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Force-frequency characteristics of multi-electrode quartz crystal resonator cluster

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ABSTRACT

The quartz resonator has a significant force-sensitive effect under the action of force. It is most commonly used in the field of modern digital measurement and control. To reduce temperature and other interference factors as well as improve force-sensitivity, a stress distribution is derived and calculated in the quartz wafer by radial force under the elastic mechanics method. A multi-electrode force-sensitive resonator cluster is designed based on the stress distribution results. The experimental results show that the sensitivity of the quartz resonator cluster is better than the traditional single electrode resonator and that the overall force-frequency coefficient of the force-frequency reaches 10,167 Hz/N. The quartz resonator cluster is a viable sensitive element for high-precision digital inertial sensors and related detection systems.

1. Introduction

The quartz crystal resonator has been widely used in system frequency stabilization devices, such as crystal filters and oscillators, due to its resonant frequency output stability and good repeatability [1–4]. After discovering the force-sensitive effect of the quartz resonator, a lot of theoretical and experimental studies have been carried out. Ratajski experimentally determined the force-frequency coefficient that subsequently became the norm for quantifying the force-frequency effect [5]. Mindlin proposed a theory for studying the high frequency vibration of the piezoelectric plate by employing the continuous high-order displacement component to analyze the complex thickness shear deformation [6]. Lee concluded a formula for calculating the frequency by employing the theory of incremental elastic deformations superimposed on initial finite deformations and established the two-dimensional governing equation of piezoelectric quartz plate vibration [7–9]. Janiaud gave the calculation method of the stress effect of the anisotropic crystal under radial force [10]. The finite element and other numerical methods have also been widely used in the study of quartz crystal resonators [11–13].

The use of a quartz crystal resonator as a sensitive sensor component in areas such as motion carrier postures, force (pressure), and quality testing is a significant research focus [14–16]. Presently, two independent resonator pairings are used to overcome the frequency change of the quartz crystal resonator caused by factors other than force (e.g., stress or strain). Here, the differential frequency output suppresses the relevant interference factors when the force sensitive characteristic of the quartz crystal resonator is used. For example, the quartz vibrating beam acceleration resonator, which operates in a flexural vibrating mode, determines its resonant frequency below 50 Khz. The force frequency characteristic of quartz vibrating beam resonator is better than that of single-electrode quartz resonator with thickness shearing mode. However, the processing technology of quartz vibrating beam resonator is strict, and the size error has great influence on the resonant frequency. It is greatly affected by temperature in application. Achieving two quartz crystal resonators for pairing whose temperature frequency characteristics are identical is difficult and adversely affects the force-frequency characteristic of the difference frequency outputs [17,18].

The force sensitivity of the quartz resonator is based on the piezoelectric effect of quartz and is related to stress change in the wafer [19]. When radial force is applied in the diameter direction of the thin wafer, then the stress distribution is not homogeneous due to crystal anisotropy. The inhomogeneity of this stress distribution causes differences in the force-sensitive effects of the resonators at different locations, which is the basis for applying the difference

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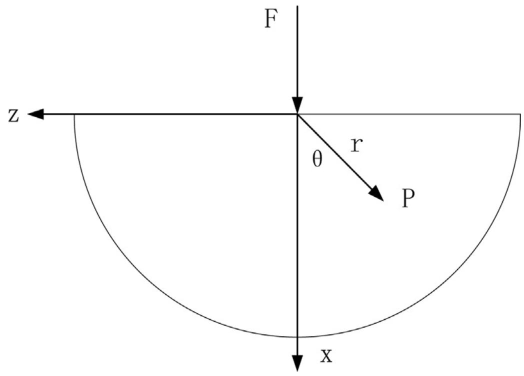

Fig. 1. Semi-infinite plane subjected to radial force.

frequency signal (due to its force-sensitive characteristics) [20,21]. A force-sensitive resonator cluster composed of multi-pair electrodes on the same circular wafer is designed. This differs from the traditional resonator design, which employs a single pair of electrodes on the quartz wafer. The optimal design position of the resonator electrode is determined by calculating the stress distribution of the quartz wafer under radial force. The output frequency signals of the resonator cluster are subtracted from each other by the common mode rejection principle to suppress the temperature and other interference factors as well improve the overall force sensitivity of the quartz crystal resonator cluster.

A digital inertial sensor system (e.g., acceleration sensor, angular velocity sensor, and horizontal attitude sensor) composed of a quartz resonator cluster with significant force sensitivity in the sensor element is small-sized, low cost, simple-structured, and easily maintained as well as being highly stable, reliable, linear, and consistent. These characteristics match the current development trend of sensor arrays, miniaturization, and information processing for computers; as such, this resonator cluster has good prospects across a broad range of applications.

