Optics and Lasers in Engineering 50 (2012) 1683–1688

Contents lists available at SciVerse ScienceDirect

Optics and Lasers in Engineering

journal homepage: www.elsevier.com/locate/optlaseng

Propagation of radially polarized beams diffracted at a circular aperture in turbulent atmosphere

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ARTICLE INFO

Article history:
Received 11 February 2012
Received in revised form
26 July 2012
Accepted 26 July 2012
Available online 9 August 2012

Keywords:
Atmospheric propagation
Apertures
Radially polarized beams

ABSTRACT

Based on the extended Huygens–Fresnel integral and the beam coherence–polarization matrix, the analytical formulae for the average intensity and the degree of polarization of the radially polarized beams diffracted at a circular aperture in turbulent atmosphere are derived, which provide a convenient approach to study the propagation and polarization properties of the apertured radially polarized beams in turbulent atmosphere. The unapertured and free-space cases can be viewed as the special cases of our general result. The analyses indicate that the average intensity and the degree of polarization are closely related to the propagation distance, the structure constant of the atmospheric turbulence, and the truncation parameter. The existence of the circular aperture weakens the influence of the atmospheric turbulence on the evolution properties of the radially polarized beams.

1. Introduction

As one particular case of the nonuniformly polarized beams introduced by Gori [1], radially polarized beams have attracted increasing interest recently. The cylindrical symmetry of the electric field vectors of radially polarized beams make them exhibit unique advantages in electron acceleration, particle trapping, optical data storage, laser cutting, material processing, and tip-enhanced Raman spectroscopy [2–7]. Various methods for generating radially polarized beams have been reported [8–13].

It is well known that the behavior of various laser beams in turbulent atmosphere has been extensively studied due to their wide applications in free-space optical communication and remote sensing [14–16]. The propagation properties of radially polarized coherent and partially coherent beams in turbulent atmosphere have been discussed in detail [17–19]. The propagation characteristics of radially polarized partially coherent beams through an optical system in turbulent atmosphere have also been analyzed [20]. However, the results have been restricted to the unapertured case. In the practical optical systems, the aperture effect usually exists. Therefore, the properties of the apertured radially polarized beams in turbulent atmosphere deserve investigation. In this paper, the propagation and polarization properties of radially polarized beams diffracted at a circular aperture in turbulent atmosphere are demonstrated. Based on the extended Huygens–Fresnel integral and the beam coherence–polarization (BCP) matrix, the analytical formulae for the average intensity and the degree of polarization are derived and illustrated by numerical examples.

2. Propagation of radially polarized beams diffracted at a circular aperture in turbulent atmosphere

In the Cartesian coordinate system, assuming that a circular aperture with radius a is located at the source plane $z=0$, the electric field of a completely coherent radially polarized beam just behind the aperture reads as follows [11,21]:

$$\mathbf{E}(x_0,y_0,0)=E_x(x_0,y_0,0)\mathbf{e}_x+E_y(x_0,y_0,0)\mathbf{e}_y \tag{1}$$

$$E_x(x_0,y_0,0)=\sqrt{2}E_0\frac{x_0}{\omega_0}\exp\left(-\frac{x_0^2+y_0^2}{\omega_0^2}\right)t(x_0,y_0) \tag{2}$$

$$E_y(x_0,y_0,0)=\sqrt{2}E_0\frac{y_0}{\omega_0}\exp\left(-\frac{x_0^2+y_0^2}{\omega_0^2}\right)t(x_0,y_0) \tag{3}$$

$$t(x_0,y_0)=\begin{cases}1 & x_0^2+y_0^2\le a^2\\ 0 & otherwise\end{cases} \tag{4}$$

where $\mathbf{e}_x$ and $\mathbf{e}_y$ are the unit vectors in the x and y directions, respectively. ω_0 is the waist width of Gaussian beam, E_0 is the amplitude constant, and $t(x_0,y_0)$ denotes the window function of the aperture. Within the framework of the paraxial approximation, the longitudinal component of the electric field compared with the transverse component can be negligible [22]. Thus, we do not consider the longitudinal component of the electric field here.

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http://dx.doi.org/10.1016/j.optlaseng.2012.07.021

The BCP matrix for the electric field of the radially polarized beams diffracted at a circular aperture has the form

$$J(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,z)=\begin{bmatrix} J_{xx}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,z) & J_{xy}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,z) \\ J_{yx}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,z) & J_{yy}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,z) \end{bmatrix} \tag{5}$$

where

$$J_{\alpha\beta}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,z)=\langle E^*_\alpha(\boldsymbol{\rho}_1,z)E_\beta(\boldsymbol{\rho}_2,z)\rangle \tag{6}$$

