig. 1, where $\alpha$ and $\theta_{inc}$ are respectively the tilt angle for the LC director and the angle of incidence for the light beam, $\varphi_d$ and $\varphi_{inc}$ are the corresponding azimuth angles, and $n_o$ and $n_e$ are respectively the ordinary and the extraordinary refractive index in the LC layer of thickness $d$. First, let us note that double refraction is produced for a beam of light transmitted into a uniaxial medium, thus both an extraordinary and an ordinary wave are produced. The phase retardation of a uniaxial medium of thickness $d$ is then given by [10,16]

$$\Gamma = \left(k_{e,z} - k_{o,z}\right)d \tag{1}$$

where $k_{e,z}$ and $k_{o,z}$ are the $z$ axis components of the wavevectors of the extraordinary and ordinary waves, respectively

$$k_{e,z} = \frac{2\pi}{\lambda}\frac{n_e\ n_o}{\varepsilon_{zz}}\left[\sqrt{\varepsilon_{zz} - \left(1 - \frac{n_e^2 - n_o^2}{n_e^2}\cos^2\alpha \sin^2\left(\varphi_d - \varphi_{inc}\right)\right)\sin^2\theta_{inc}} - \frac{\varepsilon_{xz}}{\varepsilon_{zz}}\sin\ \theta_{inc}\right] \tag{2}$$

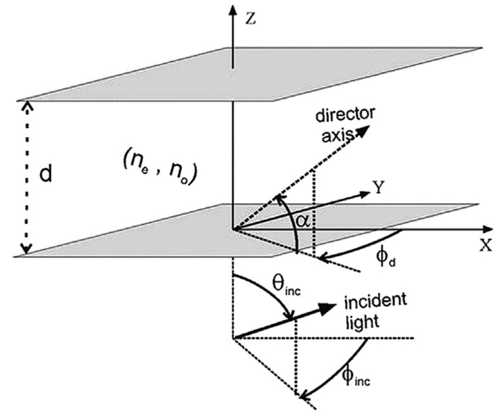


**Fig. 1.** Diagram for arbitrary light impinging on a homogeneous uniaxial medium in accordance with the rigorous approach used as a reference model for the PA-LC cell.

$$k_{o,z} = \frac{2\pi}{\lambda}\sqrt{n_o^2 - \sin^2\theta_{inc}} \tag{3}$$

and $\varepsilon_{xz}$ and $\varepsilon_{zz}$ are given by

$$\varepsilon_{xz} = \left(n_e^2 - n_o^2\right)\sin\ \alpha\ \cos\ \alpha\ \cos\left(\varphi_d - \varphi_{inc}\right) \tag{4}$$

$$\varepsilon_{zz} = n_o^2 + \left(n_e^2 - n_o^2\right)\sin^2\alpha \tag{5}$$

In the paper we restrict our attention to the basic configuration of interest [20,21] in many photonic applications with PA-LC devices where the LC director axis lays along or perpendicular to the incidence plane: typical illumination with a light beam linearly polarized parallel or perpendicular to the incidence plane stays with the SOP unchanged and providing phase-only operation when parallel to the LC director. In the simulations we consider the specific situation where $\varphi_{inc} = \varphi_d = 0°$, i.e. LC director along $XZ$ which is the incidence plane. Expressions (1)–(5) will be used as the reference against both the experimental results and the simulated results provided by the calculations with the simplified model we propose.

Let us now introduce the simplified model we propose in the paper for PA-LC devices. Its general diagram is presented in Fig. 2, where we explicitly consider a reflective cell with a cell gap $d$. We note that incidence plane and LC director are along the $XZ$ plane, which is the situation under analysis already commented with the rigorous reference model. Reflective geometry is given since the experimental results in the paper are obtained with a PA-LCoS microdisplay, which are reflective devices. Transmissive PA-LC devices can be considered as a specific case where only one passage through the LC layer is produced. Notation for the light incidence and LC director tilt angles are the same as in Fig. 1 for the reference model, i.e. angles $\theta_{inc}$ and $\alpha$. LC molecules have their director axis (optical axis) aligned at an angle $\phi$ with respect to the traversing light beam direction. $\theta_{LC}$ is the refraction angle in the LC medium. When a voltage $V$ is applied the director axis tilts an angle $\alpha$ with respect to the entrance face. This is the only voltage dependent magnitude, i.e. $\alpha(V)$. At the backplane the light beam is reflected and a second passage is produced across the LC layer whose effect is equivalent to a forward propagation at an angle $-\theta_{inc}$. We note that angles are taken positive in the counterclockwise sense, thus in the figure $\theta_{inc}$, $\theta_{LC}$ and $\phi$ are positive, whereas $\alpha$ and $\theta_{ref}$ are negative. The same convention is also valid for Fig. 1 previously presented. Some simplifications have been introduced in the diagram: we consider that the LC director orientation is homogenous across the LC cell (even when voltage is applied), no pretilt angle is taken into account, and no double refraction is considered at the interface with the LC layer. Furthermore, no Fresnel coefficients at the interfaces are taken into