2. Force distribution

2.1. Anisotropic half-plane subjected to radial force

Fig. 1 shows an anisotropic material with the unit thickness of a semi-infinite plane. The X > 0 region is a semi infinite plane and the Z axis is a semi infinite plane boundary. Here, a concentrated force, F, acts on the half-plane boundary with a vertical boundary. Per Janiaud, the normal stress at any point, P, in the half-plane is defined by the polar coordinates σ_r and $\sigma_{r\theta}$; the shear stress is $\tau_{r\theta}$.

$$\begin{cases} \sigma_r = \dfrac{1}{r}\dfrac{\lambda\cos\theta + \mu\sin\theta}{1+\alpha\cos2\theta+\beta\cos4\theta+\gamma\sin2\theta+\delta\sin4\theta} \\ \sigma_\theta = 0 \\ \tau_{r\theta} = 0 \end{cases} \tag{1}$$

where λ, μ is the constant determined by the boundary condition, such that:

$$\begin{cases} \displaystyle\int_c \sigma_r cos\theta ds = -F \\ \displaystyle\int_c \sigma_r sin\theta ds = 0 \end{cases} \tag{2}$$

α, β, γ, and δ are represented by the flexibility constants of the material and S_{ij} is an element of the material flexibility matrix, S.

$$\begin{cases} \alpha = \dfrac{4(S_{11}-S_{33})}{3(S_{11}+S_{33})+2S_{13}+S_{55}} \\ \beta = \dfrac{S_{11}+S_{33}-2S_{13}-S_{55}}{3(S_{11}+S_{33})+2S_{13}+S_{55}} \\ \gamma = \dfrac{4(S_{15}+S_{35})}{3(S_{11}+S_{33})+2S_{13}+S_{55}} \\ \delta = \dfrac{2(S_{15}-S_{35})}{3(S_{11}+S_{33})+2S_{13}+S_{55}} \end{cases} \tag{3}$$

When the force direction is not on the x-axis and the angle of the x-axis is ψ, then the material parameters are transformed into:

$$\begin{cases} \alpha_\varphi = \alpha\cos2\psi - \gamma sin2\psi \\ \beta_\varphi = \beta\cos4\psi - \delta\sin4\psi \\ \gamma_\varphi = \alpha\sin2\psi + \gamma\cos2\psi \\ \delta_\varphi = \beta\sin4\psi + \delta\cos4\psi \end{cases} \tag{4}$$

2.2. Stress distribution in a semi-infinite plane of AT-cut quartz crystal subjected to radial forces

Presently, most resonators use AT-cut quartz crystal. Stress, σ_r, in the semi-infinite plane of the AT-cut quartz crystal is subjected to radial forces, following Formula (1):

When the azimuth of the radial concentric force is $\psi = 0$, then the parameter of the force distribution formula of the quartz crystal with AT-cut quartz crystal flexibility matrix is given by:

α_0 =0.0893137, β0 = 0.206198,

The boundary condition is obtained from Formula (2), such that:

$$\begin{cases} \lambda_\varphi = -\dfrac{4F}{\pi}\left(0.488409134 + 0.01810713877\cos2\psi\right) \\ \mu_\varphi = -\dfrac{4F}{\pi} \times 0.01810713877\sin2\psi \end{cases} \tag{5}$$

When the azimuth of the radial concentric force is $\psi = 0$, then

$$\begin{cases} \lambda_0 = -\dfrac{4F}{\pi}(0.488409134 + 0.01810713877) \\ \mu_0 = 0 \end{cases} \tag{6}$$

2.3. Stress distribution in circular AT-cut quartz crystal subjected to radial forces

Fig. 2 shows a quartz wafer subjected to radial force, where F is the unit thickness of the force and the x-axis is a quartz crystal shaft. The stress of the two forces, F, in the wafer can be expressed by Eq. (7).

$$\begin{cases} \sigma_1 = \dfrac{1}{r_1}\dfrac{\lambda cos\theta_1 + \mu sin\theta_1}{1+\alpha cos2\theta_1+\beta cos4\theta_1+\gamma sin2\theta_1+\delta sin4\theta_1} \\ \sigma_2 = \dfrac{1}{r_2}\dfrac{\lambda cos\theta_2 - \mu sin\theta_2}{1+\alpha cos2\theta_2+\beta cos4\theta_2-\gamma sin2\theta_2-\delta sin4\theta_2} \end{cases} \tag{7}$$

where θ_2 is negative,

$$r_1 = \sqrt{R^2+r^2},\ r_2 = \sqrt{R^2+r^2}$$

$$tan\theta_1 = \frac{rcos\theta}{R - rsin\theta},\ tan\theta_2 = \frac{rcos\theta}{R + rsin\theta}$$

Per the superposition theorem, the stress at any point, P, in the wafer (denoted by r, θ) can be expressed as the superposition of