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A Computationally Efficient Stabilizing Model Predictive Control of Switched Systems

Hariprasad K, Sharad Bhartiya ⋆

*Department of Chemical Engineering,
Indian Institute of Technology-Bombay, Mumbai - 400 076, India*

Abstract: Many applications in engineering exhibit switching character due to discrete and continuous aspects in their dynamic behavior. Switching characteristics of hybrid systems bring discontinuity and nonlinearity in their course of operation and pose major challenges in developing stabilizing Model Predictive Control (MPC) for them. For Piecewise Affine (PWA) Systems, the MPC problem requires on-line solution of Mixed Integer Programs (MIPs) for obtaining the input profile. Since, complexity of the optimization problem that needs to be solved in MPC increases combinatorially with respect to the integer variables, on-line computing of MPC control law for large scale problems and/or problems with large horizons is expensive. In this paper we, propose a MPC formulation, under the popular framework of terminal cost - terminal set MPC, which enables tuning the complexity of the control algorithm. The proposed approach introduces an idea of a pre-terminal set, within which the inputs have enough power to trap states inside it. Since the pre-terminal set lies in the terminal mode which contains origin, this eliminates the need for binary decision variables to model mode transitions after the trajectory enters in pre-terminal set, thereby reducing the on-line complexity although at the expense of optimality. Examples are presented to illustrate the computational benefits of the proposed MPC strategy over existing MPC.

Keywords: Switched systems, Hybrid systems, Stabilizing model predictive control (MPC), Stability, Piecewise Affine Systems (PWA).

1. INTRODUCTION

Switched systems represent a class of hybrid dynamical systems, wherein occurrence of a discrete event is accompanied by a switch in the flow field or operating mode of the system. Models of switched systems consist of multiple sets of differential equations, with each set corresponding to a mode of operation. Continuous states of the switched system evolve in a given mode until certain conditions are satisfied after which the system transitions to a new mode of operation (Hariprasad et al., 2012). Switched system dynamics appear in diverse areas such as power electronic devices, manufacturing systems, communication networks, models in economics and finance and chemical process systems and many others (Chatterjee, 2007). Synthesizing regulators for such switched systems is particularly challenging since it requires a co-ordinated switching strategy, in addition to steering the states to the origin.

Model Predictive Control (MPC) has emerged as an industrially relevant control strategy in recent times. MPC can handle hard constraints on the manipulated and control variables to achieve economically optimal process operation. MPC control algorithms compute a profile of manipulated inputs by optimizing a desired open loop performance objective over a future horizon for a given initial state, and implement the first move of the profile. This procedure is repeated at each sampling time, with the updated process measurement as initial state, bringing a receding horizon nature to the control strategy (Camacho and Bourdons, 2004). The fact that different modes of operation can be expressed as constraints, makes MPC a natural choice for constrained control of switching systems.

Switching characteristics of hybrid systems bring discontinuity and nonlinearity in their course of operation, which pose major challenges while devising stabilizing MPC for hybrid systems (Mayne et al., 2000). For MPC to be an acceptable solution, for control of hybrid systems, it should have the following features: (i) nominal and robust stability, (ii) computational tractability for on line implementation. The current generation of MPC algorithms for switched systems have primarily focused on the former feature and neglected the issue of computational tractability, leaving it entirely to the solver. Consequently most practical implementations of MPC have been limited to the control of small systems. In this work, we present a model predictive control scheme of switching systems which provide computational benefits over existing formulations.

Finite horizon stabilizing MPC for hybrid systems is presented by Bemporad and Morari (Bemporad and Morari, 1999), wherein the system states were constrained to reach the origin after finite moves. Bemporad et al.(Bemporad

⋆ Corresponding Author: bhartiya@che.iitb.ac.in (Sharad Bhartiya),Tel: +91 22 25767225.

et al., 2000) presented a stabilizing infinite horizon MPC for hybrid systems which employed a terminal cost. The terminal cost is obtained as the solution of a common Lyapunov function corresponding to those modes through which the trajectory passes before reaching the origin. Mayne and Rakovic (Mayne and Rakovic, 2003) introduced a stabilizing MPC formulation for Piece Wise Affine systems (PWA) and presented algorithms for its solution. In their work, stability of the MPC is ensured by engaging a terminal cost as well as a terminal set. Algorithms to find stabilizing explicit solutions of terminal cost-terminal set MPC for constrained piecewise affine systems using multi-parametric programming are also reported in literature (Borelli et al., 2005; Grieder et al., 2005). A proof of stability along with a method to synthesize terminal cost-terminal constraint MPC of hybrid systems using common Lyapunov functions is reported by Lazar et al. (Lazar et al., 2006). Employment of terminal equality constraint for stabilization, where states are constrained to reach target after a finite number of moves, leads to aggressive control actions for both stable and unstable plants (Mayne et al., 2000). Thus the majority of the works reported in literature adopt a terminal cost-terminal set approach, where the inputs are obtained over a sufficiently large but finite horizon to ensure that states reach the terminal set at the end of the horizon, beyond which the inputs are parameterized as a state feedback control law over the infinite horizon. For Piece Wise Affine systems, this formulation lead to a mixed integer programming (MIP) or multi parametric Mixed Integer programming (mp-MIP) problem that needs to be solved over a horizon to yield optimal input profile. If the initial condition is far from the terminal set and/or if the dynamics are sluggish, the lower bound on the finite horizon length may assume a large value resulting in a MIP with large number of binary variables in the MPC optimization problem. It is well known that the worst case computational complexity of a combinatorial search algorithm such as in Branch and Bound (B & B) varies with $\mathbb{O}(2^{n_{binary}})$, where n_{binary} is the total number of binary variables involved in the optimization problem (Nemhauser and Wolsey, 1988; Nocedal and Wright, 1999). It is, therefore, of interest to limit the complexity of the control problem so as to make it practically implementable.

