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Bayesian strain modal analysis under ambient vibration and damage identification using distributed fiber Bragg grating sensors

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ABSTRACT

Strain-based damage identification approaches have been proven to be more effective than traditional displacement or acceleration measurements. Both considering the uncertainties and the spatial dependence among the distributed strain measurement, this paper aims to develop a more practical damage identification system using fiber Bragg grating as the distributed sensing system. In practical situations, the measurement of the vibration response of a structure is generally more convenient than that of the excitation source; furthermore, the measurement and modeling error are inevitable. In this paper, strain modal analysis under ambient vibration excitation and the corresponding identification of structural damage are investigated by considering inevitable measurement noise and modeling error from the perspective of Bayesian statistics. Considering the uncertainties, the strain modal parameters and associated uncertainties are identified based on the Bayesian strain spectral method by only measuring the output dynamic strain response without input excitation. The uncertainty information provides the confidence level of the identified results and can help to determine the sources causing the variation of identified parameters. An improved stochastic simulation method based on slice sampling is proposed to solve the high-dimension integral in the Bayesian formula. Considering the spatial dependence during the distributed measurement, the spatial statistical significance test of the damage indexes is used to located damage and to determine the optimal sensor configuration. Finally, a plate structure experiment simulating one part of the mechanical structure is conducted to demonstrate the proposed method.

1. Introduction

The structural health monitoring (SHM) is important to ensure the reliability of equipment in various fields, such as nuclear power, aerospace, civil engineering, and so on. Numerous methods have been proposed to detect, locate, and estimate the severity of damage, and assess structural integrity. Farrar and Worden [1] reviewed the development status of SHM from the perspective of statistical pattern recognition (SPR), and listed several current technical challenges about SHM. The vibration-based damage identification methods have been recognized as one class of effective methods and have rapidly developed in recent decades [2,3]. Adewuyi et al. [4] compared the performance of various vibration-based damage identification methods using a simply supported beam, and found that the methods based on strain measurement are more reliable than those based on displacement when considering measurement noise.

The damage identification methods based on strain measurement have several advantages compared with other methods. According to the theory of elastic mechanics, the normal strain is the derivative of displacement; hence, the variation of strain is more sensitive to small structural defects than that of displacement. For example, Li et al. [5] performed strain modal analysis of a plate structure using the Rayleigh–Ritz approach and then proposed two damage sensitive indices to locate damage based on the strain mode shapes. Recently various structural damage identification methods based on strain measurement have been reviewed by Li [6].

The fiber Bragg grating (FBG), considered to be a promising technology, has been increasingly applied to the SHM process. FBG has several advantages, such as immunity to electromagnetic interference, high sensitivity, light weight, and so on. The excellent multiplexing capability of the FBG facilitates its use as a distributed sensor system, which not only monitors the local key parts of the structure but also captures the overall dynamic information. Capoluongo et al. [7] tested the high frequency (up to kilohertz) resonant characteristics of the FBG, and then detected damage based on modal analysis using an ad hoc steel structure. Cusano et al. [8] carried out an experimental modal analysis of a wing of an aircraft model using embedded FBG sensors. Panopoulou et al. [9] developed a complete damage detection system using FBGs. The dynamic strain response data from the FBG is first measured, then the feature

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indices are extracted by various signal processing methods, and finally an artificial neural network is utilized to detect and locate damage. This system has been demonstrated by a thin composite panel and a honeycomb structure and is planned for use in a future application of an antenna reflector.

Realistically, there are many ambient factors that cause the vibration of mechanical structure, such as the effect of storms on ships and oil drilling platforms, the airflow excitation on spacecraft during flight, the moving vibration of cars, and the vibration of general machinery caused by the motor. These ambient vibration sources are usually difficult to be measured. Furthermore, the installation of specialized excitation devices for these structures is generally costly and inconvenient. Therefore, modal analysis based only on the output response under ambient excitation is an important trend in traditional modal analysis methods. The basic assumption of modal analysis based on ambient excitation is that the input excitation is the broad-band stochastic signal. There are already various methods using modal analysis based on only-output, such as the random decrement (RD) technique [10], the natural excitation technique (NExT) [11], and so on. Recently Katafygiotis and Yuen [12,13] proposed a modal analysis method based on the Bayesian spectral density estimator. The most significant advantage of this method is that it provides the associated uncertainties of identified unknown parameters, which are typically important for SHM.

This paper studies strain modal analysis under ambient excitation using distributed FBG measurement. Based on the idea of Bayesian spectral approach proposed by Katafygiotis and Yuen [12,13], the strain spectral density calculated from the FBG response signal is used to identify the strain modal parameters and associated uncertainty. For the high-dimensional integral in the Bayesian formula, an improved stochastic simulation method is proposed to obtain the sample of unknown parameters. Then, damage detection and location are considered using the results of the strain modal analysis. A new damage index based on the strain mode shapes is defined to locate damage. Finally, the experiment is carried out to demonstrate the effectiveness of this method.

