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An auto-parametrically excited vibration energy harvester

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ABSTRACT

Parametric resonance, as a resonant amplification phenomenon, is a superior mechanical amplifier than direct resonance and has already been demonstrated to possess the potential to offer over an order of magnitude higher power output for vibration energy harvesting than the conventional direct excitation. However, unlike directly excited systems, parametric resonance has a minimum threshold amplitude that must be attained prior to its activation. The authors have previously presented the addition of initial spring designs to minimise this threshold, through non-resonant direct amplification of the base excitation that is subsequently fed into the parametric resonator. This paper explores the integration of auto-parametric resonance, as a form of resonant amplification of the base excitation, to further minimise this activation criterion and realise the profitable regions of parametric resonance at even lower input acceleration levels. Numerical and experimental results have demonstrated in excess of an order of magnitude reduction in the initiation threshold amplitude for an auto-parametric resonator (~0.6 ms^{-2}) as well as several folds lower for a parametric resonator with a non-resonant base amplifier (~4.0 ms^{-2}), as oppose to a sole parametric resonator without any threshold reduction mechanisms (10's ms^{-2}). Therefore, the superior power performance of parametric resonance over direct resonance has been activated and demonstrated at much lower input levels.

1. Introduction

Vibration energy harvesting (VEH) has gained immense popularity in recent years. However, the absolute attainable power level remains an issue in practical applications. Resonant-based vibration energy harvesters have been the core of the technology to amplify the base excitation in order to maximise the total amount of mechanical energy that can be captured [1,2]. Attempts to improve the power output, based on design and mechanical mechanisms, include: system parameter optimisation [3], array addition around the same frequency range [4], bending of resonant peaks due to Duffing nonlinearities [5,6], jumping of potential wells for bi-stabile [7,8] or multi-stable systems [9], stochastic resonance [10], coupling of multiple transducers such as piezoelectric and electromagnetic transducers [11] as well as frequency up conversion of either linear [12,13] or rotational generators [14,15]. Yet, most of these either yielded no noticeable improvement or relatively modest enhancement (within a few folds) in terms of the power density for a given acceleration.

Overwhelming majority of the conventional approaches, either linear or nonlinear, have primarily relied upon directly excited resonance. The authors however, have previously demonstrated an alternative: parametric resonance at both macro-scale [16,17] and MEMS scale [18,19], outperforming the same device driven into direct resonance by over an order of magnitude in power output when subjected to the same acceleration level. Instead of forcing a direct response from a system associated with direct resonance, parametric resonance relies on the periodic modulation in one of the system parameters to internally accumulate energy. Despite the promising potential of parametric resonance over its direct counterpart, the drive acceleration must attain a damping-dependant initiation threshold amplitude prior to activating it altogether [20,21].

Previous parametric resonant harvesters developed by the authors [17,18] have included an additional initial spring structure, which serves to amplify the base excitation, coupling into the parametric resonator. However, prior to this paper, the authors have only reported non-resonant amplification of the base excitation by the initial spring. Nonetheless, this design approach has already experimentally demonstrated the ability to reduce the initiation threshold amplitude of a typical sole parametric resonant VEH prototype from 10's ms^{-2} to 1's ms^{-2}. This allows access to the profitable regions of parametric resonance at lower and more practical input acceleration levels.

This paper builds on the previous work and proposes the employment of auto-parametric resonance, as a means of resonant

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amplification of the base excitation, to further reduce the initiation threshold amplitude of the parametric resonant harvester device in an attempt to render this threshold limitation practically negligible (targeting 0.1's ms^{-2}). Contrary to the external excitation induced hetero-parametric resonance, auto-parametric resonance is triggered by aligning the longitudinal and transverse natural frequencies within the system in a 2:1 ratio. The classical example is a pendulum suspended off an elastic spring where the natural frequency of the spring is twice that of the pendulum [22].

The various configurations of resonant systems explored in this paper are delineated as the following.

- *Direct resonator:* a resonating system that responds to direct excitation
- *(Plain) parametric resonator:* a resonating system that responds to parametric excitation, but has no additional mechanical mechanisms to minimise the initiation threshold amplitude
- *Parametric resonator with non-resonant base excitation amplification:* a parametric resonating system with an initial spring. The natural frequency of the initial spring is not twice that of the parametric resonator.
- *Auto-parametric resonator:* a parametric resonating system with an initial spring. The natural frequency of the initial spring is twice that of the parametric resonator.

