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High-angular-sensitivity total-internal-reflection heterodyne interferometry for small displacement measurements

Jiun-You Lin [a,*], Xuan-Wei Lee [a], Meng-Chang Hsieh [b], Chia-Ou Chang [c]

[a] Department of Mechatronics Engineering, National Changhua University of Education, Changhua, 500, Taiwan, ROC
[b] Institute of Applied Mechanics, National Taiwan University, Taipei, 10617, Taiwan, ROC
[c] College of Mechanical Engineering, Guangxi University, Nanning, 530004, China

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ABSTRACT

This paper proposes an optical method for measuring small displacements using high-angular-sensitivity total-internal-reflection (TIR) heterodyne interferometry. In the designed system, a half-wave plate and a quarter-wave plate that display specific optic-axis azimuths are combined to form a phase shifter. When an isosceles right-angle prism is placed between the phase shifter and an analyzer that displays suitable transmission-axis azimuth, it shifts and enhances the phase difference of the *s*- and *p*- polarization states at one TIR. The enhanced phase difference is a function of the prism incident angle, whose variation depends on the displacement of a target; therefore the displacement can be easily and precisely measured by estimating the phase-difference variation. The feasibility of our method was demonstrated with measurement resolution and sensitivity levels superior to 0.025 μm and 4.0°/μm, respectively in a measurement range of 10 μm.

1. Introduction

The accurate measurement of small displacements plays a crucial role in many research and industrial fields, such as the semiconductor industry, precise manufacture, and metrological instruments. Nondestructive, non-contact, and real-time methods are strongly required for displacement measurements. Several optical methods have been proposed for this purpose, including the oblique ray [1,2], optical fibre reflection intensity [3,4], and optical interferomtric [5–11] methods. Among these methods, optical interferometry is predominantly adopted, since it can transform small displacement information into accurately measurable phase shifts of interference signals, resulting in high sensitivity and resolution. Based on the relation between the displacement and phase shift, Michelson-type heterodyne [5,6], speckle pattern [7,8] and reflection-type [9–11] interferometric techniques were developed. Regarding the Michelson-type heterodyne interferometry, the small displacement of an object is inferred from the phase difference between the test and reference interference beams. Regarding the speckle pattern interferometry, the desired displacement is obtained with the phase shift of the interference fringe pattern formed by the superposition of the diffuse object and the reference beams. Both the two approaches can yield sensitive and real-time measurements, and excellent measurement resolution, but they are also highly sensitive to environmental disturbances and the stability of the light source due to their non-common-path configuration. The reflection-type measurement techniques mainly include surface plasmon resonance (SPR) [9,10] and multiple total-internal-reflection (MTIR) methods [11]. The two methods use a lens or a mirror system to convert displacement information of a target into a variation of the incident angle of the test light on a SPR or a MTIR apparatus; the angular variation produces a significant phase variation of the interference test signal, thus attaining accurate measurement results. The SPR and MTIR approaches both are resistant to environmental disturbances and light source instability, because of their common-path configuration. However, for the SPR method, if the sensitivity and resolution are desired to be further enhanced, another coating process is required to alter the thickness or the material of the metal film on the SPR apparatus, increasing the inconvenience in measurement; for the MTIR method, an elongated parallelogram prism is needed to attain excellent sensitivity and resolution, but it also makes the system bulky.

This study develops an alternative method for measuring small displacements based on high-angular-sensitivity total-internal-reflection (TIR) heterodyne interferometry and the angular amplification principle proposed by Chiu et al. [10]. A heterodyne beam reflected from a plane mirror propagates through a phase

* Corresponding author.
E-mail address: jylin@cc.ncue.edu.tw (J.-Y. Lin).

