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# Theory of fluxgate sensor: Stability condition and critical resistance

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ABSTRACT

This article established a useful method of solving the state equation of fluxgate sensor in which only basic detection circuit parameters were used. The solution of the equation was given, and the stability of the solution as well as critical resistance was discussed carefully. We also introduced stability condition of fluxgate by an inequality, the physical meaning of this inequality was discussed by simplifying it with maximum energy transfer condition. Experiments on different fluxgate sensors were performed, and the results show good agreements with calculation results.



## 1. Introduction

Fluxgate sensor has been widely used for aeromagnetics, geophysical prospecting, and space exploration, because its advantages such as low noise, low zero drift and long-term stability. The theory of fluxgate is an essential part for designing and developing fluxgate sensor, and it has been developed by a lot of pervious works. Primdahl et al. [1] studied current output of short-current fluxgate. Russell et al. [2] described the theory and numerical results of a capacitively loaded fluxgate sensor, and developed the theory of its stability in al. [3]. The condition of parameters, that makes the response of current-output fluxgate sensors maximum, was developed by al. [4].

Detection circuit of fluxgate sensor is not a typical LRC circuit because its inductance is not a constant. So the equation of detection circuit is a nonlinear differential equation. Most of pervious works are based on Serson–Hannaford approximation [1], which is not necessary, and they need to involve some simplified parameters or only have numerical results. In this paper we use another method to get the theoretical solution of detection circuit state, and found the stability condition as well as critical resistance. But the theoretical result is complex, so some discussion of this article is based on the METC in al. [4] in order to show a simplified result.

From the aspect of energy, the steady-state of fluxgate sensor means the energy being transferred into detection circuit and dissipated in it are equal. Ignoring the dissipation in magnetic core, which is usually very small (the equivalent resistance is no more than 5% of the total resistance), only the resistance of detection circuit can dissipate the energy. It means that for any fluxgate sensor, once the resistance is large enough, it can be stable. Thus for any fluxgate sensor there must exist a resistance, which we call as *critical resistance*, can keep the fluxgate sensor stable if the resistance of detection circuit is greater than it. This resistance was measured accurately by experiments in this paper. And theoretical results show that the critical resistance is related to all the parameters of fluxgate sensor. This relationship is very complex, but if the phase matches the maximum energy transferred condition (METC), it will become very simple. And the physical meanings of this relationship will be discussed.

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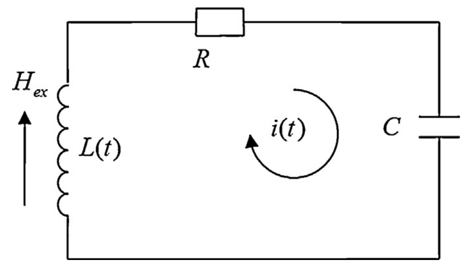


**Fig. 1.** The detection circuit scheme.

## 2. Derivation of the fluxgate equation

A typical fluxgate sensor detection circuit is shown in Fig. 1. The inductance of the detection coil *L(t)* has two value $L_s$ and $L_n$, which represents its saturated value and non-saturated value, respectively. The variation of *L(t)* is shown in Fig. 3a. *R* is the sum of DC resistance of the detection coil and the load. The magnetic flux in the detection coil is $\Phi(t)$ [4],

$$\Phi(t) = L(t)\left[i(t) + i_{\mathrm{ex}}\right] \tag{1}$$

where $i_{ex}$ is the equivalent current of the external magnetic field that interacts with the detection coil, which can be expressed as:

$$i_{\mathrm{ex}} = \frac{lH_{\mathrm{ex}}}{N} \tag{2}$$

where *l* is the effective length of the detection coil, *N* is the number of turns of detection coil. So the equation of the circuit in Fig. 1 is:

$$\frac{\mathrm{d}}{\mathrm{d}t}\Phi(t) + \frac{1}{C}q(t) + Ri(t) = 0 \tag{3}$$

Substituting (1) into (3) and considering the two states of fluxgate, assuming that the fluxgate is non-saturated in $0^+ \sim t_1^-$ and saturated in $t_1^+ \sim T^-$, leaves the result [4]:

$$\begin{cases} L_n q''(t) + Rq'(t) + \dfrac{1}{C}q(t) = 0 \quad t = 0^+ : t_1^- \\ L_s q''(t) + Rq'(t) + \dfrac{1}{C}q(t) = 0 \quad t = t_1^+ : T^- \end{cases} \tag{4}$$

where *T* is excitation period, which means the period of the inductance variation. Because the detection coil flux and the capacitor charge must be continuous, considering about (1):

$$\begin{pmatrix} q\left(t_1^+\right) \\ i\left(t_1^+\right) \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & \dfrac{L_n}{L_s} \end{pmatrix} \begin{pmatrix} q\left(t_1^-\right) \\ i\left(t_1^-\right) \end{pmatrix} - \begin{pmatrix} 0 \\ \left(1 - \dfrac{L_n}{L_s}\right) i_{\mathrm{ex}} \end{pmatrix} \tag{5}$$

$$\begin{pmatrix} q(T^+) \\ i(T^+) \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & L_s/L_n \end{pmatrix} \begin{pmatrix} q(T^-) \\ i(T^-) \end{pmatrix} - \begin{pmatrix} 0 \\ (1 - L_s/L_n)i_{\mathrm{ex}} \end{pmatrix} \tag{6}$$

It is difficult to solve (3), because of its non-linearity. However, in any single period of (4) it is not very hard to solve it. After considering the continuous condition (5) (6), the solution in one period as well as the initial value of the next period can be calculated. So the entire solution of any interval can be gotten by gathering all the periods in this interval.

The initial value of $(k+1)$th period can be seen as a series:

$$a_{k+1} = \beta a_k + c \tag{7}$$

This expression is derived in Appendix A.

And (7) can be rewritten as:

$$a_{k+1} + \left(\beta - 1\right)^{-1} c = \beta \left[a_k + \left(\beta - 1\right)^{-1} c\right] \tag{8}$$

which suggests that:

$$a_k = \beta^k \left[a_0 + \left(\beta - 1\right)^{-1} c\right] - \left(\beta - 1\right)^{-1} c \tag{9}$$

This is the initial state of the *k*th period, the solution of this period can be gotten by the same way as we did in previous steps.

The stability condition can be gotten by (9). Because *c* is a constant vector, for nonzero $i_{ex}$ it is easy to demonstrate that it is not zero nor infinite, so the stability of the solution just depends on the terms $(\beta\text{-}1)^{-1}$ and $\beta^k$. They look like two different terms, but in fact the stability of them is the same, we will demonstrate it and derive the stability condition in Appendix B.

According to (9) and Appendix B the stability condition is:

$$\ln\left[\gamma + \sqrt{\gamma^2 - 1}\right] - \alpha_s(T - t_1) - \alpha_n t_1 \le 0 \tag{10.a}$$