Here $\alpha,\beta=x,y$, $\boldsymbol{\rho}_1=(x_1,y_1)$ and $\boldsymbol{\rho}_2=(x_2,y_2)$ are the position vectors at the receiver plane. The asterisk stands for the complex conjugate and the angle brackets represent an ensemble average over the medium statistics. By using Eqs. (1)–(6), the BCP matrix of the completely coherent radially polarized beams through a circular aperture at the source plane $z=0$ is expressed as

$$J(\boldsymbol{\rho}_{01},\boldsymbol{\rho}_{02},0)=\frac{2E_0^2}{\omega_0^2}\exp\left(-\frac{\rho_{01}^2+\rho_{02}^2}{\omega_0^2}\right)t^*(x_{01},y_{01})t(x_{02},y_{02})\times\begin{bmatrix} x_{01}x_{02} & x_{01}y_{02} \\ y_{01}x_{02} & y_{01}y_{02}\end{bmatrix} \tag{7}$$

where $\boldsymbol{\rho}_{01}=(x_{01},y_{01})$ and $\boldsymbol{\rho}_{02}=(x_{02},y_{02})$ are the position vectors at the source plane.

On the basis of the extended Huygens–Fresnel integral [17,23], the elements of the BCP matrix of the apertured radially polarized beams propagating in turbulent atmosphere are given by

$$J_{\alpha\beta}(\boldsymbol{\rho},\boldsymbol{\rho},z)=\left(\frac{k}{2\pi z}\right)^2\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}J_{\alpha\beta}(\boldsymbol{\rho}_{01},\boldsymbol{\rho}_{02},0)\times\exp\left[-\frac{ik}{2z}(\boldsymbol{\rho}_{01}-\boldsymbol{\rho})^2+\frac{ik}{2z}(\boldsymbol{\rho}_{02}-\boldsymbol{\rho})^2\right]\times\exp\left[-\frac{(\boldsymbol{\rho}_{01}-\boldsymbol{\rho}_{02})^2}{\rho_0^2}\right]dx_{01}dy_{01}dx_{02}dy_{02} \tag{8}$$

where k is the wavenumber, $\rho_0=(0.545\,C_n^2k^2z)^{-3/5}$ is the coherence length of a spherical wave propagating in turbulent atmosphere, and C_n^2 is the structure constant of the refractive index and describes the turbulence level.

In order to obtain the analytical expressions of the elements of the BCP matrix, the aperture function is expanded as the sum of complex Gaussian functions with finite terms [24]:

$$t(x_0,y_0)=\sum_{n=1}^{N}A_n\exp\left[-\frac{B_n}{a^2}(x_0^2+y_0^2)\right] \tag{9}$$

where the complex constants A_n and B_n are the expansion and Gaussian coefficients, respectively, which can be obtained by optimization computation. Substituting Eq. (9) into Eq. (7) yields

$$J(\boldsymbol{\rho}_{01},\boldsymbol{\rho}_{02},0)=\frac{2E_0^2}{\omega_0^2}\sum_{s=1}^{N}\sum_{j=1}^{N}A_s^*A_j\exp\left(-\frac{B_s^*\rho_{01}^2+B_j\rho_{02}^2}{a^2}\right)\times\exp\left(-\frac{\rho_{01}^2+\rho_{02}^2}{\omega_0^2}\right)\begin{bmatrix} x_{01}x_{02} & x_{01}y_{02} \\ y_{01}x_{02} & y_{01}y_{02}\end{bmatrix} \tag{10}$$

By using Eq. (10) and the integral formulae [25]

$$\int_{-\infty}^{\infty}x\exp(-px^2+2qx)dx=\frac{q}{p}\sqrt{\frac{\pi}{p}}\exp\left(\frac{q^2}{p}\right) \tag{11}$$

$$\int_{-\infty}^{\infty}x^2\exp(-\mu x^2+2\upsilon x)dx=\frac{1}{2\mu}\sqrt{\frac{\pi}{\mu}}\left(1+\frac{2\upsilon^2}{\mu}\right)\exp\left(\frac{\upsilon^2}{\mu}\right) \tag{12}$$

$$\int_{-\infty}^{\infty}\exp(-p^2x^2\pm qx)dx=\frac{\sqrt{\pi}}{p}\exp\left(\frac{q^2}{4p^2}\right) \tag{13}$$

Eq. (8) reduces to

$$J_{xx}(\boldsymbol{\rho},\boldsymbol{\rho},z)=\left(\frac{kE_0}{2z\omega_0}\right)^2\sum_{s=1}^{N}\sum_{j=1}^{N}A_s^*A_j\left(\frac{1}{p_1p_2}\right)^2\times\left\{\frac{1}{\rho_0^2}-\frac{1}{2}\left(\frac{kx}{z}\right)^2\left(\frac{1}{\rho_0^2p_1}-1\right)\left[1+\frac{1}{\rho_0^2p_2}\left(\frac{1}{\rho_0^2p_1}-1\right)\right]\right\}\times\exp\left\{-\left(\frac{k\rho}{2z}\right)^2\left[\frac{1}{p_1}+\frac{1}{p_2}\left(\frac{1}{\rho_0^2p_1}-1\right)^2\right]\right\} \tag{14}$$