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shifter, comprising a half-wave and a quarter-wave plate, subsequently entering into an isosceles right-angle prism at an angle larger than the critical angle. The light in the prism undergoes one TIR, finally passing through an analyzer to extract the interference signal of the p- and s-polarized states. When the azimuth angles of the phase shifter wave plates and the analyzer are properly chosen, the final phase difference exhibited by the prism incident angle of the interference signal is significantly improved, thereby leading to a high angular sensitivity. As the mirror generates a small displacement, the reflected beam from the mirror is deflected at a slightly different angle. The deflection of the reflected beam produces a small variation in the prism incident angle, inducing an obvious phase-difference variation of the interference signal. The phase-difference variation can be precisely measured with heterodyne interferometry. The desired displacement is therefore attainable by substituting the measured results into the derived equations. The experimental results of the phase differences and displacements demonstrated the feasibility of this technique, which yielded the measurement sensitivity and resolution levels greater than 4.0°/μm and 0.025 μm in the measurement range of 10 μm. This method has the merits of both common-path interferometry and heterodyne interferometry.

2. Principle

2.1. Phase difference resulting from high-angular-sensitivity TIR apparatus

Fig. 1(a) shows the optical configuration of the high-angular-sensitivity TIR apparatus. For convenience, the + z axis is set in the direction of the propagation of light and the x axis is perpendicular to the plane of the paper. A linearly polarized light whose light polarization plane is properly set at an angle θ_p from the x axis exhibits the following Jones vector:

$$E_i = \begin{pmatrix} \cos\ \theta_p \\ \sin\ \theta_p \end{pmatrix} \tag{1}$$

The light is transmitted through a phase shifter, composed of a half-wave plate H_1 (fast axis at a $\gamma/2$ angle to the x-axis) and a quarter-wave plate Q_1 (fast axis at 45° with respect to x-axis), and is subsequently incident at θ_i on one side of an isosceles right-angle prism. The light beam enters the prism at an incidence angle of θ_1 onto the prism/air interface, and is completely reflected inside the prism. The light output from the prism passes through an analyzer AN_t (with the transmission axis being β to the x-axis) for interference. The final electric field E_t can be expressed as follows:

$$\begin{aligned} E_t &= \begin{pmatrix} \cos^2\beta & \sin\beta\cos\beta \\ \sin\beta\cos\beta & \sin^2\beta \end{pmatrix} \begin{pmatrix} t_p t'_p \exp(-i\delta/2) & 0 \\ 0 & t_s t'_s \exp(i\delta/2) \end{pmatrix} \\ &\frac{1}{2}\begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix} \times \begin{pmatrix} \cos\gamma & \sin\gamma \\ \sin\gamma & -\cos\gamma \end{pmatrix} \begin{pmatrix} \cos\theta_p \\ \sin\theta_p \end{pmatrix} \\ &= \left(A_t^+ \cos\theta_p \exp(i\varphi) + A_t^- \sin\theta_p \exp(i\pi/2)\right) \begin{pmatrix} \cos\beta \\ \sin\beta \end{pmatrix} \end{aligned} \tag{2}$$

where the amplitudes A_t^+ and A_t^- can be written as follows:

$$A_t^{\pm} = \left\{ \frac{1}{2}[(t_p t'_p \cos\beta)^2 + (t_s t'_s \sin\beta)^2 \pm t_p t'_p t_s t'_s \sin 2\beta \cdot \sin(2\gamma+\delta)] \right\}^{1/2} \tag{3}$$

and the phase difference φ can be expressed as follows:

$$\begin{aligned} \varphi &= \tan^{-1}[-\tan(45^\circ - \sigma)\cdot\tan(\gamma + \delta/2 - \pi/4)] \\ &+\tan^{-1}[\tan(45^\circ + \sigma)\cdot\tan(\gamma + \delta/2 - \pi/4)] \end{aligned} \tag{4}$$

