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Direct inference of parameters for piezoresistive micromechanical resonators embedded in feedthrough

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ABSTRACT

This paper reports a means of directly inferring all the characteristic lumped parameters (including the quality factor) for a piezoresistive micromechanical resonator with an electrical transmission heavily buried in parasitic capacitive feedthrough. This is particularly useful when the height of the resonant peak is less than 3 dB such that the quality factor cannot be directly obtained. In this limit, the magnitude of the resonance is comprised mainly of the feedthrough contribution and hence unrepresentative of the motional output from the resonator. To validate the accuracy of this method of parameter inference, we have applied the method to electrical transmission measurements for an electrostatically actuated square-extensional bulk mode resonator whose motional current is sensed piezoresistively. Due to substantial feedthrough, the height of the resonant peak is less than 0.22 dB such that the quality factor cannot be obtained through the −3 dB bandwidth since this information is not directly available. The values of parameters estimated using the reported method compare well (to within 5%) with those obtained by a more protracted and accurate extraction procedure that involves the subtraction of the modeled feedthrough contribution from the measured data followed by a model curve fit.

1. Introduction

Micromechanical resonators constitute the basic vibratory unit in a number of MEMS applications today which include energy harvesters [1], sensors [2,3], and RF passives for frequency control [4]. The presence of parasitic elements distorts the pass band, and in severe cases the resonant peak is reduced to less than 3 dB above the floor such that direct inference of the quality factor is not possible. On this note, we have previously demonstrated a method to directly infer all the parameters of a resonator for the case where the resonator is electrostatically actuated and sensed through a capacitive transducer [5].

More recently, piezoresistive sensing has been shown to offer a number of advantages over conventional capacitive sensing. For example, piezoresistive sensing generally provides better (or more efficient) electromechanical transduction compared to capacitive sensing. This is beneficial for reducing the motional impedance of the device, which is desirable for lowering insertion loss in filter applications and phase noise for oscillator applications. We have recently shown that by injecting 5 mA of current through a piezoresistive resonator, the motional resistance could be lowered by an order of magnitude compared to capacitive sensing [6]. Using 10 mA current and 40 V DC bias drive, van Beek achieved an enhancement of over 130 times [7] in the piezoresistive output compared to using capacitive sensing. Furthermore, van Beek's analysis [8] has shown that the miniaturization of device size toward higher operating frequencies in the GHz range scales in favor of piezoresistive sensing over capacitive sensing. Moreover, since the piezoresistive property is inherent to silicon, the piezoresistors come as part of (i.e. built-in) the lithographically defined structure. As such, no additional fabrication steps are required to create the piezoresistive transducers. In fact, the enhancement in transduction afforded by piezoresistive sensing is underscored by a recent demonstration of a piezoresistive MEMS oscillator operating in air [9]. The condition for oscillation in the reported case could not have been realized otherwise using capacitive sensing.

To follow up on this development of using piezoresistive-sensing in MEMS resonators for the purpose of enhanced transduction, this paper presents a variant of the previous direct extraction method applicable to capacitive resonators, herein modified for application to the case of MEMS resonators that employ piezoresistive sensing.

When the motion of a resonator is sensed through a modulation in the capacitive transducer gap, the motional current produced is given by:

$$i_{mc} = \left(\frac{dC}{dx}\right) V_P \dot{x} = j \left(\frac{dC}{dx}\right) V_P \omega x \quad (1)$$

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V_P is the DC bias voltage applied across the capacitive transducer, dC/dx is the normalized transduction factor associated with a transducer capacitance of C and displacement x, and ω denotes the angular frequency. From (1), it can be seen that since the capacitive motional current is given by the velocity, it is therefore $\pi/2$ out of phase with the displacement. The frequency response function (FRF), which is seen as the admittance, is given by:

$$Y(\omega) = \frac{j\omega C_m}{[1-(\omega/\Omega)^2] + j(\omega/\Omega)(1/\Omega)} \quad (2)$$

C_m is the motional capacitance when the resonator is modeled as a series LRC circuit [10], ω is the angular frequency, Ω is the angular resonant frequency, and Q is the quality factor. This is likened to the mobility (ratio of the velocity response to the harmonic force) of a single-degree of freedom (SDOF) mechanical system [11].

On the other hand, when the motion of a resonator is sensed through modulation of stress in the device resulting in changes in the resistance due to the piezoresistive effect, a modulated current is produced when a voltage is applied across the resistor, R:

$$i_{mp} = I_d\left(\frac{\Delta R}{R}\right) \quad (3)$$

I_d is the nominal drain current that flows through the piezoresistor which experiences a resistance change of ΔR as the resonator vibrates. Since ΔR is either in or out of phase with the stress depending on the resonator design as well as the direction of the current I_d. The underlying stress in turn is also either in or out of phase with the displacement. Therefore, it may be seen that the piezoresistive motional current will be in or out of phase with the displacement. Hence the FRF, which is seen as the transconductance, is given by:

$$G(\omega) = \frac{\Pi}{[1-(\omega/\Omega)^2] + j(\omega/\Omega)(1/\Omega)} \quad (4)$$

