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Experimental verification of a quasi-trapped degenerate mode magnetic acoustic resonator

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ABSTRACT

This paper presents an elastic resonator exhibiting modal degeneracy and quasi-energy trapping of the displacement field. Electromagnetic acoustic based coupling is employed to excite a pair of degenerate SH dominated modes in an aluminium plate. A circular mesa is machined symmetrically on the plate for the purpose of localising the displacement field. Numerical modelling of the modal properties are compared with experimental measurements. Mapping of the modeshapes using 3D laser vibrometry shows that displacement field to be highly localised to the region defining the mesa in agreement with the numerical model. The numerical model shows that significant out of plane displacement outside the mesa region is expected due to the propagating SV + P waves of the plate. It is expected that design modification will reduce the out of plane contributions thus improving the energy trapping. The high Q-factor of the degenerate trapped resonator has potential applications as a gyroscopic sensor or mass sensor.

1. Introduction and motivation

Trapped and quasi-trapped mode resonators have a long history [1–3] and have formed the basis for two hugely successful technologies. High frequency crystal filters often operate in a trapped shear mode configuration in order to exploit the high intrinsic mechanical quality factor and high stability. Similarly, the Quartz Crystal Microbalance (QCM) also operates in a trapped shear configuration for the same reasons. Thus far there is no evidence in the literature of degenerate trapped or quasi-trapped shear mode resonators. They offer significant advantages in mass sensing applications and offer alternative designs for Coriolis gyroscopes.

In the case of Coriolis gyroscopes, modal degeneracy between the driven and sensed modes of vibration results in a resonant response to the applied input rate. With Quality factors typically of the order of 10^4 for conventional MEMS gyroscopes, the benefits of degeneracy are an increase in rate sensitivity by several orders of magnitude over non degenerate designs [4]. In addition, the degeneracy between the modes permits reduced excitation levels of the driven mode thus reducing the signal noise stemming from the drive signal by a factor equal to the Quality-factor of the degenerate mode pair.

Mass sensing using the absolute frequency shift of a single mode of vibration of a resonator, [5,6], is susceptible to environmental effects that can cause changes to the resonant frequency of that single mode. An alternative approach, which is largely insensitive to the unwanted causes of frequency shift, is made possible through using the well-known degenerate modal properties of cyclically symmetric structures [7]. Independent cyclic modes which vary ω_n circumferentially as $\sin\left(n\theta\right)$ and $\cos\left(n\theta\right)$, $n \neq 0$, share a common natural frequency ω_n. When mass is added to these resonators, in a way which disrupts this symmetry, the degeneracy is broken and the single natural frequency 'splits' to yield two, close, natural frequencies ω_{n1} and ω_{n2}. This frequency split is used to determine the added mass. Common mode effects e.g. temperature and liquid effects are removed by this differential measurement [8,9]. Previous work [10] demonstrated that degenerate mode sensing offered a factor of around 70 improvement in resolution for degenerate mode sensing compared with an absolute frequency measurement.

Mass sensing under liquid presents a significant challenge to mechanical resonators. It has been reported, [5], that out of plane surface displacement causes severe damping rendering resonant based mass detection impossible. In this study the objective is to configure the resonator geometry to support **highly localised, degenerate** modes with dominant in-surface shear vibrations at its surface. The study will include characterisation of the surface displacements to compare the relative size of the in-plane and out of

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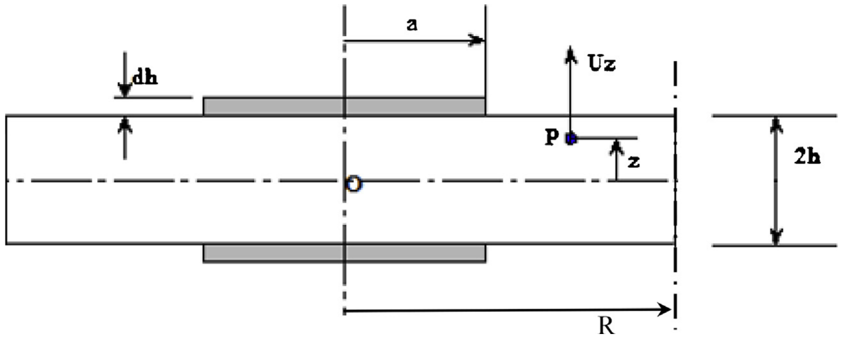

Fig. 1. Schematic of the plate and mesa. The radii of the mesa and plate are defined as (a) and (R), respectively. The mesa and plate thicknesses are defined as (dh) and (2 h), respectively. The out of plane displacement component of an arbitrary point P within the plate is defined as Uz.

plane displacements. The resonator will be expected to possess an inherently high quality factor due to the localisation of the vibration and permit mass detection under liquid through differential measurement of a pair degenerate resonant frequencies [8]. The localisation investigated in this study results in trapping of the in-plane displacements components.

