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Displacement measurement with multi-level spiral phase filtering in speckle interferometry

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ABSTRACT

A multi-level spiral phase filter is proposed in electronic speckle pattern interferometry (ESPI) for out-of-plane displacement measurements. This filter generates a particular kind of speckle pattern that results from the convolution of standard speckles with the filter point spread function (Fourier transform). We shall refer to it as a vortex-filtered speckle pattern (VF-SP). It is shown here that if unresolved and fully developed VF-SPs are generated, then each speckle contains embedded phase terms which can be controlled by the multi-level spiral filter rotation. This mechanism effectively allows the application of standard phase extraction procedures for displacement measurements. Numerical simulations of an interferometer working with VF-SPs were done to verify this technique. Experimental validation was achieved with an out-of-plane electronic speckle pattern interferometer, in which an Liquid Crystal on Silicon (LCoS) was used to generate the multi-level spiral phase filters.

1. Introduction

Speckle interferometry is a technique for displacement measurements that uses rough surfaces illuminated by laser light, assuming that such surfaces are capable of generating fully developed speckle patterns [1]. Several interferometric techniques for displacement measurements have been devised for various sensitivities and applications, including in-plane, out-of-plane and shearing methods [2]. Their arrangements include an iris diaphragm in the pupil plane of the system to control the speckle size. Further developments of these techniques have recently added $4f$ optical correlator systems for various purposes. These include increasing the field of view of speckle shearing interferometers [3,4], and introducing tilts and shears in the pupil plane with a spatial light modulator (SLM), allowing thus the compensation of rigid movements of the object under test [5]. In all of these cases, the imaging and field lenses were placed in front of the $4f$ system. Knowing this, what we propose here is using a $4f$ optical correlator system, but for yet another end. We will show that phase stepping can be attained in out-of-plane speckle interferometry by spatial filtering. For this end, we will use a multi-level vortex spatial filter oriented at 0°, 90°, 180° and 270°. To our knowledge, the use of vortex spatial filters is presently unexplored in speckle interferometry applications involving phase stepping.

Given the random nature of the speckle phase, there are chances that optical vortices occur naturally in speckle interferometry, and its density has been found to depend on the roughness of the samples [6]. Tracking of the vortex positions has allowed applications for nanometric displacement measurement [7] and dynamic speckle from biological samples [8]. Recently, continuous phase vortices have been used as spatial filters for phase contrast microscopy [9,10]. Microscopic specimens are typically smooth, so that they can be regarded as weak diffusers, producing undeveloped speckle patterns, that is, speckle patterns where the random phase excursion of the field is $\ll 2\pi$. When a vortex-filtered, undeveloped speckle pattern is combined with a smooth reference wave, it yields an interference pattern with spiral fringes, and phase extraction becomes an involved task, specific for this kind of fringe patterns [10]. Fortunately, in ESPI we deal with objects which are optically rough (random phase excursions of the field $\geq 2\pi$), producing fully developed speckle patterns. This is also valid for VF-SPs, which means that we can apply to them the usual phase extraction algorithms of ESPI, as we shall prove in Section 2.

Continuous phase vortices, like those used in microscopy, are difficult to generate and unsuitable for high speed speckle correlation interferometry. Therefore, a multi-level spiral phase vortex is proposed here to solve these problems, and also to reduce the complexity of the phase extraction process. Our aim was to process fully developed speckle patterns of an out-of-plane speckle interferometer which includes a multi-level spiral phase filter, and finding ways of extracting the juxtaposed phase in the VF-SP. Since the technique requires only one phase vortex in the Fourier domain, we assumed that a modification of this vortex would

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simultaneously affect the phase of each speckle in the spatial domain. In particular, it is shown that phase stepping in the whole spatial domain can be easily implemented for each random phase of unresolved speckles by the simple expedient of rotating the proposed multi-level spiral filter included in the ESPI system. Furthermore, it is also shown that the correlation of VF-SPs makes unnecessary deconvolution methods, such as that used in microscopy, for the purpose of phase extraction. Since vortex rotation can be controlled with high accuracy, the required phase stepping is achieved with comparable precision. Therefore, piezo-electric transducer (PZT) problems, such as wobbling and hysteresis, are avoided with this technique [11].

2. Multi-level phase filtering and phase stepping in ESPI

2.1. Multi-level phase filtering

In polar coordinates (ρ, θ), the complex amplitude of a phase vortex of order one, with its angular discontinuity at θ_n, is $\exp[i(\theta - \theta_n)]$, where $0 \le \theta - \theta_n < 2\pi$. When this continuous spiral phase object is used as a spatial filter, like in the case of microscopy [10], the spatial filtering operation can be represented as

$$U_i^{(n)}(x_i, y_i) = \mathscr{F}^{-1}\{\mathbb{H}^{(n)}(u,v)\mathscr{F}\{P(x_o, y_o)U_o(x_o, y_o)\}\} \tag{1}$$

where $\mathscr{F}$ and $\mathscr{F}^{-1}$ are, respectively, the forward and inverse Fourier transform operators,

$$U_o(x_o, y_o) = A(x_o, y_o)\exp[i\varphi(x_o, y_o)] \tag{2}$$

is the object complex amplitude distribution, $P(x_o, y_o)$ is the generalized pupil function, and

$$\mathbb{H}^{(n)}(u,v) = \mathrm{circ}(\rho/\rho_0)\exp[i\psi_n(u,v)], \tag{3}$$

where

$$\mathbb{H}^{(n)}(u,v) = \mathbb{H}^{(n)}\left(\frac{x_f}{\lambda f}, \frac{y_f}{\lambda f}\right) \tag{4}$$