2. Design and modelling

2.1. Design and analytical model

The design employed is shown in Fig. 1a, resembling an inverted T-shape. The primary parametric resonator is the vertically upright cantilever beam resting on a horizontal clamped-clamped initial spring. When a vertical driving force is applied, the initial spring is directly excited while the cantilever can be parametrically excited under the right conditions. For a configuration where the natural frequency of the initial spring ω_1 is twice the natural frequency of the cantilever ω_2, auto-parametric resonance, and therefore direct resonant amplification of the base excitation to feed into the parametric resonator, can be achieved. On the other hand, for configurations where $\omega_1 \neq 2\omega_2$, only non-resonant amplification of the base excitation can be achieved to drive the principal parametric resonance, as reported in previous studies [17,18].

Fig. 1b represents the mass-spring-damper equivalent of the system. Here, c_s, k_s, c_p, k_p, m_p, y and x denote damping of the initial spring, stiffness of the initial spring, damping of the parametric resonator, stiffness of the parametric resonator, effective mass of the system, displacement of the initial spring and displacement of the parametric resonator respectively.

The initial spring is able to transmit energy absorbed from the mechanical excitation to the subsidiary cantilever spring. However, x must possess an initial displacement to allow orthogonal propagation of vibration into the parametric resonator. This criterion can be bypassed by a vertically upright end mass configuration, drawing parallels from an inverted pendulum, to place the zero-displacement rest position in an unstable equilibrium. This study employs a clamped–clamped beam design as the initial spring for a cantilever-based parametric resonator. However, other spring designs, such as cantilever beams or membranes, for both the initial spring and the parametric resonator, are all theoretically possible.

The natural frequency of a typical firmly clamped cantilever beam is given by Eq. (1) [23].

$$\omega_0^2 = \frac{3EI}{(0.24m_b + m_l)l^3} \tag{1}$$

where, ω_0 is the natural frequency, E is the elastic modulus, I is the area moment of inertia, m_b is the beam mass, m_l is the proof mass and l is the beam length. The effective natural frequency of the cantilever in this T-shaped system will be a fraction of this relationship depending on the spring stiffness of the initial spring.

The natural frequency of a typical clamped-clamped beam with centred mass is given by Eq. (2) [23] and of an asymmetrically placed mass is given by Eq. (3).

$$\omega_0^2 = \frac{48EI}{\left(\frac{48m_b}{\pi^4} + m_l\right) l^3} \tag{2}$$

$$\omega_0^2 = \frac{9\sqrt{3}lEI}{ml_b(l^2 - l_b^2)^{3/2}} \tag{3}$$

where, l_a and l_b are two segments of the doubly clamped beam separated by the proof mass, $l_a + l_b = l$ and the effective mass m is placed at distance l_a from the origin measured from the clamped side of l_a. The x displacement of the subsidiary cantilever beam shifts the centre of mass for the clamped–clamped beam and affects its natural frequency accordingly, as given in Eq. (4). Though, this natural frequency variation is minimal and can be neglected in most simplified cases.

$$\omega_0^2 = \frac{9\sqrt{3}lEI}{m\,(l_b - x(t))\left(l^2 - (l_b - x(t))^2\right)^{3/2}} \tag{4}$$

The equations of motion for such a two degrees-of-freedom auto-parametric system with the direct-to-parametric

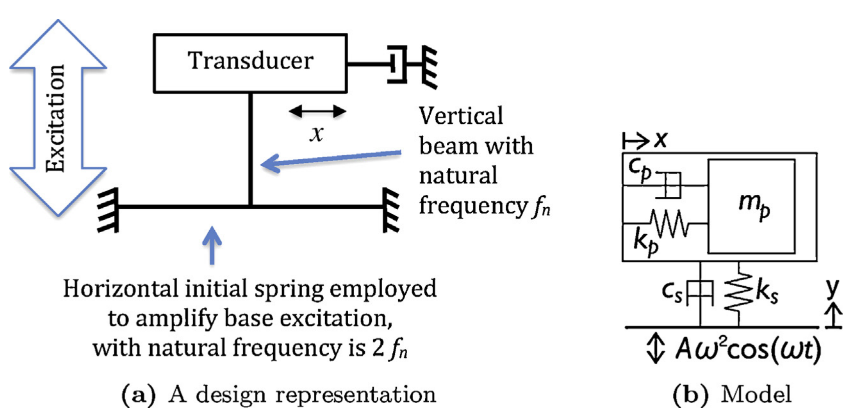

Fig. 1. Vertical cantilever parametric resonator with added horizontal initial spring to amplify the base excitation in order minimise the initiation threshold amplitude to access parametric resonance. For a direct resonator, the excitation is parallel to the displacement. Whereas, the excitation and displacement of parametric resonators are typically orthogonal to each other. A plain parametric resonator has no initial spring (c_s and k_s). For an auto-parametric resonator, the natural frequencies of the initial spring and the cantilever have a ratio of 2:1. A parametric resonator with non-resonant base excitation amplification does not have a 2:1 internal frequency ratio.