In the current study, we propose a MPC formulation for linear switched systems, which can tune the computational complexity of the problem resulting in a trade off between performance and computational burden. In particular, the current study uses a notion of a pre-terminal set, a maximal mode admissible set, wherein inputs have enough power to ensure that the state trajectory does not leave the terminal mode. This allows us to omit the binary variables that represent the state space partition, over that part of the horizon, where the states are constrained to stay inside the terminal mode of operation. This, in turn, reduces binary decision variables in the MPC optimization problem, thereby reducing the computational complexity of the control problem. The price paid is that the trajectory of evolution is constrained to lie in the pre-terminal set thereby making the performance suboptimal. The notion of multiple terminal sets in not new and has been used in two previous contexts of MPC : (i) Move blocking MPC (Gondhalekar et al., 2009; Oldewurtel et al., 2009) where a controlled invariant feasible set is used as a pre-terminal set to achieve feasibility in MPC formulation that uses move blocking to mitigate computational complaints; and (ii) Robust MPC (Chisci et al., 2001; Mayne et al., 2005), where invariant reachable sets are used along the prediction horizon. Our work uses two terminal sets in context of switched systems which has not been presented in literature previously to the best of author's knowledge. The proposed MPC uses completely parameterized inputs over the horizon and no attempt is made towards input blocking. Thus the novel element of the algorithm is the ability to tune computational complexity of the algorithm and performance.

The paper is organized as follows: preliminaries of terminal cost-terminal constraint MPC are presented in section 2, section 3 present the details of the proposed MPC formulation, its properties and algorithm; section 4 presents one simulation case study to illustrate features of the present work; and section 5 gives concluding remarks.

2. PRELIMINARIES

Assume that the state space is partitioned into compact polyhedral regions called modes M_j, $j \in J$, an index set representing the modes of operation. Consider the discrete time evolution of trajectories of the switched system in M_j (Liberzon, 2003)

$$\mathbf{x}_{k+1} = f_j(\mathbf{x}_k, \mathbf{u}_k), j \in J \tag{1}$$
$$\mathbf{x} \in \mathbb{X} \subset \mathbb{R}^n, \mathbf{u} \in \mathbb{U} \subset \mathbb{R}^m \tag{2}$$

where $\mathbf{x} \in \mathbb{X} \subset \mathbb{R}^n$ represent continuous states, $\mathbf{u} \in \mathbb{U} \subset \mathbb{R}^m$ represent continuous inputs. $\mathbb{X}$ and $\mathbb{U}$ represent state and input constraint sets, and are assumed to be nonempty compact and polyhedral, containing the origin in their interiors. As the hybrid state space spans over different modes of operation, $\mathbb{X}$ can be partitioned in to $\mathbb{X}_j \triangleq (\mathbb{X} \cap M_j), j \in J$, with $\mathbb{X}_0$ corresponds to the constraints of the origin containing mode. Switching triggers the transition from the current mode $j \in J$ to a new mode $j' \in J$ of the system with the following switching rule: $\sigma_k : \mathbb{X} \mapsto J$ such that,

$$\sigma_k \triangleq j, \text{ if } \mathbf{x}_k \in M_j \tag{3}$$

. For a fixed horizon $N > 1$, $\overline{\mathbf{x}}_k(\mathbf{x}_k, \overline{\mathbf{u}_k}) \triangleq (\mathbf{x}_{1/k}, \ldots, \mathbf{x}_{N/k}) \in \mathbb{X} \times \cdots \times \mathbb{X} \triangleq \mathbb{X}^N$ represents the state sequence generated by applying the input sequence $\overline{\mathbf{u}_k} \triangleq (\mathbf{u}_{0/k}, \ldots, \mathbf{u}_{N-1/k}) \in \mathbb{U} \times \cdots \times \mathbb{U} \triangleq \mathbb{U}^N$ to the system of Eqs. (1,2).

Definition 1. **Maximal state feedback positive invariant set ($\mathcal{X}_\infty$)(Blanchini, 1999; Borelli et al., 2012):** A set $\mathcal{X} \subset \mathbb{X}_0$ is said to be positively invariant for the system Eqs. (1,2) under state feedback law $\mathbf{u}_{k+i/k} = g(\mathbf{x}_{k+i/k}) \in \mathbb{U}$, if $\mathbf{x}_k \in \mathcal{X} \subset \mathbb{X}_0 \Rightarrow f_0(\mathbf{x}_{k+i/k}, g(\mathbf{x}_{k+i/k})) \in \mathcal{X}, \forall i \geq 0$. Then the maximal state feedback positive invariant set ($\mathcal{X}_\infty$) is defined as the union of all such state feedback positive invariant sets, $\mathcal{X}$.

The set $\mathcal{X}_\infty$ is assumed to be compact and polyhedral.

Definition 2. **Admissible input sequence $\overline{\mathbf{U}}_f(\mathbf{x}_k)$:** The class of admissible input sequence for the system of Eqs. (1,2) is defined with respect to $\mathcal{X}_\infty$ and $\mathbf{x}_k$ as follows: $\overline{\mathbf{U}_f}(\mathbf{x}_k) \triangleq \{\overline{\mathbf{u}}_k \in \mathbb{U}^N | \overline{\mathbf{x}}_k(\mathbf{x}_k, \overline{\mathbf{u}_k}) \in \mathbb{X}^N, \mathbf{x}_{k+N/k} \in \mathcal{X}_\infty\}$.