$$J_{yy}(\boldsymbol{\rho},\boldsymbol{\rho},z)=\left(\frac{kE_0}{2z\omega_0}\right)^2\sum_{s=1}^{N}\sum_{j=1}^{N}A_s^*A_j\left(\frac{1}{p_1p_2}\right)^2\times\left\{\frac{1}{\rho_0^2}-\frac{1}{2}\left(\frac{ky}{z}\right)^2\left(\frac{1}{\rho_0^2p_1}-1\right)\left[1+\frac{1}{\rho_0^2p_2}\left(\frac{1}{\rho_0^2p_1}-1\right)\right]\right\}\times\exp\left\{-\left(\frac{k\rho}{2z}\right)^2\left[\frac{1}{p_1}+\frac{1}{p_2}\left(\frac{1}{\rho_0^2p_1}-1\right)^2\right]\right\} \tag{15}$$

$$J_{xy}(\boldsymbol{\rho},\boldsymbol{\rho},z)=J_{yx}(\boldsymbol{\rho},\boldsymbol{\rho},z)=-\frac{xy}{8}\left(\frac{k^2E_0}{z^2\omega_0}\right)^2\sum_{s=1}^{N}\sum_{j=1}^{N}A_s^*A_j\left(\frac{1}{p_1p_2}\right)^2\times\left(\frac{1}{\rho_0^2p_1}-1\right)\left[1+\frac{1}{\rho_0^2p_2}\left(\frac{1}{\rho_0^2p_1}-1\right)\right]\times\exp\left\{-\left(\frac{k\rho}{2z}\right)^2\left[\frac{1}{p_1}+\frac{1}{p_2}\left(\frac{1}{\rho_0^2p_1}-1\right)^2\right]\right\} \tag{16}$$

where

$$p_1=\frac{1}{\omega_0^2}+\frac{B_s^*}{a^2}+\frac{ik}{2z}+\frac{1}{\rho_0^2} \tag{17}$$

$$p_2=\frac{1}{\omega_0^2}+\frac{B_j}{a^2}-\frac{ik}{2z}+\frac{1}{\rho_0^2}-\frac{1}{p_1\rho_0^4} \tag{18}$$

From Eqs. (14)–(18), we can get the average intensity and the degree of polarization of the apertured radially polarized beams propagating in turbulent atmosphere,

$$I(\boldsymbol{\rho},z)=TrJ(\boldsymbol{\rho},\boldsymbol{\rho},z)=J_{xx}(\boldsymbol{\rho},\boldsymbol{\rho},z)+J_{yy}(\boldsymbol{\rho},\boldsymbol{\rho},z) \tag{19}$$

$$P(\boldsymbol{\rho},z)=\sqrt{1-\frac{4Det\,J(\boldsymbol{\rho},\boldsymbol{\rho},z)}{[Tr\,J(\boldsymbol{\rho},\boldsymbol{\rho},z)]^2}} \tag{20}$$

where *Det* and *Tr* denote the determinant and the trace of the BCP matrix, respectively. To the best of our knowledge, previous study dealt with the effect of the aperture on the cosh-Gaussian beams in turbulent atmosphere [26], but it has not been extended to inhomogeneous polarization. Eqs. (14)–(18) are the basic results obtained in this paper, which depict the propagation of the radially polarized beams diffracted at a circular aperture in turbulent atmosphere. According to Eqs. (14)–(20), one finds that the axial symmetry is preserved for the average intensity and the degree of polarization of the apertured radially polarized beams during propagation.

For the unapertured case $a\to\infty$, Eqs. (14)–(16) simplify to

$$J^0_{xx}(\boldsymbol{\rho},\boldsymbol{\rho},z)=\left(\frac{kE_0}{2z\omega_0p_{01}p_{02}}\right)^2\times\left\{\frac{1}{\rho_0^2}-\frac{1}{2}\left(\frac{kx}{z}\right)^2\left(\frac{1}{\rho_0^2p_{01}}-1\right)\left[1+\frac{1}{\rho_0^2p_{02}}\left(\frac{1}{\rho_0^2p_{01}}-1\right)\right]\right\}\times\exp\left\{-\left(\frac{k\rho}{2z}\right)^2\left[\frac{1}{p_{01}}+\frac{1}{p_{02}}\left(\frac{1}{\rho_0^2p_{01}}-1\right)^2\right]\right\} \tag{21}$$

$$J^0_{yy}(\boldsymbol{\rho},\boldsymbol{\rho},z)=\left(\frac{kE_0}{2z\omega_0p_{01}p_{02}}\right)^2\times\left\{\frac{1}{\rho_0^2}-\frac{1}{2}\left(\frac{ky}{z}\right)^2\left(\frac{1}{\rho_0^2p_{01}}-1\right)\left[1+\frac{1}{\rho_0^2p_{02}}\left(\frac{1}{\rho_0^2p_{01}}-1\right)\right]\right\}$$