Π here represents a lumped parameter that describes the electromechanical conversion in a piezoresistive resonator. A comparison between (2) and (4) illustrates a phase difference of $\pi/2$ between the respective FRFs of a piezoresistive and capacitive resonator. This may also be seen by comparing the respective Nyquist plots for each FRF shown in Fig. 1. This form of the FRF is likened to the receptance (ratio of the displacement response over the harmonic force) of an SDOF mechanical system [11]. Now it can be seen that since the forms of the respective FRFs for the piezoresistive and capacitive resonators differ between each other, the effect of parasitic feedthrough on the FRFs will be different. As such, estimating a piezoresistive resonator's parameters directly from the electrical transmission embedded in feedthrough requires a different treatment. This is the motivation and objective of this paper: reporting a means of directly estimating a resonator's parameters for a receptance FRF buried in feedthrough which characterizes the transmission of a piezoresistive resonator.

The next section (Section 2) describes the theoretical basis of this direct inference method for a micromechanical resonator whose output is due purely to the piezoresistive effect or at least dominated by this mechanism. This is achieved by examining both the susceptance frequency response and the Nyquist plot. Thereafter in Section 3, for the purpose of validation, the method is applied to the measured electrical transmission (heavily buried in substantial feedthrough) of a square-plate resonator excited in the breathing mode to allow for a piezoresistive pick off. Section 4 discusses the limits of the reported direct inference approach before concluding this work in Section 5.

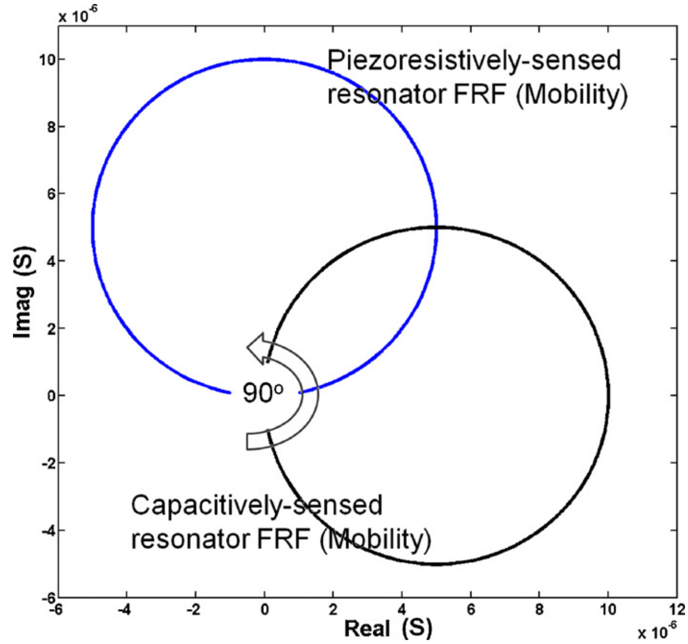

Fig. 1. Comparison between the Nyquist plots for a capacitively sensed and piezoresistively sensed resonator.

2. Methodology

The FRF of a resonator using a capacitive pick-off is described by a circle for all values of Q on a Nyquist plot. For a piezoresistive pick-off, the locus of the FRF does not perfectly fit a circle as illustrated by Fig. 2 since it does not begin at the origin but at the point with coordinates $(-\Pi, 0)$. This deviation from a circle fit decreases with increasing Q, as shown from comparisons between Fig. 2(a) and (d). Hence for our purpose with regard to MEMS resonators where the Q of interest is typically at least in the 1000 s, we approximate the locus of the FRF to a circle. As shown by the Nyquist plot in Fig. 3a (plotted for a resonator with a Q of 8000), the diameter of the circle is then given by the product ΠQ and centered at the coordinates $(0, \Pi Q/2)$. The resonance then occurs at the coordinates $(0, \Pi Q)$, while the -3 dB points occur between the origin and resonance at coordinates $(-\Pi Q/2, \Pi Q/2)$ and $(\Pi Q/2, \Pi Q/2)$.

With reference to (4), the addition of capacitive feedthrough between the input and output port results in the following expression:

$$G(\omega) = \frac{\Pi}{[1-(\omega/\Omega)^2] + j(\omega/\Omega)(1/\Omega)} + j\omega C_F \quad (5)$$

C_F represents the parasitic feedthrough capacitor seen between the input and output ports. Fig. 3a shows the effect of adding the capacitive feedthrough component ωC_F which is a shift in the circle's position along the imaginary axis by a magnitude of ΩC_F. Here, ΩC_F was set to be twice of ΠQ. In the plot of the susceptance frequency response, this positional shift shows up as a uniform upward shift in the curve by the same magnitude of ΩC_F (see Fig. 3b). In the limit of heavy feedthrough (such that $\Omega C_F > \Pi Q$), since the FRF comprises primarily of the susceptance which is characterized by a resonant peak with no anti-resonance valley, the resonant peak observed in the feedthrough-embedded FRF magnitude would correspond to the mechanical resonance. This agrees with similar observations from the corresponding Nyquist plot. On the Nyquist plot, the maximum magnitude (i.e. the resonant peak in the FRF) is given by the point on the locus that is furthest from the origin. From the Nyquist plot in Fig. 3a, this corresponds to the mechanical resonance.