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Non-linear deflection of a circular diaphragm-type piezoactuator under loads of voltage and pressure

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ABSTRACT

Analytical non-linear equations are formulated to predict the deflection of a circular diaphragm-type piezoactuator, which consists of a passive layer, a bonding layer and a PZT layer. Previous similar analytical solutions presented in the literature are based on thin plates with small deflections (linear problem), however the linear solutions fail to predict the deflection of the piezoactuator when the driven loads, such as voltage and pressure loads, are large. In this research, a non-linear analytical solution for the piezoactuator deflection under loads of voltage and pressure is derived using the principal of minimum energy and the Rayleigh-Ritz method. Each of the three layers in the piezoactuator is considered as an individual layer. The energy associated with the solution includes elastic potential energy of the deformed piezoactuator, electric potential energy in the piezodisc, and the work done by the uniform pressure force. The proposed non-liner solution is validated via static deflection measurements, and it approves that the non-linear analytical results are found to be in a good agreement with the measurements while the linear solution is invalid when the loads are large. Based on the non-linear equations, the effects of the piezoactuator dimensions and the imposed loads on the actuator performance (stroke volume) are also investigated.

1. Introduction

Piezoactuators or piezoelectric actuators, converting electrical energies into mechanical ones, are extensively employed in the micro electro mechanical systems (MEMS), such as the micropumps [1], ejectors [2] and drug deliveries [3], due to their advantages of high stroke force, fast reaction and high efficiency. The diaphragm-type piezoactuators are popular [4] because it has a larger transverse displacement and a lower price, and they commonly consist of the passive layer, bonding layer and PZT layer, which is polarized along the thickness. When a voltage is imposed on the piezodisc, its contraction or expansion in the radial and lateral directions induces a bending moment to the actuator, and results in a deformation in the transverse direction. Such actuators with a piece of piezodisc bonded on one side or both sides of the passive layer are termed as unimorph or bimorph, respectively. Mostly, the diameters of the piezodisc and bonding layers are generally smaller than the diameter of the passive layer to generate larger deflections, and the piezoactuator is tightly clamped around its periphery [5–7].

Accurate analytical solution to the deflections of such diaphragm-type piezoactuators in terms of the actuator dimensions, material properties, and loads is valuable for optimal design. Currently, many linear analytical equations have been proposed through different methods for the deflection analysis of the diaphragm-type piezoactuators. Dobrucki and Pruchnicki [8] investigated the deflection of a piezoelectric bimorph in the form of elastic shell of revolution based on the constitutive equations of piezoelectricity and the classic beam equations, which were solved with finite element method (FEM). Based on the moment balance between the composite three-layer and the rest of the passive layer, Li and Chen [9] developed an analytical solution for static deflections of an edge-clamped circular piezoelectric unimorph by assuming a linear strain distribution along the thickness. Through a different method, Prasad et al. [10] constructed an analogical equivalent electrical circuit to predict the electromechanical behavior of a clamped axisymmetric piezoelectric transducer (unimorph structure) based on the lumped parameter system. Based on the theory of thin plates with small deflections, Dong et al. [11] formulated a simple analytical solution for the transverse bending deformation of a circular piezoactuator with the elastic diaphragm entirely covered by piezoceramic. But more detailed analytical closed form equations for a circular piezoelectric unimorph with piezoceramic partially covered on the elastic diaphragm were presented by Desh-

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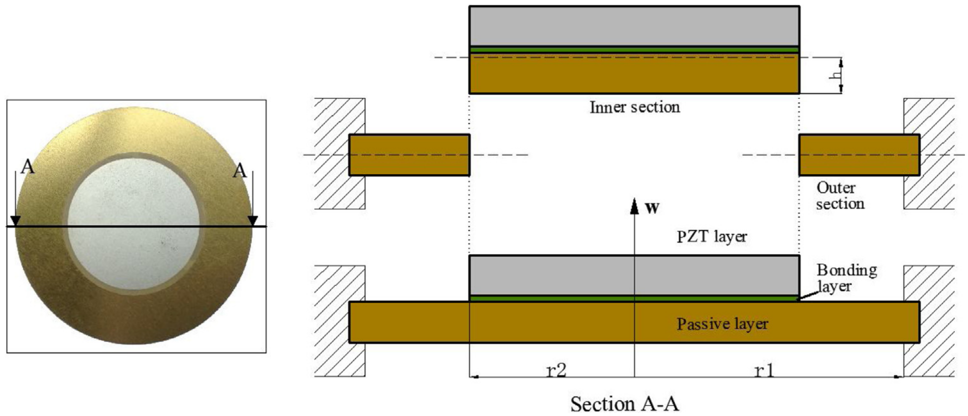

Fig. 1. Schematic of the piezoelectric unimorph actuator.

