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Scaling properties of resonant cavities driven by piezo-electric actuators

L. de Luca*, M. Girfoglio, M. Chiatto, G. Coppola

Department of Industrial Engineering, Aerospace Sector, Universitá degli Studi di Napoli "Federico II", Naples, Italy

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ABSTRACT

Acoustic-structural properties of piezo-electric driven resonant cavities usually employed to generate the so-called synthetic jets are theoretically and numerically investigated in order to characterize the performances of such devices. It is shown that the actuator behaves as a two-coupled oscillators system and the dimensionless form of the governing equations allows one to identify various operating conditions, in particular those leading to their decoupling. The theoretical predictions are validated through analytical, numerical and experimental findings for devices having different mechanical and geometrical characteristics, designed to achieve an increasing coupling effect. Considerations about the strength of jet formation at the Helmholtz frequency are made as well.

1. Introduction

The literature about the synthetic jet actuators is huge and includes a wide field of applications such as flow control, heat transfer from small size surfaces, overall enhancement of mixing between fluid currents, generation of micro-thrust for propulsion or attitude control of Micro Aerial Vehicle (MAV). We limit to cite here the review papers of Glezer and Amitay [1] and Cattafesta and Sheplak [2]. Regarding present authors, previous contributions dealt with the direct numerical simulation of jet vectoring, as described by Mongibello et al. [3] and the formulation and the experimental validation of a lumped element physical model of the actuator operation devoted to the prediction of its frequency response (de Luca et al. [4]). More recently, Girfoglio et al. [5] developed a detailed modeling able to predict the efficiency of piezo-electric actuators, which was also validated experimentally.

The overall design of the actuator needs practical modeling tools, which are generally based on reduced order, lumped element physical models. A significant lumped element model of piezo-electrically-driven synthetic jet device is described by Prasad [6] and Prasad et al. [7], who gave detailed relationships for the transverse deflection of the inner and outer regions of the composite membrane in the simultaneous presence of applied voltage and pressure load. They resorted to the approach based on the equivalent electric circuit. Following the same approach, Gallas et al. [8] noted that in a lumped model one may use the acoustic compliance of the shim only, reduced by a proper factor depending on the ratio of the radius, thickness and Young's modulus of the piezo-ceramic and shim materials. Later on, Sharma [9] proposed a different model directly based on the equations of fluid dynamics, where the oscillating membrane is considered as a single-degree-of-freedom mechanical system, while the cavity and the orifice are described by means of proper forms of the continuity and Bernoulli's unsteady equations, respectively. Sharma [9] validated his model on the very same experimental data of Gallas et al. [8].

Chaudhari et al. [10] carried out systematic measurements about the effects of the excitation frequency on the ejection and suction velocities, by varying the geometrical parameters of the cavity. Krishnan and Mohseni [11] studied the characteristics of the flow field produced by a round synthetic jet by using detailed numerical simulations of the turbulent Navier–Stokes equations. Seeley et al. [12] described a simplified fluid-structure interaction model based on the implementation of commercial Finite Elements codes, and proved its validity at relatively low frequency, namely well below the Helmholtz frequency. Persoons [13] proposed a low-order model of prediction of the frequency response of synthetic jet actuators driven by electromagnetic or piezo-electric supply. Based on the equivalent circuit approach, its model yields analytical expressions for the two resonance frequencies, as a function of the structural and Helmholtz resonance frequencies.

de Luca et al. [4] presented a fluidic-type lumped element modeling, which has been inspired by the Sharma's work [9], yielding the frequency response of the resonant cavity in terms of internal

* Corresponding author.

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pressure, membrane displacement and external jet velocity. The model, validated against systematic experimental measurements, gave also simple but accurate analytical relationships for the two resonance frequencies characterizing the overall system response. The present contribution is a follow-up inspection of the previous investigation and is devoted to gain new insights on the acoustic-structural interaction occurring during the operation of a typical piezo-electrically driven resonant cavity. The analysis hereafter presented is based on the dimensionless form of the equations governing the behavior of the two coupled oscillators, the diaphragm and the Helmholtz one. It is corroborated by a parallel experimental investigation carried out on specifically designed actuators.

2. Model formulation

The model described hereafter is the same as the one presented by de Luca et al. [4], in turn inspired by the Sharma's work [9]. It refers to the three basic elements of the actuator: the oscillating membrane (diaphragm or wall, constituted by a thin round metal shim on which a smaller diameter piezo-ceramic disk is bonded), the cavity, the orifice.