The following paper is organized as follows. Section 2 introduces the theoretical background of this paper. First the expression of the dynamic strain response under ambient vibration excitation is derived. Then, the strain modal parameters identification method, based on Bayesian strain spectral density, is researched. Lastly, the computation issue in the Bayesian formula is discussed and the idea of group slice sampling is presented. Section 3 is the experimental verification section. The experimental apparatus and procedures are initially introduced in Section 3.1. The signal analysis of the dynamic response and the corresponding strain spectra are discussed in Section 3.2. The results of the strain modal identification and damage identification are shown in Sections 3.3 and 3.4, respectively. A few conclusions are discussed in Section 4.

2. Theoretical background

In the actual measurement, the measured dynamic strain response, denoted by $\boldsymbol{\varepsilon}\mathbf{m}(t)$, is expectably different from the theoretical strain response $\boldsymbol{\varepsilon}(t)$ due to the presence of prediction error $\mathbf{e}(t)$, which is given as

$$\boldsymbol{\varepsilon}\mathbf{m}(t) = \boldsymbol{\varepsilon}(t) + \mathbf{e}(t) \tag{1}$$

Including measurement noise and modeling error, the prediction error $\mathbf{e}(t)$ is usually modeled as zero mean Gaussian white noise with the covariance matrix $\boldsymbol{\delta}$. By explicitly considering this prediction error, the strain spectral approach based on the Bayesian identification framework [13] is used to identify the strain modal parameters in this paper.

In the next subsection, the strain mode decomposition of the multiple degrees of freedom (DOF) system is discussed, and then the theoretical dynamic strain response of the structure under ambient excitation is derived. Due to the inevitable prediction error, the measured dynamic strain response is modeled as a zero-mean Gaussian vector process, and the corresponding discrete Fourier transform follows the complex multivariate normal distribution. Furthermore, the averaged strain spectral density follows the central complex Wishart distribution. According to the definition of the Wishart distribution, the probability density distribution of the power spectral density is calculated, which can be seen the likelihood function of the model parameters. Lastly, the posterior probability distribution of the model parameters is constructed according to the Bayesian formula. The mean of the Wishart distribution is the expectation of the measured strain spectral density, which joins the strain modal parameters and the measured dynamic strain though the prediction error.

The high-dimensional integral in the Bayesian formula is then discussed in Section 2.3. Various solving methods including the stochastic simulation are reviewed, then the idea of slice sampling is introduced, and finally, the corresponding improved method is proposed.

2.1. Strain response under ambient vibration excitation

According to Eqs. (32) and (41) of Appendix A, the displacement mode decomposition and the strain mode decomposition of the N_d DOFs system can be expressed as following, respectively:

$$\mathbf{x}(t) = \boldsymbol{\Phi}\mathbf{q}(t) \tag{2}$$

$$\boldsymbol{\varepsilon}(t) = \boldsymbol{\psi}\mathbf{q}(t) \tag{3}$$

where $\mathbf{x}(t)$, $\boldsymbol{\varepsilon}(t)$, and $\mathbf{q}(t)$ are the displacement vector, normal strain vector, and modal coordinates of the system, respectively; $\boldsymbol{\Phi}$ is the displacement mode shape; and $\boldsymbol{\psi}$ is the strain mode shape. It is worth noting that the strain mode shape components from the nth measured DOF are selected to be normalized as:

$$\psi_n^{(m)} = 1, \quad m = 1, 2, \ldots, N_d \tag{4}$$

The selection of the measured DOF to be normalized is arbitrary, but is not a node of the mth mode, which is similar to the displacement modal analysis [13,14].

When comparing Eqs. (2) and (3), it can be seen that the expression of the strain modal is similar to the traditional displacement modal. The uncoupled modal equation from the dynamic strain response is the same as from the displacement response, so the modal frequency and damping ratio can be identified though strain modal analysis. Unlike the displacement mode shape, the strain mode shape reflects the amplitude of the strain response at the measured points, which represents the differential in the normal direction of the displacement mode shape. Therefore, the strain mode shape could capture the structural defects more effectively than the displacement mode shape. Furthermore, each component of the strain mode shape corresponding to different measured DOF includes the spatial information of damage, which can be used to locate damage.

Next the dynamic strain response of the structure under ambient excitation is considered. There are various environmental factors that cause structural vibration, such as traffic, storms, airflow, and so on. These excitation sources are usually the broad-band stochastic signal, which can be modeled as stationary Gaussian white noise. In this case, the excitation force vector, $\mathbf{f}(t)$, after undergoing modal decomposition in Appendix A can be seen as a zero-mean Gaussian signal, and the corresponding spectral intensity matrix is $\mathbf{S}_f(w) = \mathbf{S}_{f0}$. Assume that the lowest N_m modes are considered, $\mathbf{S}_{f0}$ is a symmetric matrix of N_m by N_m.