is the transfer function of the filter, (x_f, y_f) are the Cartesian coordinates at the spatial frequency plane, f is the focal length of the Fourier transforming lens, and λ is the illumination wavelength. The amplitude $A(x_o, y_o)$ and the phase $\varphi(x_o, y_o)$ in Eq. (2) are random functions if rough surfaces are used to generate $U_o(x_o, y_o)$. In Eq. (3) $\rho = \sqrt{u^2 + v^2}$ and $\rho_0 = l_{y_f}/(\lambda f)$ are the radius of the filter in spatial frequency units, where $2\ l_{y_f}$ is the smaller side of the SLM—typically, its height. This radius, however, can be reduced with an iris diaphragm if we want to increase the speckle size in the output intensity distribution [12]. Finally, in the same equation $\psi_n(u,v)$ is the vortex phase as function of the spatial frequencies $u = \rho\cos\theta$ and $v = \rho\sin\theta$ [13]. As usual, $\mathrm{circ}(\rho/\rho_0) = 1$ if $\rho \le \rho_0$, and zero otherwise. In this description we assume that the object is placed at the front focal plane of the Fourier transforming lens, and that vignetting is neglected.

The filtering operation can be easily implemented in a 4*f* optical system, in which the phase vortex is inserted, and its radius adjusted, in the frequency plane (x_f, y_f), as shown in Fig. 1. In this system, the Fourier transform of the complex amplitude distribution $P(x_0, y_0)\ U_o(x_o, y_o)$, generated by coherent light scattered from a rough surface at the plane (x_o, y_o), and bounded by the pupil function $P(x_0, y_0)$, is obtained at the back focal plane (x_f, y_f) of the lens L_1. The vortex filter, shown with its angular discontinuity $0-2\pi$ oriented in the positive direction of the u-axis, adds its phase to the phase of the spectrum $\mathbb{G}_f(u,v) = \mathscr{F}\{P(x_o, y_o)U_o(x_o, y_o)\}$, and the resulting complex amplitude is the Fourier transformed by the lens L_2 to generate the VF-SP at the plane (x_i, y_i).

In practice, small angular phase increments, like those required to produce acceptable vortices, are difficult to make with digital devices. They have been made with deformable mirrors [14], and also with digital SLMs, mainly LCoSs [15], and more recently with digital micro-mirror devices (DMD) [16].

To understand the effects to which the phase transformations at the frequency plane (u, v) give rise, we shall consider a discrete phase vortex made up of four steps, one for each quadrant of the Cartesian coordinate system. For a vortex of order one, it implies a phase increment of $\pi/2$ between two contiguous steps, except at the original phase discontinuity, where the step size becomes $3\pi/2$. A similar approach to generate a phase vortex, but with polarizing elements, was reported in [17].

The phase configurations of the four discrete vortices that we need, one for each orientation of the major vortex discontinuity, can be represented as

$$\begin{aligned}\boldsymbol{\psi}(u,v) &= \left[\psi_1(u,v),\quad \psi_2(u,v),\quad \psi_3(u,v),\quad \psi_4(u,v)\right]^{\mathrm{T}} \\ &= (\pi/2)\mathbf{R}\mathbf{q}(u,v),\end{aligned} \tag{5}$$

where the symbol T denotes transposition,

$$\mathbf{q}(u,v) = [H(u)H(v),\quad H(-u)H(v),\quad H(-u)H(-v),\quad H(u)H(-v)]^{\mathrm{T}}, \tag{6}$$

and

$$\begin{aligned}\mathbf{R} &= [r_{i,j}] \\ &= [\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \mathbf{r}_4]^{\mathrm{T}} \\ &= \begin{pmatrix} 0 & 1 & 2 & 3 \\ 3 & 0 & 1 & 2 \\ 2 & 3 & 0 & 1 \\ 1 & 2 & 3 & 0 \end{pmatrix}\end{aligned} \tag{7}$$

we shall refer to $\mathbf{q}(u,v)$ as the quadrant vector (a vector with functions as components), and to $\mathbf{R}$ as the vortex rotation matrix.

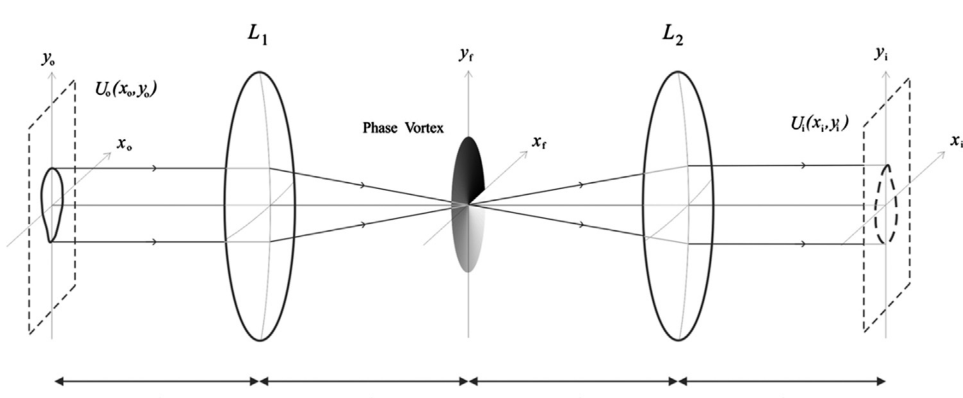

Fig. 1. Schematic diagram of a 4*f* optical correlator with a phase vortex filter represented in gray levels.