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Three-port equivalent circuit of multi-layer piezoelectric stack

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ABSTRACT

Multi-layer piezoelectric transducers (MPS) is a transducer stacked by numerous thin piezo layers, which can used as an actuator, sensor, or energy harvester. This paper points out the limitations of existing equivalent circuits for the effective modelling of MPS and proposes a 3-port equivalent circuit of MPS which is inspired by the idea of network theory and formulated exactly on the basis of the simplified fundamentals. The 3-port equivalent circuit, which separates the MPS into two acoustic ports and one electrical port through an electro-mechanical transformer, offers an exact and explicit representation of electro-mechanical coupled interaction of MPS. It is very straightforward to apply and effectively simplifies and facilitates the analysis, modelling, and calculation of free and loaded vibration of MPS in either transmitter mode (actuator) or receiver mode (sensor or energy harvester). Compared with existing circuit models in literature, the proposed circuit can be extended to any electrical and mechanical condition. Besides, as the proposed circuit elements are explicitly and exactly derived in terms of material and dimension information rather than determined from measured information, the proposed circuit model allows one to predict behaviors with material properties and structural dimension. For validation, a simple case study is carried out. For both free and loaded vibrations of MPS in the case study, the effectiveness of the proposed circuit has been validated by 3D FEA models.

1. Introduction

Multi-layer piezoelectric stack (MPS) is a transducer stacked by numerous thin piezo layers which can convert the electrical energy into mechanical energy (actuators) and can also convert the mechanical energy into electrical response (sensors and energy harvesters). In many MPS-based applications such as design of free and loaded resonators, determination of unknown material property and design of harvester scavenging energy from ambient vibrations, modelling vibration behaviours of MPS is a key step. To facilitate analysis, modelling, and calculation of free and loaded vibrations of MPS in either transmitter mode (actuator) or receiver mode (sensor or energy harvester), a three-port equivalent circuit of MPS is proposed in this paper.

Section 2 reviews the existing equivalent circuits in literature. Their limitations are pointed out and a novel equivalent circuit is proposed. Section 3 presents the fundamentals of MPS, which provide a basis for the construction of the proposed equivalent circuit presented in Section 4. To validate the formulated equivalent circuit, 3D FEA models of MPS are developed in commercial software 'ANSYS' in Section 5. Simulation results between the two models are compared and the limitations of the proposed model are discussed. Section 6 presents the summary.

2. Existing equivalent circuits

For the facility in describing behaviours of piezo-electrically excited vibration system, equivalent electrical circuits are often used. A widely used equivalent electrical circuit, which has been standardized in IEEE [1], is the Van Dyke circuit [2] shown in Fig. 1, which consists of a lump capacitance C_0 connected in parallel with an inductance L_1, a resistance R_1, and a capacitance C_1. The Van Dyke circuit can be used to represent a piezo-electrically excited mechanical vibration system with one dominant resonant mode captured. However, a piezo-electrically excited mechanical vibration system is actually a continuous system rather than discrete systems, which shows multiple resonant modes in practices. To capture more modes, the Van Dyke model was modified and extended with additional RCL branches in parallel shown in Fig. 2 [1]. Other similar equivalent electrical circuit models were also explored to model multiple resonant modes and improve the accuracy [3,4]. The electrical parameters in those equivalent electrical

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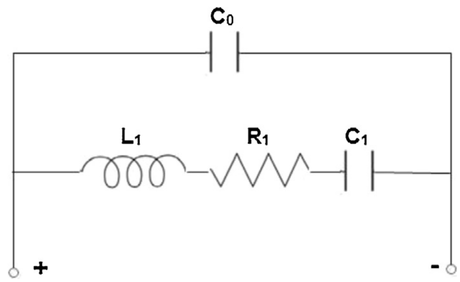

Fig. 1. The Van Dyke equivalent circuit.

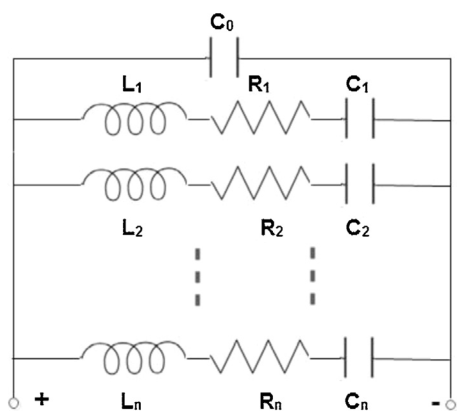

Fig. 2. The extended Van Dyke model.

circuits can be determined by various types of system identification methods based on measured information [1–4]. Although these equivalent circuits can be used to describe the behaviours of piezoelectrically excited system, the electromechanical interaction and physically mechanical quantities are obscure. Therefore, they are incapable to predict the behaviours, when mechanical boundary conditions are changed.