pande and Saggere [12]. The equations were also based on the theory of thin plates with small deflections and the results well agreed with the experimental data and FEM results when the imposed loads were small. Based on the same theory, Cui et al. [13] analyzed a similar type of piezoactuator in a valveless micropump using the principal of minimum energy, and the influence of actuator dimensions and material properties on the static deflections were further investigated through FEM simulations. To develop more accurate analytical equations, Wang and Huo [14] considered the bonding layer as an individual layer and validated their equations through experimental data. Under loads of pressure, Herz et al. [15] derived a linear analytical solution for the transverse and radial displacement of an edge simple supported circular piezoelectric unimorph with piezoceramic entirely and partially covered on the elastic diaphragm. The linear static transverse deflections of circular piezoactuators with annular piezoceramic used in the nebulizers or atomizers were investigated by Samuel [16], and performance of the piezoactuator under different packaging boundary constraints was studied.

The linear analytical solutions for the static deflections of the piezoactuator are appropriate and accurate when the deflections or driving loads are small, however there is a significant discrepancy between the linear analytical results and experimental results when the loads are large. Yoon et al. [17] experimentally investigated a piezoactuated gas microcompressor with check valves, and center displacement of the piezoactuator was measured as non-linear when the driving loads are large. The non-linear experimental data of the static piezoactuator deflections was also presented by Wang and Huo [14]. Therefore, it is necessary to develop a non-linear analytical solution that is accurate in predicting the static deflections of the diaphragm-type piezoactuators.

There are three main non-linear effects for the deflections of a piezoactuator, electrostrictive, electroelastic and geometric effects [18–20]. The first two non-linear effects are dominated by the material of the piezodisc, while the last one is resulted from the large deflection of the piezoactuator. Yao et al. [21] introduced the electrostrictive and electroelastic coefficients to the constitutive equations for a piezoelectric cantilever actuator, and indicated that the non-linear analysis had been more accurate than the linear analysis. Van den Ende et al. [22] incorporated the non-linear mechanical behavior of the a bimorph bender through introducing a non-linear stiffness function of strain in the layers. Panda [23] considered geometrically non-linear strain-displacement relations in finite element analysis of an annular substrate plate integrated with the piezoelectric fiber reinforced composite (PFRC) material.

In the case of theory of thin plates with small deflections (geometric linear problem), three assumptions are made [24]: (i) elements remain perpendicular to the neutral plane of the multilayer plate after bending; (ii) the normal stresses in the transversal direction are neglected; (iii) there is no deformation in the neutral plane of the plate. The assumption (i) is generally satisfactory when the plate has no holes, and the assumption (ii) holds well for thin plates since the top and bottom are free edges. As for the assumption (iii), it holds when the deflections or loads are small, and turns into invalid when forces develop in the neutral plane. However, in actual deformation of the piezoactuator, strains and stresses are introduced in the neutral plane when the deflections or the driving loads are large. These stresses should be considered in deriving the analytical equations for the deflections of the piezoactuator, thus the solution turns into a non-linear problem and more complicated.

This research makes efforts on the non-linear analytical solution for a circular diaphragm-type piezoactuator under voltage and pressure loads considering the strains and stresses in the neutral plane. The principal of minimum energy is used in the analysis, that is, the piezoactuator reaches equilibrium state with minimum total energy. The Rayleigh-Ritz method is employed as well, which solves boundary value problems through reformulating the given problems to a minimization problem. Therefore, a deflection function of power series is assumed so as to satisfy the clamped boundary condition and minimization of total energy is used to determine the unknown coefficients in the assumed function. Each of the three layers in the piezoactuator is considered as an individual layer in the calculations. The total energy associated with the solution includes elastic potential energy of the deformed actuator, electric potential energy in the piezodiscs and the work done by the uniform pressure force. The non-linear static deflections are validated through experimental measurements. Based on the proposed non-linear equations, the effects of the piezoactuator dimensions on the actuator performance are investigated. Moreover, the stroke volume of the piezoactuator under voltage and pressure loads is also discussed under the optimum actuator dimensions.

2. Non-linear deflection modeling

The diaphragm-type piezoelectric unimorph is an elastic plate (passive layer) with one piece of piezodisc (PZT layer) bonded to one surface. The bonding layer tightly glues the passive layer and the PZT layer together, thus radial and lateral contraction or expansion of the piezodisc under the voltage load is converted to a large bending deformation of the piezoactuator in the transverse direction. In the modeling, the piezoactuator is divided into two parts, shown in Fig. 1, the inner section is a three-layer circular composite structure, and the outer section is an annular passive layer clamped around its periphery. The radius of the PZT layer and bonding layer are r_2, and the clamped radius of the passive layer is r_1, the thicknesses of the passive layer, bonding layer and PZT layer are t_p, t_b,