2. Description of the system

An analysis of the antisymmetric SH wave dispersion characteristics of time-harmonic waves in elastic waveguides, as described in [11], illustrates how highly spatially localised resonant responses can result from the appropriate choice of geometry. The highly localised resonant responses can be characterised either as pertaining to “trapped” modes or “quasi-trapped” modes of the elastic structure. In trapped modes all components of the displacement field are highly localised and thus the energy associated with the resonance is conserved or “trapped”. In contrast, quasi-trapped modes do radiate energy as one component of the displacement field is not localised. Fig. 1 shows the structure of such a plate. Localisation, including both quasi-trapping, of antisymmetric thickness shear waves is made possible due to surface loading on the plate caused by the mesa. The mesa is circular to maintain axisymmetry but this is not essential for localisation. In the case considered the mesa and the plate are made from identical materials. Piezoelectric actuation using single crystal plates is commonplace in high frequency filtering and QCMs. However, the anisotropic elasticity of the piezoelectric crystals would break the axisymmetry of the design required for degenerate resonant responses. Magnetic acoustic coupling provides an alternative excitation method which permits the use of elastically isotropic material thus maintaining the desired degeneracy [12–17]. The material must however be electrically conductive to support the generation of eddy currents essential for excitation via this method. The material was chosen to be aluminium due to its high electrical conductivity. Fig. 2 illustrates the aluminium plate, mesa and the permanent magnet

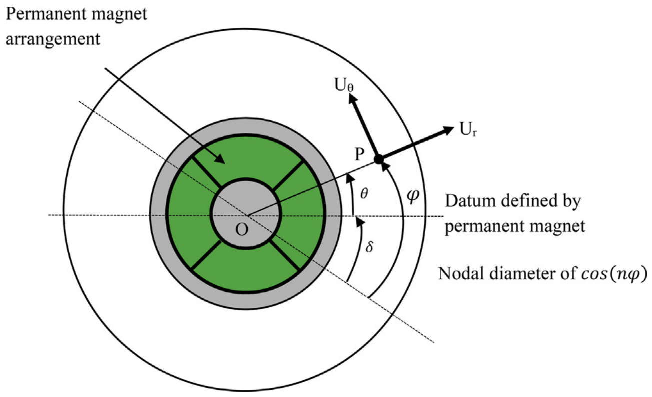

Fig. 2. Plate, mesa and permanent magnet configuration. The geometric centre of the plate, mesa and magnet configuration is point O. The angular coordinate θ defines the angular position of the arbitrary point P on the plate with respect to the datum defined by permanent magnet configuration. The orientation of the nodal diameter of the $cos\,(n\varphi)$ mode with respect to the datum is defined by the angular coordinate δ. The angular coordinate φ defines the angular position of the point P with respect to the nodal diameter. The radial and tangential displacement components of the point P are U_r and $U\theta$, respectively.

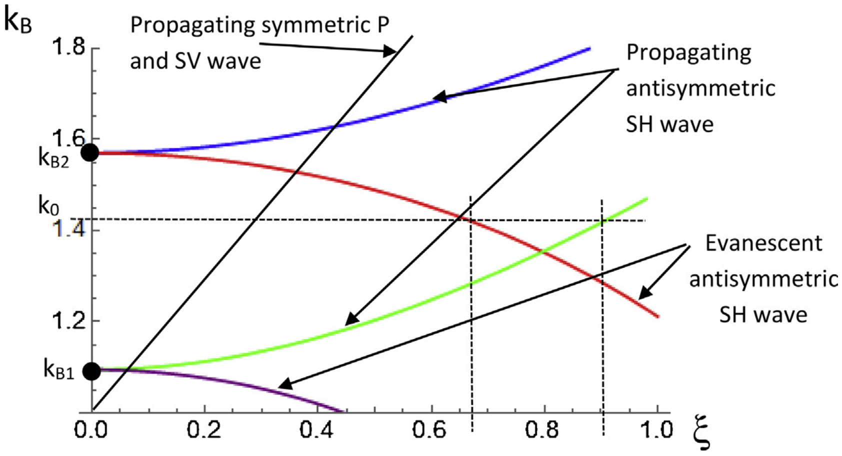

Fig. 3. Dispersion plot for layered plate. The parameters k_{B1} and k_{B2} are the Bechmann numbers for the plate and mesa, respectively and define the nondimensional frequencies in the plate and mesa corresponding to case where the radial wave number ξ is zero. The Bechmann numbers define the cutoff frequencies for the SH waves of the structure. The term k_0 defines the nondimensional design frequency and it must located with the cutoff frequencies defined by the Bechmann numbers.