To describe the electro-mechanical coupled interaction of MPS, a widely recognized lumped model is proposed by Goldfarb and Celanovic [5]. This model is composed of an electrical side, a mechanical side, and the ports which couples the electrical and mechanical models. The mechanical side is modelled as a lumped mass-spring-damper system with an electrical port, which can introduce a lumped piezo force from applied voltage in the electrical side, based on the inverse piezoelectric effect. The electrical side is modelled by a lumped capacitance with a mechanical port, which can introduce a counter charge from the induced piezo displacement in the mechanical side (similar to counter EMF).

The lumped model can be easily implemented with some non-linear models [6] for positioning control purpose. However, lumped-models are just approximated models, as a MPS is actually a distributed system. Adriaens et al. [6] and Chen et al. [7] further elaborated this lumped electromechanical model of MPS into distributed-parameter models. In their models, the MPS is proposed to be taken as an equivalent distributed normal solid with no piezoelectricity and the action of voltage is taken as equivalent force acting on the MPS. However, their proposition is not well proved. Besides, the related equivalent constants, which are required to be determined from experiments, make the models hard to predict behaviours, when the experiments are not available.

The 3-port equivalent circuit proposed herein offers an explicit representation of electro-mechanical coupled behaviours of MPS, which separates the MPS into two acoustic ports and one electrical port through an electromechanical transformer. Compared with existing circuit models in literature, the proposed circuit model can

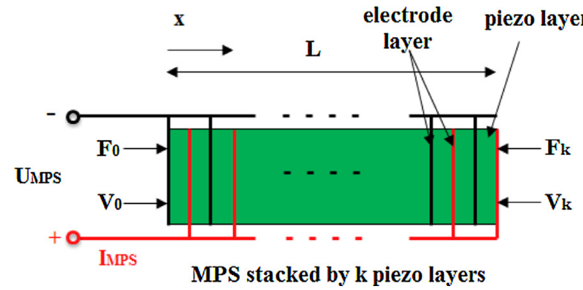

Fig. 3. Schematic of multi-layer piezoelectric stack.

be extended to any electrical and mechanical condition. Besides, as the proposed circuit elements are explicitly and exactly derived in terms of material and dimension information rather than determined from measured information, the proposed circuit model allows one to predict behaviours with material properties and structural dimension. The configuration of the proposed circuit is inspired by the idea of network theory and each electrical element in the proposed equivalent circuit is derived exactly on the basis of simplified fundamental of MPS presented in the previous paper of the authors [8].

3. Fundamentals of MPS

The schematic of MPS is shown in Fig. 3, where each piezo layer is mechanically connected in series and electrically connected in parallel. By applying voltage/charge to MPS, it can produce force/displacement in the longitudinal direction of MPS (x direction shown in Fig. 3) and vice versa. The former is called transmitter mode which can be used as an actuator and the latter is called receiver mode which can be used as a sensor or energy harvester. In transmitter mode, the stack configuration of MPS allows a large displacement output with a low voltage input in a compact size. In receiver mode (sensors and energy harvesters), the stack configuration enables a large charge output for equivalent force inputs.

The proposed equivalent circuit is based on the simplified fundamentals of MPS, which is presented and justified in the previous paper of the authors [8]. For the completeness of the formulation of the equivalent circuit, the simplified fundamentals of MPS are reproduced here.

To facilitate the model formulation, define MPS with overall length L stacked by k piezo layers and assign an x axis along the length. Since, MPS is designed for its longitudinal vibration (the stack is more than three times longer than wide), the vibration is associated with a uniaxial stress state (i.e. the lateral stress is zero). For the operation of MPS, the electrical field is only applied to the direction of vibration (i.e. the electrical field in lateral directions is zero). Based on these two conditions, the constitutive equations of each piezo layer in MPS along the longitudinal direction can derived from the 3-D piezoelectric constitutive equations of IEEE standards [1], shown in Eq. (1).

$$T_3 = c'S_3 - e'E_3, \tag{1.a}$$

$$D_3 = e'S_3 + \epsilon' E_3, \tag{1.b}$$

where T, S, E, and D are respectively stress, strain, electrical field, and electrical displacement. c',e', and ϵ' are piezoelectric coefficients ($c' = 1/s_{33}^E$, $e' = d_{33}/s_{33}^E$, and$\epsilon' = \epsilon_{33}^T - {d_{33}}^2/s_{33}^E$). s_{33}^E,d_{33}, and ϵ_{33}^T are the standardized piezoelectric coefficients [1]. The subscript '3' stands for the direction of polarization (i.e. x direction in this case). For the convenience, this subscript '3' will be omitted in the later notations.