For the sake of convenience, the three differential equations which describe the dynamics of the actuator are summarized below.

$$\ddot{x}_w + 2\zeta_w\omega_w\dot{x}_w + \omega_w^2 x_w = \omega_w^2 \Delta x_w \sin\omega t - \frac{p_i A_w}{m_{wt}} \tag{1}$$

$$\frac{V_c}{\gamma p_o}\frac{dp_i}{dt} - A_w\dot{x}_w = -A_o U \tag{2}$$

$$\ddot{U} + \frac{K}{l_e}|U|\dot{U} + \omega_h^2 U = \frac{A_w}{A_o}\omega_h^2\dot{x}_w \tag{3}$$

The dynamics of the membrane is described through the motion equation of a one-degree of freedom forced-damped spring-mass system, Eq. (1), where x_w is the (average) membrane displacement, t is time, p_i is the cavity (internal) differential pressure, m_{wt} is the diaphragm total mass, including shim, piezo-element and air added mass, ω_w is the natural frequency of membrane, ω is the operating frequency, Δx_w is the average linear membrane displacement due to the application of a certain voltage to the piezo-element, A_w is the membrane surface area, ζ_w is the diaphragm damping ratio, dot denotes time derivative.

The second equation of the model is the conservation of mass in the cavity under the assumption of zero-dimensional (lumped) system, Eq. (2). By relating the density and pressure variations by means of an isentropic compression/expansion transformation, the continuity equation can be formulated as above written, where V_c is the cavity volume, p_o is the ambient pressure, γ is the specific heat ratio, U is the instantaneous orifice jet-flow velocity, A_o is the orifice area.

The application of the unsteady Bernoulli's equation between a point inside the cavity where the flow velocity is practically null and a point, just outside the cavity, representing the location where the pressure matches the unperturbed external ambient value, yields the third equation of the model, Eq. (3), where ω_h is the natural Helmholtz frequency, K is the head loss coefficient, and l_e is the effective orifice length, i.e. the distance between the two points of application of the Bernoulli's equation.

It should be pointed out that the (first mode) structural circular frequency of the membrane is given by:

$$\omega_w = \sqrt{\frac{k_w}{m_{wt}}} \tag{4}$$

and represents the uncoupled natural frequency of the membrane oscillator, where k_w is the equivalent spring stiffness of the membrane. This last can be obtained as:

$$k_w = m_w(2\pi\widetilde{f_w})^2 \tag{5}$$

where $\widetilde{f_w}$ is the frequency of the principal mode of vibration of a rigidly clamped disk. Although the presence of the piezo-ceramic element bonded to the metal shim enhances the flexural rigidity of the membrane (and in principle the very thin layer of glue should be taken into account as well), for standard operating conditions $\widetilde{f_w}$ can be referred to the first fundamental mode of the shim only (that is the membrane structural element actually clamped) and calculated by using the standard formula reported in many textbooks (de Luca et al. [4]). Here m_w is the diaphragm mass taking into account both shim and piezo-ceramic disk, but not including the dynamic contribution of the air added mass.

The uncoupled natural frequency of the acoustic oscillator is the so called Helmholtz frequency ω_h, which is usually recognized to be:

$$\omega_h = \sqrt{\frac{\gamma A_o^2 p_o/V_c}{\rho_a l_e A_o}} = \sqrt{\frac{k_a}{M_a}} \tag{6}$$

where k_a and M_a are, respectively, the equivalent stiffness of the air inside the cavity, $k_a = \gamma A_o^2 p_o/V_c$, and the effective mass of the air at the orifice, $M_a = \rho_a l_e A_o$.

It is worth to stress that the membrane dynamics is forced by the acoustic oscillator via the cavity pressure term as well as by the piezo-electric effect due to the applied sine voltage. The amplitude of this forcing, F_o, is expressed conveniently as:

$$F_o = \frac{k_w d_A V_a}{A_w} = k_w \Delta x_w \tag{7}$$

where d_A is the effective acoustic piezo-electric coefficient that represents the ratio of the cavity volume variation ΔV to the applied voltage V_a, when the driving differential pressure is null [7]. In Eq. (1) such a forcing appears as $\omega_w^2\Delta x_w$, being normalized respect to the diaphragm mass. Note that in the previous Eq. (7) the cavity volume variation $\Delta V = d_A V_a$ is divided by the membrane area A_w in order to obtain the average linear membrane displacement Δx_w (to be multiplied by k_w to obtain the driving force):

$$\Delta x_w = \frac{d_A V_a}{A_w} \tag{8}$$

The coefficient d_A could be evaluated analytically by means of the distribution of the transverse displacement of the composite diaphragm, as made by Prasad and Prasad et al. [6,7]. This procedure is not practical due to the difficulty of determining the required physical parameters. An alternative procedure consists in determining the acoustic compliance of the membrane C_{ac} which, through a dual definition of d_A, is given by the ratio of the volume variation ΔV to a uniformly distributed pressure load p, in condition of electrical short-circuit [7]. Of course the evaluation of C_{ac} would require the same difficulties. However, one can refer to the acoustic compliance of a homogeneous circular plate (namely, having the properties of the piezo-ceramic disk) that yields insight into the scaling behavior of the diaphragm, and ultimately to obtain d_A by means of the relationship:

$$d_A = C_{ac}\phi_a \tag{9}$$

in which ϕ_a is the electroacoustic transduction coefficient [7]. As introduced by de Luca et al. [4], the electroacoustic transduction coefficient is assumed to be a fitting parameter of the computer code, to be evaluated by experimental comparison.

In summary, the behavior of the synthetic jet actuator can be modeled by the dynamics of two mutually coupled oscillators: the