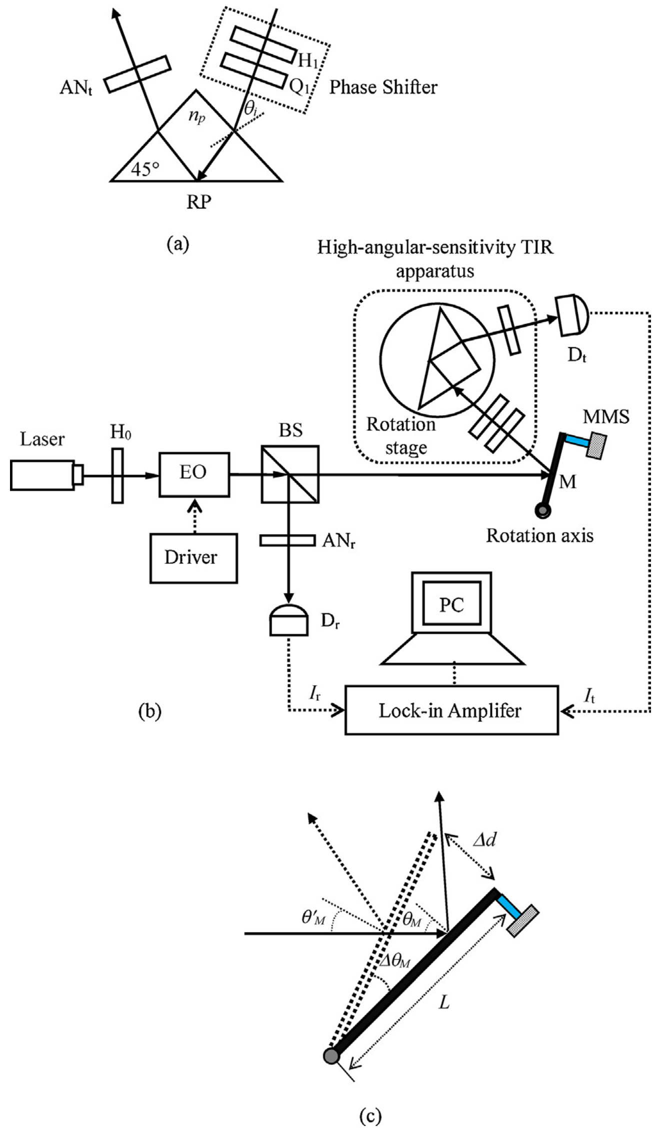

Fig. 1. (a) The high-angular-sensitivity TIR apparatus, (b) the schematic diagram of the small displacement measurements, and (c) Schematic representation of the mirror displacement and the angular variation.

In Eqs. (3) and (4), δ is the phase difference between the s- and p-polarizations of one TIR at the base of the prism; (t_p, t_s) and (t'_p, t'_s) are the transmission coefficients at the air-prism and prism-air interfaces, respectively; γ and σ are the parameters introduced using the phase shifter and the analyzer AN_t, respectively. δ and σ can be derived based on Fresnel's equations and Jones matrix calculation [12]:

$$\delta = 2\tan^{-1}\left\{ \frac{\left[\sin^2\left[45^\circ + \sin^{-1}\left(\sin\theta_i/n_p\right)\right] - (1/n_p)^2\right]^{1/2}}{\tan\left[45^\circ + \sin^{-1}(\sin\theta_i/n_p)\right]\cdot\sin\left[45^\circ + \sin^{-1}(\sin\theta_i/n_p)\right]} \right\}, \tag{5}$$

$$\sigma = \tan^{-1}\left(\frac{t_s t'_s}{t_{p'} t'_p}\tan\beta\right) \tag{6}$$

and the transmission coefficients are given as follows:

$$t_p = \frac{2\cos\theta_i}{n_p\cos\theta_i + \left[1 - (\sin\theta_i/n_p)^2\right]^{1/2}} \tag{7}$$

$$t_s = \frac{2\cos\theta_i}{\cos\theta_i + n_p\left[1 - (\sin\theta_i/n_p)^2\right]^{1/2}} \tag{8}$$