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Acoustic damping on flexural mechanical resonators

Guillaume Aoust*, Raphaël Levy, Béatrice Bourgeteau, Olivier Le Traon

ONERA—The French Aerospace Lab, F-91123 Palaiseau Cedex, France

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ABSTRACT

Flexural resonators operated within a fluid are most of the time limited by viscous damping. Nonetheless, depending on the resonator geometry, significant acoustic damping has also to be considered. After reviewing the main theoretical approaches proposed in the literature to evaluate acoustic losses, we provide useful and simplified expressions to quantify their corresponding quality factor. Original results are obtained by extending well-known methods to different shapes. We also present numerical simulations and experiments to compare and assess the validity of our predictions, and show that analytical models combining both viscous and acoustic damping can give reliable values for the observed quality factors.

1. Introduction

Flexural mechanical resonators are most of the time operated in vacuum in order to minimize any coupling with the surroundings that could affect their excellent resonance characteristics. Some applications however require a complete immersion within a fluid medium. For example, they are used for highly sensitive chemical sensing platform [1], but also for in-situ force measurements such as the photo acoustic detection [2], the resonant optothermoacoustic detection [3] or precise temperature measurements [4].

In those cases, the surrounding fluid strongly modifies the characteristics of the resonance and can dramatically reduce the sensors performances compared with vacuum operation. It is now established that the damping occurring within the fluid can be attributed to several mechanisms, among which the most relevant ones are viscous damping and acoustic radiation damping. These two effects, originating both from a fluid-structure interaction, rely on two distinct physical properties of the fluid that are viscosity and compressibility, and their contributions behave differently with respect to the dimensions of the resonator.

Acoustic radiation damping is not frequently considered in high quality flexural resonators. One of the main reasons is that a huge majority of them are designed for applications in the inertial or time-frequency domains, *i.e.* under vacuum operation where no fluid-structure occurs. For other applications requiring an immersion within a fluid, viscous damping has been shown to be negligible for macroscopic resonators because of their low surface over volume ratios. Their acoustic properties have been investigated to obtain reliable expressions for the total quality factor [5–7]. For smaller resonators, the number of geometries reported up to now remains quite limited; and acoustic damping is most of the time hardly noticeable if not negligible. Nevertheless, acoustic damping is still possible with specific geometries and can become comparable with viscous damping; therefore a more detailed study is of significant importance for any optimization purposes.

Finite element simulation softwares can quantitatively and accurately predict the mechanical behavior of any type of resonator. However, we will focus on analytical models in order to keep a good physical insight. In the following, we will restrain to the case of resonators made of beams with a circular or rectangular cross section, since it almost covers all kind of existing structures.

A first complete analytical model including both viscosity and compressibility has already been developed [8]. The result is exact but only deals with infinitely thin cantilevers and fails to provide either a direct physical insight of the compressibility effect or a simplified formula for the quality factor. Moreover, the approach does not include a possible static wall nearby, which is necessary to deal with viscous damping for resonators made of several beams such as tuning forks [9].

Other existing analytical treatments currently available in the literature generally assume that the surrounding fluid acts as an uncompressible medium [9,10]. The damping is then reduced to the fluid viscous effects, for which analytical results give quite accu-

* Corresponding author. Fax: +33 146734824.
E-mail address: guillaume.aoust@onera.fr (G. Aoust).

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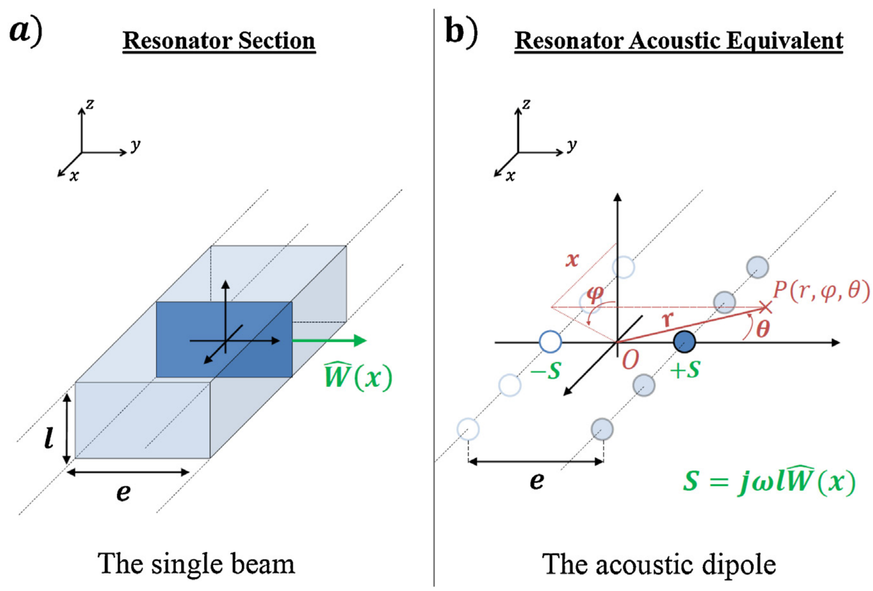

