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# Full-field digital image correlation with Kriging regression

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ABSTRACT

A full-field Digital Image Correlation (DIC) method with integrated Kriging regression is presented in this article. The displacement field is formulated as a best linear unbiased model that includes the correlations between all the locations in the Region of Interest (RoI). A global error factor is employed to extend conventional Kriging interpolation to quantify displacement errors of the control points. An updating strategy for the self-adaptive control grid is developed on the basis of the Mean Squared Error (MSE) determined from the Kriging model. Kriging DIC is shown to outperform several other full-field DIC methods when using open-access experimental data. Numerical examples are used to demonstrate the robustness of Kriging DIC to different choices of initial control points and to speckle pattern variability. Finally Kriging DIC is tested on an experimental example.



## 1. Introduction

Over the past three decades several different methods have been developed and successfully applied in Digital Image Correlation (DIC). These methods belong to two general classes, that is, local (subset-based) methods and global (full-field) techniques, both of which have been used extensively in different applications. The local approach is perhaps the better established of the two because of its simplicity and suitability to parallel computation [1], but lacks inter-subset continuity and is more sensitive to measurement noise than the global approach. Consequently it is necessary to apply smoothing as a post-processing operation to measured displacements before computing strain results [2]. Alternatively, the global approach imposes certain constraints and treats the Region of Interest (RoI) as a whole, thereby enabling smooth displacement fields to be achieved together with good sub-pixel accuracy. The same level of sub-pixel accuracy is achievable by the global approach, more efficiently than the local approach, which requires subset overlapping [3] with multiple processing of the same data and increased computational cost.

Full-field DIC methods include: Finite Element (FE) based methods [4–9]; the Extended FE method, known as XFEM, [10–13]; B-Spline methods (NURBS) [14,15] and Spectral methods based on spatial Fourier transforms [16–18]. DIC techniques aim to produce an accurate and reliable displacement field through the computed correlation of deformed speckle patterns with a reference image. This process requires the use of shape functions to describe the displacement field in terms of grey-scale values determined from individual pixel intensities within a subset or RoI. Of course, it is generally not possible to design a shape function that perfectly matches the actual displacement field in a particular application. However, the Kriging prediction has the advantage that it is based not only upon regressing certain parameters on discrete measurements, but also on the correlation of neighbouring samples. The fitting residual is represented by a Gaussian random process resulting in a best linear unbiased prediction. This represents lack of knowledge of the true displacement field and is not related to measurement error. The choice of a Gaussian random process is analogous to the choice of a Gaussian random variable in statistics: it is analytically tractable, flexible and frequently correct. Unlike other full-field shape functions that normally require an artificial control grid, the Kriging formula can generate the control grid for a RoI automatically on the basis of its estimated Mean Square Error (MSE). In addition it is possible to adapt the Kriging formula to account for imperfect sample-point data due to measurement noise that would otherwise be reproduced exactly (by conventional Kriging interpolation) because of perfect correlation of the sample point with itself. This adaptation, known as Kriging regression, will be described in detail in Section 3.

In this article Kriging regression is integrated into the classical full-field DIC algorithm. The full-field displacement estimate is obtained by training the Kriging model using increasing numbers of sample (or control) points at each step until the MSE at untried sites (between the control points) is deemed to be acceptably small. At the end of this process the displacements at the untried sites are found in terms of the complete system of control-point displacements. Kriging

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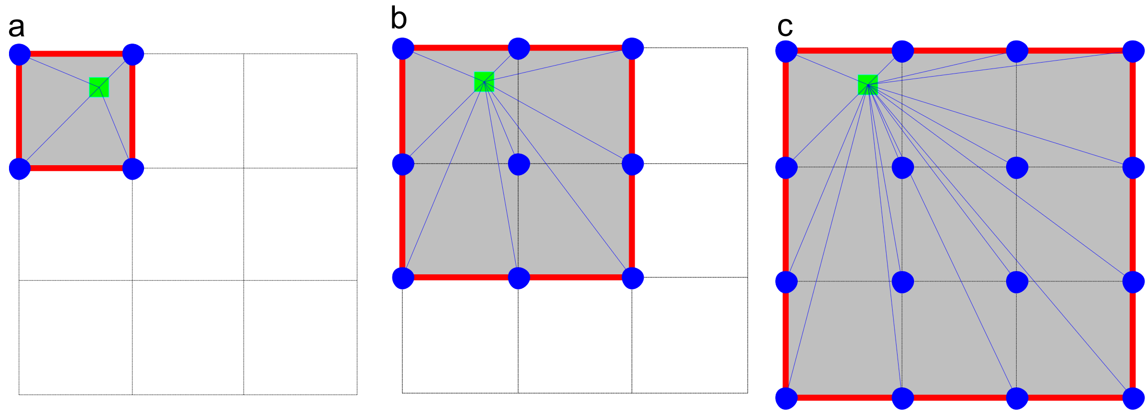


**Fig. 1.** Dependency relationship of one inner point (green square): (a) Q4-FE, (b) Cubic Spline, (c) Kriging – control points shown as blue circles.

regression generally outperforms the classical FE and B-spline methods where untried-site displacements are determined only in terms of several neighbouring control points. Fig. 1(a) illustrates this point, where it is seen that the inner-point displacement is determined by only four nodal displacements when using the Q4-FE shape function [19], possibly resulting in abrupt ridges at the element boundaries. When using the B-spline method, the inner-point displacement, shown in Fig. 1(b), is given in terms of a greater number of nodal displacements, but shape-function remains local to the inner point. The Kriging shape function is genuinely global, as shown in Fig. 1(c) where the inner-point displacement is given in terms of control points distributed over the entire RoI.

