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Nonlinear cc-beam microresonator model for system level electrical simulations: Application to bistable behavior analysis

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ABSTRACT

Microelectromechanical resonators nonlinearities can be exploited in many ways to obtain a set of diverse new applications. In particular, some applications of bistable behavior includes threshold mechanical switches, memory cells, energy harvesting and chaotic signal generators. A key step for practical and efficient design for bistability behavior involves accounting for accurate and efficient models. In this paper we present a nonlinear electromechanical model for capacitive clamped-clamped beam resonators implemented in an analog hardware description language (AHDL) enabling system level electrical simulations. The model accounts for nonlinearities from variable resonator-electrode gap, residual fabrication stress, fringing field contributions as well as an accurate resonator deflection profile in contrast to parallel plate approximations. The model has been applied to derive for first time accurate analytical expressions for bistability design conditions. The work includes FEM analysis and experimental data that corroborates the correctness of the model in describing the required bias voltage conditions for bistability.

1. Introduction and motivation

The nonlinear behavior of electrostatically actuated microelectromechanical (MEMS) resonators has been extensively reported [1–4]. Although MEMS resonators have been traditionally operated in the linear regime, some novel applications take advantage of such intrinsic nonlinearities to propose new type of devices or to enhance their features like in nonlinear mass sensing, mechanical memories, energy harvesting and chaotic signal generators [5–8].

Chaotic behavior has been observed to arise from the inherent mechanical and electrostatic nonlinearities in MEMS/NEMS resonators [6]. The use of simple resonant structures, like a doubly clamped beam structure (cc-beam), provides additional benefits in terms of scalability down to nanometer dimensions and monolithic integration making them potential candidates as chaotic signal generators in system-on-chip applications [7,8]. In a previous work, we reported on the cc-beam operating conditions as a bistable (cross-well motion) to obtain rich and sustained chaotic behavior. Bistability (two-well potential distribution) can be seen as the first step to attain chaotic behavior in MEMS resonators having some other applications like threshold switches, memory cells, relays, valves, etc [9]. Bistable behavior requires relative large beam displacements making the resonator to electrode parallel plate approximation ([7,8]) inaccurate to determine its final design and the required biasing voltages among other parameters, especially in narrow beams. In addition, fringing field effects and residual stress may modify significantly the resonator-electrode capacitance and its resonant frequency respectively [10].

An accurate model considering cc-beam microresonator nonlinearities with a low computational cost suited for system level simulations is demanded to embed such devices into the IC design flow. In this work a nonlinear compact model with accurate near-real resonator deflection profile based on finite difference method is derived. The model also includes the mechanical stiffness cubic nonlinearity term, the intrinsic electrostatic nonlinearities, the fringing field contributions and the residual fabrication stress [11,12]. Only time-derivative equations are used to allow the model to be included within typical electrical simulators. The model has been implemented in Verilog-A for electromechanical simulations using Spectre within the CADENCE IC design framework. The implemented Verilog-A nonlinear model can be used to determine parameters like DC sweep and operating point, transient small signal (AC) and large signal (PSS), among others, as well as performing simulations of electrical and non-electrical variables like resonator position or velocity.

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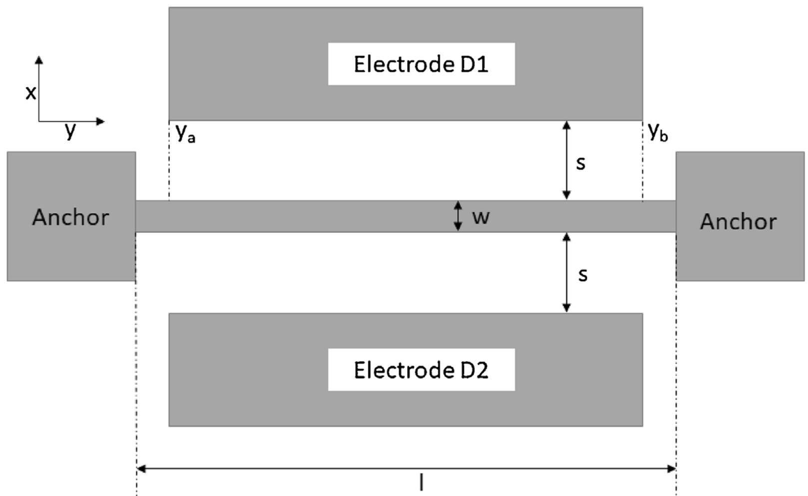

Fig. 1. Schematics of a cc-beam showing the geometrical parameters: length (l), width (w) and beam-driver gap distance (s). The thickness (t_h) dimension is normal to the plane x-y. Material properties are determined by Young modulus (E) and mass density (ρ).