Fig. 1. Schematic of the acoustic equivalence for the single beam resonator. (a) The geometrical cross section of the oscillating beam of width e and thickness l. The amplitude of the displacement along the y axis is denoted $\hat{W}(x)$. (b) The corresponding point sources modeling of the beam, with S the strength of the source.

rate formulas for the quality factor. However, such an assumption is not valid in some cases. It has been evidenced in the study of tuning forks in gas [11], and more recently in cryogenic fluids [12,13]. In the latter example, it has been shown that commercial quartz tuning forks immersed within liquid or gaseous helium at very low temperature can provide various parameters such as viscosity, pressure or temperature [14]. Under such conditions, acoustic damping is not a negligible and has to be taken into account.

From a theoretical point of view, both viscous and acoustic damping effects are accounted for in the Navier–Stokes equations. However, no general analytical solution has yet appeared if both sources of damping are considered simultaneously. Indeed, the coupled equation system governing the behavior of the fluid on the first hand and the motion of the resonator on the other hand cannot be reduced into two independent equations.

A simplified approach consists in solving the two problems in a sequential way, which is an approximate way to force the independence. The motion of the resonator is solved first assuming that the surrounding fluid is uncompressible, leading to an analytical expression of the damping only due to viscous effects. Then, the motion of the resonator is used as a boundary initial condition for the purely acoustic problem neglecting viscosity.

In this paper, we will generalize this sequential approach to any flexural mechanical resonator made of beams with arbitrary rectangular or circular cross section, as well as investigate the validity of our results with numerical simulations and experiments. We also provide expressions for the quality factors, as well as simplified expressions to capture more easily the physics behind fluid damping on resonators.

2. Theoretical model

2.1. The single beam model

We first consider a single beam oscillating within a homogeneous fluid unbounded in space. Notations used in the following are presented in Fig. 1. The motion of the resonator subject to viscous damping only has already been solved using the Euler–Bernoulli bending theory, so we will use directly the results obtained in this previous work [9]. Let us remind the main assumptions concerning the beam, which generally hold true for any flexural resonator:

- The beam's cross section is rectangular and uniform over its entire length L.
- The length of the beam L greatly exceeds its transverse dimensions e and l.
- We consider only flexural modes of vibration in the y-direction, whose amplitudes are supposed far smaller than dimensions e and l. Plane sections hence remain plane and normal to the axis of the beam.
- The material composing the beam is assumed isotropic and homogeneous. Its young modulus E and density ρ_b are constants.

Considering the surrounding fluid, we suppose that:

- The fluid is incompressible with a homogeneous density ρ_f and dynamic viscosity μ.
- The fluid is in the continuum regime [15] and unbounded in space. The latter regime is defined by its Knudsen number K_n smaller than 0, 01, ensuring that the Navier-Stokes equation are valid.

Under the previous assumptions, the position of the oscillating beam $\hat{W}(x,\omega)$ in the y direction can be written on its nth eigen angular frequency ω_n as Eq. (1).

$$\hat{W} = \hat{W}(\omega)\phi_n(x) = \left[\int_0^L \mathrm{d}x\widehat{F_{drive}}(x,\omega)\phi_n(x)\right] \frac{12jL^4 Q_v(\omega_n)}{Ee^3 l\alpha_n^4 \int_0^L \phi_n^2(x)\,\mathrm{d}x}\phi_n(x) \quad (1)$$

where α_n and ϕ_n are respectively the characteristic coefficient and the normalized deformation of the beam when oscillating on its nth mode of vibration. Their values can be found in any textbook about dynamic beam flexure, and two useful cases are recalled in Table 1. The viscous quality factor Q_v has been derived in the previous study [9]. Function $\widehat{F_{drive}}$ is the excitation force acting on the beam in the y direction other than the viscous forces.

In this second step, we suppose that the motion of the beam expressed in Eq. (1) is given. The strategy to derive the acoustic