In this study, three case studies are used to test the performance of the proposed Kriging DIC method. In the first example, a two-directional rigid body translation is applied and a fixed regular control grid is used. This example provides the basis for a comparison of Kriging DIC with Q4-FE DIC and Cubic Spline DIC. In the second case study using irregular, adaptive control grids and FE-generated displacement fields, based on different numerical generation of speckle patterns, the robustness of Kriging DIC to initially chosen control points and speckle pattern variability is tested. Finally, in the third case study Kriging DIC is applied to a full-scale experimental structure and results are compared to those obtained from a commercial DIC system.

## 2. Review of the full-field DIC method

Full-field DIC is considered for the case of a two-dimensional image where the unknown displacement field $(u(x,y),v(x,y))$ is to be determined at spatial coordinate $(x,y)$. The displacement $(u(x,y),v(x,y))$ may also be understood as the optical flow from a reference image $f(x,y)$ of speckle-pattern intensity to its corresponding deformed image $g(x,y)$. Then the displacement field may be estimated by minimising the objective function,

$$\eta(u,v)=\int_{\Theta}(g(x+u(x,y),y+v(x,y))-f(x,y))^2\,\mathrm{d}\Theta \tag{1}$$

where $\Theta$ denotes the region of interest (RoI) in the reference image.

In practice, the continuous displacement field $(u(x,y),v(x,y))$ may be approximated by a linear combination of basis functions of finite dimension $n$, expressed as

$$\begin{aligned} u(x,y) &\approx \sum_{j=1}^{n}\mu_j(x,y)\rho_{u_j} \\ v(x,y) &\approx \sum_{j=1}^{n}\mu_j(x,y)\rho_{v_j} \end{aligned} \tag{2}$$

where $\mu_j(x,y)$; $j=1,2,\ldots,n$ are the kernel functions and $\rho_{u_j},\rho_{v_j}$; $j=1,2,\ldots,n$ are the combination coefficients. Since $g(x+u(x,y),y+v(x,y))$ is an implicit function of $(u(x,y),v(x,y))$, the Newton method may be applied to solve the minimisation problem. Therefore, an approximate solution of the full-field displacement, $(u(x,y),v(x,y))$, may be obtained by iteration [5,6,15,20]

$$\mathbf{M}_w^i\left(\boldsymbol{\rho}_w^{i+1}-\boldsymbol{\rho}_w^i\right)=\mathbf{b}_w^i;\quad w\in\{u,v\} \tag{3}$$

where $\mathbf{M}_u^i,\mathbf{M}_v^i$ are $n\times n$ matrices and $\mathbf{b}_u^i,\mathbf{b}_v^i$ are $n\times 1$ vectors, with components given by

$$\begin{aligned}(m_{jk})_w^i=&\int_{\Theta}\left(\mu_j(x,y)\frac{\partial g(x+u^i,y+v^i)}{\partial z}\right)\\ &\times\left(\mu_k(x,y)\frac{\partial g(x+u^i,y+v^i)}{\partial z}\right)\mathrm{d}\Theta\end{aligned} \tag{4}$$

and

$$\begin{aligned}(b_j)_w^i=&\int_{\Theta}\mu_j(x,y)\frac{\partial g(x+u^i,y+v^i)}{\partial z}\\ &\times\left(f(x,y)-g(x+u^i,y+v^i)\right)\mathrm{d}\Theta\end{aligned} \tag{5}$$

where $z\in\{x,y\}$ when $w\in\{u,v\}$ respectively and $j,k=1,2,\ldots,n$.

The gradient $\frac{\partial g(x+u^i,y+v^i)}{\partial z}$ in Eqs. (4) and (5) is in principle updated at each iteration. However, as proposed by Sutton [21,22], the grey-level gradients may be calculated from the reference image rather than the deformed image without loss of accuracy, that is, $\frac{\partial g(x+u^i,y+v^i)}{\partial z}=\frac{\partial f(x,y)}{\partial z}$.

The interpolation functions in Eq. (2) are generally local piecewise functions [14,23], for example, cubic spline or finite element shape functions. The combination coefficients then represent the displacements of a set of control points (or nodes). In this article, a different linear modelling approach for the displacement field is investigated, known as Kriging regression.

## 3. Kriging-DIC

The obtained displacement from Eqs. (2) and (3) is an approximate solution on a linear subspace. In this Section, the approximation residual $e_w(x,y)=w(x,y)-\sum_{j=1}^{n}\mu_j(x,y)\rho_{w_j}, w\in\{u,v\}$, in the DIC algorithm will be modelled as a Gaussian random field.

The true displacement field $w(x,y)$ may be modelled as a realisation of a random function $W(x,y)$ which combines a deterministic regression model and a zero-mean stochastic field as [24],

$$W(x,y)=\sum_{\ell=1}^{m}c_{\ell}(x,y)\beta_{\ell}+Z(x,y) \tag{6}$$

where $c_{\ell}(x,y),\ell=1,\ldots,m$, are the regression functions, $\beta_{\ell}$ denotes the $\ell^{th}$ regression parameter and $Z(x,y)$ is a Gaussian stochastic field with zero mean and covariance $\mathrm{cov}(Z(x_j,y_j),\ Z(x_k,y_k))$ between