The model has been applied to predict the design parameters and biasing conditions to achieve two-well potential distribution (or bistable behavior) in a narrow cc-beam resonator reported in previous works [13,14]. Moreover, the accuracy of the model has been validated through extensive FEM simulations and experimental data.

Section 2 carries an analytical approach to the system features, while Section 3 describes the model based on a finite difference method. Model parameters are analyzed by FEM simulations and experimental data in Section 4, and Section 5 develops the Verilog-A macro-model and presents the results obtained in the bistability of narrow cc-beam resonators analysis. Finally, conclusions are given in Section 6.

2. Nonlinear electromechanical analysis

The electromechanical system consists of a doubly clamped beam (cc-beam) resonator placed symmetrically in-plane between two electrodes (Fig. 1) with electrostatic actuation (AC excitation) and capacitive readout. A bias voltage (DC) applied to the beam structure induces a capacitive motional current that is sensed and converted to a voltage signal by an on chip transimpedance amplifier [14]. This topology reduces the parasitic feedthrough current in contrast to other reported topologies; moreover, the use of independent AC and DC voltage sources is more suitable for operating as a self-sustained oscillator [15].

The system dynamics is described analytically with a single degree of freedom equation in the x-direction (Fig. 1). The beam deflection under a punctual force in such direction (F_x) applied at the beam center is [16]:

$$\omega_F(y) = \frac{F_x y^2}{4EI}\left(\frac{y}{3} - \frac{l}{4}\right) \tag{1}$$

for beam length values between 0 and $l/2$, with $I = t_h w^3/12$ being the first moment of area for a rectangular cross-section, and E the material Young modulus. Considering a lumped model that assumes the beam effective mass concentrated at its middle point ($y = l/2$ in Fig. 1), the displacement of that point is taken as the reference displacement x, and the linear stiffness constant according to the Hooke law is:

$$k_1 = \frac{16Et_h w^3}{l^3} \tag{2}$$

In a cc-beam, the cubic nonlinear stiffness (k_3) cannot be neglected, especially for large deformations. In [1], an approximate analysis of the nonlinear additional anharmonic force caused by large deformations is used to estimate the nonlinear stiffness of a cc-beam accordingly to Eq. (3), an expression quite similar to that reported in [17–19] that has been validated from FEM simulations in this work.

$$k_3 = 0.767\frac{k_1}{w^2} \tag{3}$$

The electrostatic actuation is understood as a distributed phenomenon along the span that causes the beam deformation; such deformation in turn impacts the electrostatic actuation. Since the beam parallel plane deformation proposed in previous works is found to be not realistic enough, especially for large displacements of the middle point, the cc-beam deflection profile under a uniform load (q) is considered in this work and, according to [16], is obtained as

$$\omega_q(y) = \frac{q}{EI}\left(-\frac{l}{12}y^3 + \frac{1}{24}y^4 + \frac{l^2}{24}y^2\right) \tag{4}$$

The maximum deflection occurs at the beam center ($\omega_q(l/2)$), whose displacement corresponds to the x parameter, and is used to find the normalized deflection profile for the cc-beam (Eq. (5)), that is, the deflection at each position in terms of such maximum. Thus, the obtained expression is independent of the load value.

$$\omega_q(x, y) = \frac{384x}{l^4}\left(-\frac{l}{12}y^3 + \frac{1}{24}y^4 + \frac{l^2}{24}y^2\right) \tag{5}$$

The assimilation of the electrostatic force to a distributed load case is found to be accurate enough as shown in Fig. 2, illustrating the agreement between the deflection profiles obtained by FEM simulations of a cc-beam under electrostatic force and by analytical Eq. (5). When the electrostatic coupling is not applied over the